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IYGB GCE

Mathematics SYN

Advanced Level

Synoptic Paper G

Difficulty Rating: 4.0275

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This synoptic practice paper follows closely the Advanced Level Pure Mathematics Syllabus, suitable for first assessment Summer 2018.

The standard booklet “Mathematical Formulae and Statistical Tables” may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 21 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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Question 1

The point $P(3,k)$ lies on the curve with equation

$$y = x^2 + ax - 4,$$

where a and k are constants.

Given that the gradient at P is 3 determine the value of a and the value of k. **(6)**

Question 2

$OABC$ is a parallelogram and the point M is the midpoint of AB.

The point N lies on the diagonal AC so that $AN : NC = 1:2$.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

a) Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{c}$, for each of the vectors $\overrightarrow{AC}$, $\overrightarrow{AN}$, $\overrightarrow{ON}$ and $\overrightarrow{NM}$. **(5)**

b) Deduce, showing your reasoning, that O, N and M are collinear. **(1)**

Question 3

It is given that for all values of x

$$5x^2 + Ax + 7 = B(x-2)^2 + C, \quad x \in \mathbb{R}.$$

Determine the values of each of the constants A, B and C. **(4)**

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Question 4

A circle C has equation

$$x^2 + y^2 - 6x + 14y + 33 = 0$$

a) Determine the coordinates of the centre and the radius of C. **(4)**

b) Show that the circle lies entirely below the x axis. **(2)**

The point $P(6,k)$, where k is a constant, lies outside the circle.

c) By considering the distance of P from the centre of the circle, or otherwise, determine the range of the possible values of k. **(6)**

Question 5

$$f(\theta) = 2\cos\theta + 3\sin\theta, \ \theta \in \mathbb{R}.$$

a) Express $f(\theta)$ in the form $R\cos(\theta - \alpha)$, $R > 0$, $0 < \alpha < \frac{\pi}{2}$.
Give the value of α correct to 3 decimal places. **(3)**

b) State the maximum value of $f(\theta)$ and find the smallest positive value of θ for which this maximum occurs. **(3)**

The temperature T °C in a warehouse is modelled by the equation

$$T = 16 + 2\cos\left(\frac{\pi t}{12}\right) + 3\sin\left(\frac{\pi t}{12}\right), \ \ 0 \le t < 24,$$

where t is the time in hours measured since midnight.

c) State the maximum temperature in the warehouse and a value of t when this maximum temperature occurs. **(3)**

d) Find the times, to the nearest minute using 24 hour clock notation, when the temperature in the warehouse is 17 °C. **(6)**

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Question 6

Given that the exact value of $\tan 20° = t$, show that

$$\tan 10° = \frac{-1+\sqrt{t^2+1}}{t}.$$ **(8)**

Question 7

A sequence a_1, a_2, a_3, a_4, … is given by

$$a_{n+1} = p + q a_n,$$

where p and q are non zero constants.

It is given that $a_1 = 250$, $a_2 = 220$ and $a_3 = 196$.

a) Determine the value of p and the value of q. **(4)**

b) Show clearly that the sequence converges to 100. **(3)**

Question 8

$$y = \sqrt{x^2+16}, \quad x \in \mathbb{R}.$$

a) Describe the geometric transformation which maps the graph of $y = \sqrt{x^2+16}$ onto the graph of $y = 4\sqrt{x^2+1}$. **(3)**

When the graph of $y = \sqrt{x^2+16}$ is translated by the vector $\begin{bmatrix} k \\ 0 \end{bmatrix}$, where k is a non zero constant, the image of the transformed graph passes through the point $(6,5)$.

b) Determine the possible values of k. **(5)**

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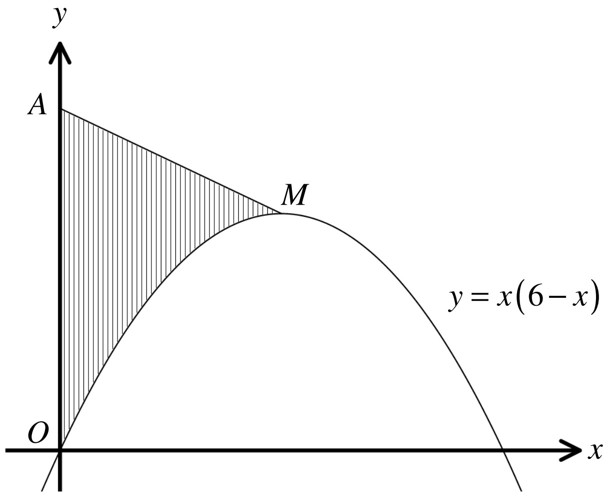
Question 9



The figure above shows the curve C with equation

$$y = x(6-x),\ x \in \mathbb{R}.$$

The point M is the maximum point of C and the point A has coordinates $(0,12)$.

Find the exact area of the shaded region, bounded by the curve, the y axis and the straight line segment from A to M. **(8)**

Question 10

$$f(x) = (2-3x)^2(1+4x)^7.$$

Find the coefficient of x^2 in the polynomial expansion of $f(x)$. **(6)**

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Question 11

Two curves C_1 and C_2 are defined for all values of x and have respective equations

$$y_1 = 7^x \quad \text{and} \quad y_2 = 2\times 5^x .$$

Show that the x coordinate of the point of intersection of the two curves is given by

$$\frac{1}{\log_2 7 - \log_2 5} .$$

(5)

Question 12

A triangle has vertices at $A(1,0)$, $B(9,4)$ and $C(k,6)$.

a) Given that $\measuredangle ABC = 90°$, show that $k = 8$. **(3)**

b) Find an equation of the straight line BC, giving the answer in the form $ax + by = c$, where a, b and c are integers. **(3)**

The line BC meets the x axis at the point D.

c) Find the area of the triangle ABD. **(3)**

d) Hence, or otherwise, determine the area of the triangle ABC. **(3)**

Question 13

The 3rd, 4th and 5th term of a geometric series are given in terms of a constant x.

$$U_3 = (x+5),\ U_4 = (4x-1) \quad \text{and} \quad U_5 = (2x+3) .$$

Find the sum to infinity of the series. **(10)**

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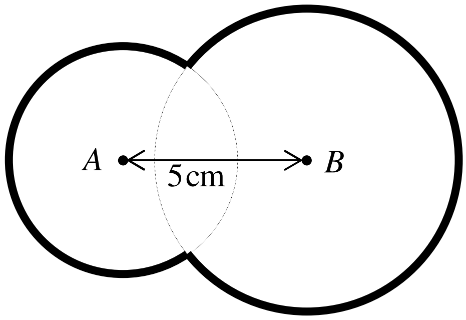
Question 14



The figure above shows the design for an earring.

The design consists of a part of a circle of radius 3 cm centred at A and another part of a circle of radius 4 cm centred at B.

The circles overlap in such a way so that the distance AB is 5 cm.

Find, to three significant figures, the perimeter of the design. **(7)**

Question 15

Use the substitution $u = 1 + x\mathrm{e}^{\sin x}$ to find an exact simplified value for the following definite integral.

$$\int_0^{\pi} \frac{1 + x\cos x}{x + \mathrm{e}^{-\sin x}} \, dx .$$

(10)

Question 16

A quartic curve C has equation

$$y = x(x-2)^3, \; x \in \mathbb{R} .$$

Show that there is only one point on C where the gradient is 10. **(12)**

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Question 17

A body is moving and its distance, x metres, is measured from a fixed point O at different times, t seconds.

The body is moving in such a way, so that the rate of change of its distance x is inversely proportional to its distance x at that time.

When $t = 0$, $x = 50$ and when $t = 4$, $x = 30$.

Determine the time it takes for the body to reach O. **(11)**

Question 18

A curve C is given implicitly by

$$2x^2 + xy - y^2 - 4x - y + 20 = 0.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{4x + y - 4}{2y - x + 1}.$$ **(5)**

b) Find the coordinates of the stationary points of C. **(6)**

c) Show further that

$$4 + 2\frac{dy}{dx} - 2\left(\frac{dy}{dx}\right)^2 + (x - 2y - 1)\frac{d^2y}{dx^2} = 0.$$ **(3)**

d) Hence determine the nature of the stationary points of part **(b)**. **(3)**

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Question 19

The function f is defined as

$$f : x \mapsto 6 - \ln(x+3), \ \ x \in \mathbb{R}, \ x \geq -2 .$$

Consider the following sequence of transformations T_1, T_2 and T_3.

$$\ln x \xrightarrow{T_1} \ln(x+3) \xrightarrow{T_2} -\ln(x+3) \xrightarrow{T_3} -\ln(x+3)+6 .$$

a) Describe geometrically T_1, T_2 and T_3, and hence sketch the graph of $f(x)$. Indicate clearly any intersections with the axes and the graph's starting point. **(7)**

b) Find, in its simplest form, an expression for $f^{-1}(x)$, stating further the domain and range of $f^{-1}(x)$. **(4)**

The function g satisfies

$$g : x \to e^{x^2} - 3, \ \ x \in \mathbb{R} .$$

c) Find, in its simplest form, an expression for the composition $fg(x)$. **(3)**

Question 20

A curve C is defined by the parametric equations

$$x = 2t + 4 \ , \ \ y = t^3 - 4t + 1 , \ \ t \in \mathbb{R} .$$

a) Show that an equation of the tangent to the curve at $A(2,4)$ is

$$2y + x = 10 . \qquad \textbf{(6)}$$

The tangent to C at A re-intersects C at the point B.

b) Determine the coordinates of B. **(6)**

Question 21

Liquid is pouring into a container which initially contains 8.1 litres of liquid.

When the height of the liquid in the container is h cm, the volume of the liquid, V cm^3, is given by

$$V = 36h^2 .$$

The rate at which the water is pouring into the container is $2t \text{ cm}^3\text{s}^{-1}$, where t s is the time since the liquid started pouring in.

Determine the rate at which the height of the liquid in the container is rising 2 minutes after the liquid started pouring in.

$\left[1 \text{ litre } = 1000\text{cm}^3\right]$ **(10)**

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