Differentiation

Whenever you see a question about gradients, tangents or normals, you should immediately think, "Oho, that'll be differentiation." It's also handy for curve sketching and for finding maximum and minimum values of functions.

1 Given that $y = x^7 + \frac{2}{x^3}$, find:

a) $\frac{dy}{dx}$

....

(2 marks)

b) $\frac{d^2y}{dx^2}$

....

(2 marks)

2 A curve has equation $y = 3x + 4 + x^4$.

The point A (2, 26) lies on the curve. Find the gradient of the curve at point A.

....

(4 marks)

3 The curve C is given by the equation $y = 2x^3 - 10x^2 - 4\sqrt{x} + 12$.

a) Find the gradient of the tangent to the curve at the point where $x = 4$.

....

(4 marks)

b) Hence find an equation for the normal to the curve at this point.
Give your answer in the form $ax + by + c = 0$.

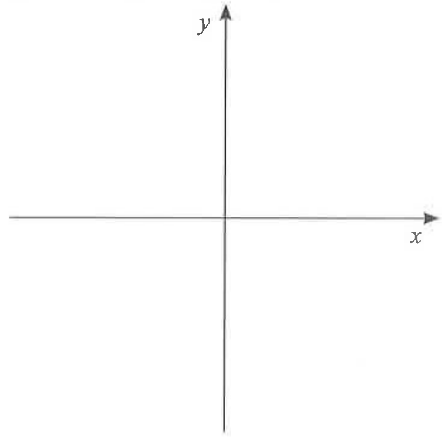
....

(4 marks)

Differentiation

4 $f(x) = x^3 - 7x^2 + 8x + 9$

a) Sketch the graph of $y = f'(x)$, showing clearly the points of intersection with the axes.



(4 marks)

b) Explain the significance of the x-intercepts on your sketch with regards to the graph of $y = f(x)$.

....

....

(1 mark)

5 A curve has equation $y = kx^2 - 8x - 5$, for a constant k. The point R lies on the curve and has an x-coordinate of 2. The normal to the curve at point R is parallel to the line with equation $4y + x = 24$.

a) Find the value of k.

$k =$ ....

(5 marks)

b) The tangent to the curve at R meets the curve $y = 4x - \frac{1}{x^3} - 9$ at the point S. Find the coordinates of S.

....

(5 marks)

Differentiation

6 For the curve $y = x^5 - 4x^3 + \frac{1}{x}$, show that the tangents to the curve at $x = a$ and $x = -a$ are parallel for all values of a.

(3 marks)

7 For $f(x) = 8x^2 - 1$, prove from first principles that $f'(x) = 16x$.

(4 marks)

8 Differentiate $f(x) = 5x^3$ from first principles.

(4 marks)

9 Given that the curve $y = 2x^3 + ax - 5$ is stationary at the point $(3, b)$:

a) Find the values of a and b.

a = ... b = ...

(5 marks)

b) Determine whether $(3, b)$ is a maximum or minimum point.

....

(2 marks)

Differentiation

10 Joe claims that the function $f(x) = 3x^3 + 9x^2 + 25x$ is an increasing function for all values of x.

Show that Joe's claim is correct.

(4 marks)

11 The function $f(x) = 2x^4 + 27x$ has one stationary point.

a) Find the coordinates of the stationary point.

....

(3 marks)

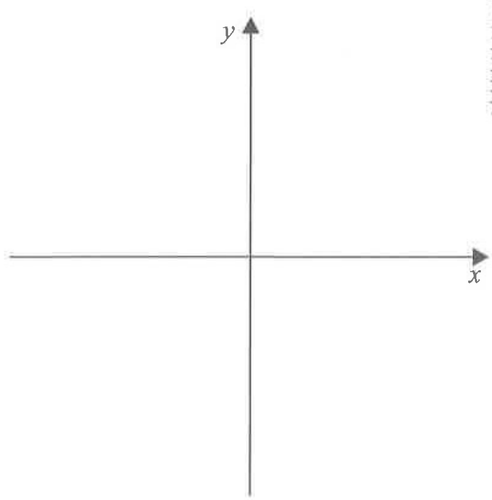
b) Find the range of values of x for which the function is increasing and the range of values of x for which it is decreasing.

Increasing for: ...., decreasing for: ....

(2 marks)

c) Hence sketch the curve $y = f(x)$, showing where it crosses the axes.

To check what happens to the curve as x gets big, factorise $f(x)$ by taking the highest power of x outside the brackets. Then if you imagine x getting very big, you can see what $f(x)$ will tend towards.



(3 marks)

Differentiation

12 An ice cream parlour needs an open-top stainless steel container with a capacity of 40 litres, modelled as a cuboid with sides of length x cm, x cm and y cm, as shown in Figure 1.

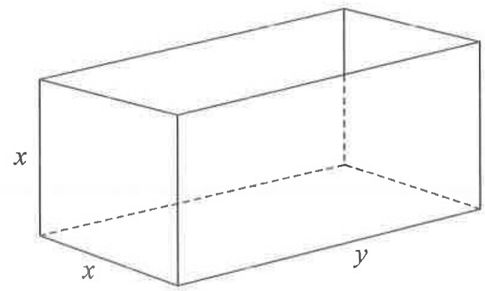


Figure 1

a) Show that the external surface area, A cm^2, of the container is given by $A = 2x^2 + \frac{120\,000}{x}$

Find y in terms of x using the info given in the question.

(4 marks)

b) Find the value of x to 3 s.f. at which A is stationary, and show that this is a minimum value of A.

(6 marks)

c) Calculate the minimum area of stainless steel needed to make the container. Give your answer to 3 s.f.

.... cm^2

(2 marks)

d) Comment on the validity of this model.

....

....

(1 mark)

EXAM TIP

Work carefully through differentiation questions, as it's easy to get a bit mixed up. Remember that tangents have the same gradient as the curve, and normals are perpendicular to the curve. To find stationary points, differentiate once, then differentiate again to determine the nature of the point — don't forget that negative means maximum and positive means minimum.

Score

74