Areas between curves and lines

Gold

Given that:

$$\mathrm{f}(x) = -3x^2 + ax + b, \quad \int_0^3 \mathrm{f}(x)\,\mathrm{d}x = 22.5$$

and that $\mathrm{f}(x) \geq 3 - x$ for $0 \leq x < 4$, find the area in the region bounded by $\mathrm{f}(x)$, $y = 3 - x$, $x = 3$ and $x = 0$.

Silver

Find the area of the region between the curve:

$$\mathrm{f}(x) = -x^2 + 4x + 5$$

and the line $y = 5 - x$ between the points where the line intersects the y-axis and x-axis.

Bronze

Find the area under the curve:

$$\mathrm{f}(x) = x^3 - 2x + 5$$

and between the lines $x = -1$ and $x = 2$.

(It may help to know that $\mathrm{f}(x)$ is positive between $x = -1$ and $x = 2$.)