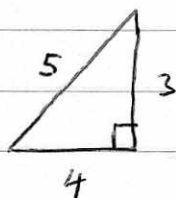


1)

direction $4i + 3j$

$$\sqrt{4^2 + 3^2} = 5$$

$$\left[\frac{15}{5} = 3 \right]$$

$$\text{Speed} = 15 \text{ ms}^{-1} \quad \therefore \text{velocity} = 3(4i + 3j) \\ = 12i + 9j$$

s

$$u = 12i + 9j$$

$$v = -12i + 6j$$

$$a = ?$$

$$t = 6$$

$$v = u + at$$

$$-12i + 6j = \cancel{12i} + 9j + 6a$$

$$-24i - 3j = 6a$$

$$\underline{a = -4i - \frac{1}{2}j} \quad \text{ms}^{-2}$$

2/ s

$$u = u$$

$$v = u + at$$

$$v = -6i + j$$

$$-6i + j = u + 3(2i - 5j)$$

$$a = 2i - 5j$$

$$-6i + j = u + 6i - 15j$$

$$t = 3$$

$$-12i + 16j = u$$

$$\text{speed} = \sqrt{12^2 + 16^2} \\ = \underline{\underline{20 \text{ ms}^{-1}}}$$

3a)

$$r = r_0 + vt$$

$$r_0 = 4i - 3j$$

$$t = 3$$

$$r = -5i + 9j$$

$$-5i + 9j = 4i - 3j + 3v$$

$$-9i + 12j = 3v$$

$$v = \underline{\underline{-3i + 4j}} \text{ ms}^{-1}$$

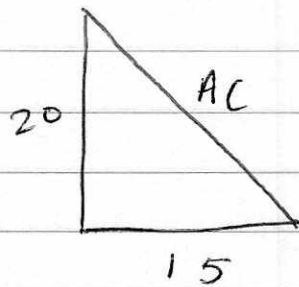
b) when $t = 5$

$$r_c = 4i - 3j + 5(-3i + 4j)$$

$$= 4i - 3j - 15i + 20j$$

$$= -11i + 17j$$

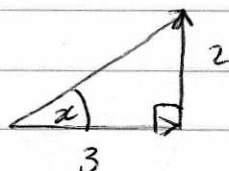
$$r_A = 4i - 3j \quad r_c = -11i + 17j$$



$$AC = \sqrt{20^2 + 15^2}$$

$$= \underline{\underline{25 \text{ m}}}$$

4a/



$$\tan x = \frac{2}{3}$$

$$x = \tan^{-1}\left(\frac{2}{3}\right)$$

$$= \underline{\underline{33.7^\circ}} \quad 3\text{sf}$$

b/

$$r = r_0 + vt$$

$$r = 2i + 11j + 2(3i + 2j)$$

$$r = 2i + 11j + 6i + 4j$$

$$r = 8i + 15j$$

$$\begin{aligned} \text{distance} &= \sqrt{8^2 + 15^2} \\ &= \underline{\underline{17\text{ m}}} \end{aligned}$$

5a/

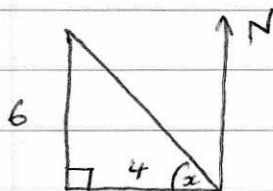
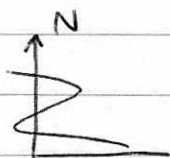
$$r = r_0 + vt$$

$$-8i + 10j = 2i - 5j + 2.5v$$

$$-10i + 15j = 2.5v$$

$$\underline{\underline{v = -4i + 6j \text{ ms}^{-1}}}$$

b/



$$\tan x = \frac{6}{4}$$

$$\begin{aligned} x &= \tan^{-1}\left(\frac{6}{4}\right) \\ &= 56^\circ \end{aligned}$$

$$\begin{aligned} \text{Bearing} &= 270 + 56 \\ &= \underline{\underline{326^\circ}} \end{aligned}$$

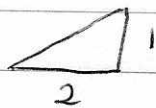
c/

$$r = r_0 + vt$$

$$= 2i - 5j + t(-4i + 6j)$$

$$= (2 - 4t)i + (6t - 5)j$$

6a)

acceleration direction $2i + j$ 

$$\sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\frac{3\sqrt{5}}{\sqrt{5}} = 3 \quad \therefore \text{acceleration} = 3(2i + j) = \underline{\underline{6i + 3j \text{ ms}^{-2}}}$$

$$\begin{aligned} v &= u + at \\ &= i - 5j + t(6i + 3j) \\ &= i - 5j + 6ti + 3tj \\ &= 6ti + i + 3tj - 5j \\ &= \underline{\underline{(6t + 1)i + (3t - 5)j \text{ ms}^{-1}}} \end{aligned}$$

b/

$$\begin{aligned} \text{speed} &= \sqrt{(6t + 1)^2 + (3t - 5)^2} \\ &= \sqrt{36t^2 + 12t + 1 + 9t^2 - 30t + 25} \\ &= \sqrt{45t^2 - 18t + 26} \\ \text{speed}^2 &= 45t^2 - 18t + 26 \end{aligned}$$

[complete the square or differentiate.]

$$y = 45t^2 - 18t + 26$$

$$\frac{dy}{dt} = 90t - 18$$

Min where $\frac{dy}{dt} = 0$

$$90t - 18 = 0$$

$$90t = 18$$

$$t = \frac{18}{90}$$

$$= \frac{1}{5} = 0.2 \text{ seconds.}$$

$$= \underline{\underline{\cancel{12 \text{ minutes}}}}$$

7a/ A: $r_0 = 5i$
 $v = -6i + 5j$

B: $r_0 = 10j$
 $v = -4i + j$

$$r_A = 5i + t(-6i + 5j)$$

$$r_A = 5i - 6ti + 5tj$$

$$= \underline{(5 - 6t)i + 5tj}$$

$$r_B = 10j + t(-4i + j)$$

$$r_B = 10j - 4ti + tj$$

$$= \underline{-4ti + (t + 10)j}$$

b/ B $r_B - r_A$

$$\begin{pmatrix} -4t \\ t + 10 \end{pmatrix} - \begin{pmatrix} 5 - 6t \\ 5t \end{pmatrix} = \begin{pmatrix} 2t - 5 \\ 10 - 4t \end{pmatrix}$$

$$r_B - r_A = (2t - 5)i + (10 - 4t)j$$

c/ Boats collide if i and j equal zero for the same value of t .

i// $2t - 5 = 0$

$$2t = 5$$

$$t = \underline{\underline{2.5}}$$

i// $10 - 4t = 0$

$$10 = 4t$$

$$t = \underline{\underline{2.5}}$$

1430

$$8 \quad A: \quad \vec{r}_0 = 12\mathbf{i} - 11\mathbf{j} \\ \quad \quad \vec{v} = 9\mathbf{i} - 6\mathbf{j}$$

$$B: \quad \vec{r}_0 = 40\mathbf{i} - 39\mathbf{j} \\ \quad \quad \vec{v} = -12\mathbf{i} + 15\mathbf{j}$$

$$\begin{aligned} \vec{r}_A &= 12\mathbf{i} - 11\mathbf{j} + t(9\mathbf{i} - 6\mathbf{j}) & \vec{r}_B &= 40\mathbf{i} - 39\mathbf{j} + t(-12\mathbf{i} + 15\mathbf{j}) \\ &= 12\mathbf{i} - 11\mathbf{j} + 9t\mathbf{i} - 6t\mathbf{j} & &= 40\mathbf{i} - 39\mathbf{j} - 12t\mathbf{i} + 15t\mathbf{j} \\ &= (12 + 9t)\mathbf{i} + (-6t - 11)\mathbf{j} & &= (40 - 12t)\mathbf{i} + (15t - 39)\mathbf{j} \end{aligned}$$

b/ collide if i and j are equal at the same time

$$\begin{aligned} i// \quad 12 + 9t &= 40 - 12t & j// \quad -6t - 11 &= 15t - 39 \\ 21t &= 28 & 28 &= 21t \\ t &= \frac{4}{3} \quad (1 \text{ hr } 20) & t &= \frac{4}{3} \end{aligned}$$

They will collide at 7:20 am.

$$\begin{aligned} c/ \text{ position of A at 7 am} &= (12 + 9)\mathbf{i} + (-6 - 11)\mathbf{j} \\ &= 21\mathbf{i} - 17\mathbf{j} \end{aligned}$$

$$\begin{aligned} \vec{r}_A &= 21\mathbf{i} - 17\mathbf{j} + \frac{1}{3}(9\mathbf{i} - 6\mathbf{j}) \\ &= 21\mathbf{i} - 17\mathbf{j} + 3\mathbf{i} - 2\mathbf{j} \\ &= \underline{24\mathbf{i} - 19\mathbf{j}} \end{aligned}$$

$$\begin{aligned} \vec{r}_B &= \left(40 - 12\left(\frac{4}{3}\right)\right)\mathbf{i} + \left(15\left(\frac{4}{3}\right) - 39\right)\mathbf{j} \\ &= \underline{16\mathbf{i} - 19\mathbf{j}} \end{aligned}$$

$$\text{Distance} = \underline{\underline{8 \text{ km}}}$$

9a/

$$\begin{aligned} r_A &= 2i - 5j + t(-4i + 6j) \\ &= (2 - 4t)i + (6t - 5)j \end{aligned}$$

$$\begin{aligned} r_B &= -3i - 9j + t(i + 8j) \\ &= (t - 3)i + (8t - 9)j \end{aligned}$$

$$r_A - r_B$$

$$\begin{pmatrix} 2 - 4t \\ 6t - 5 \end{pmatrix} - \begin{pmatrix} t - 3 \\ 8t - 9 \end{pmatrix} = \begin{pmatrix} 5 - 5t \\ 4 - 2t \end{pmatrix}$$

$$\underline{(5 - 5t)i + (4 - 2t)j}$$

b/ Due west when j 's are equal

$$4 - 2t = 0$$

$$t = 2$$

12 noon

c/

$$37 = \sqrt{(5 - 5t)^2 + (4 - 2t)^2}$$

$$1369 = 25 - 50t + 25t^2 + 16 - 16t + 4t^2$$

$$1369 = 29t^2 - 66t + 41$$

$$0 = 29t^2 - 66t - 1328$$

$$t = \frac{-(-66) \pm \sqrt{(-66)^2 - 4(29)(-1328)}}{2(29)}$$

$$t = 8$$

$$t = -5.7$$

t cannot be negative

$$\therefore \underline{t = 8}$$

$$\therefore \underline{6 \text{ pm}}$$