IYGB - MP1 PAPER L - QUESTION 1

a) SOLVING THE LINEAR INEQUALITY

$$\Rightarrow \frac{x+2}{3} < 3x-1$$

$$\Rightarrow x+2 < 3(3x-1)$$

$$\Rightarrow x+2 < 9x-3$$

$$\Rightarrow -8x < -5$$

$$\Rightarrow x > \frac{5}{8}$$

b) SOLVING THE QUADRATIC INEQUALITY

$$\Rightarrow x+6(x^2+x) > 20$$

$$\Rightarrow x+6x^2+6x > 20$$

$$\Rightarrow 6x^2+7x-20 > 0$$

FACTORIZE OR USE THE QUADRATIC FORMULA TO FIND THE CRITICAL VALUES

$$\Rightarrow (3x-4)(2x+5) > 0$$

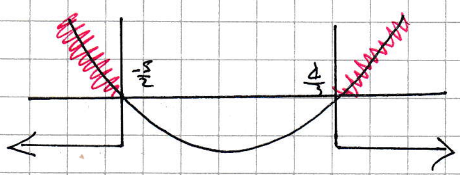
$$\Rightarrow C.V = \begin{cases} \frac{4}{3} \\ -\frac{5}{2} \end{cases}$$

$$x = \frac{-7 \pm \sqrt{7^2-4\times6\times(-20)}}{2\times6}$$

$$x = \frac{-7 \pm \sqrt{529}}{12}$$

$$x = \frac{-7 \pm 23}{12}$$

$$x = \begin{cases} 4/3 \\ -5/2 \end{cases}$$



$x < -\frac{5}{2}$ OR $x > \frac{4}{3}$

IYGB - MP1 PAPER L - QUESTION 2

USING THE STANDARD BINOMIAL EXPANSION FORMULA

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n}a^0 b^n$$

$$\left(x+\frac{2}{x}\right)^4 = \binom{4}{0}(x)^4\left(\frac{2}{x}\right)^0 + \binom{4}{1}(x)^3\left(\frac{2}{x}\right)^1 + \binom{4}{2}(x)^2\left(\frac{2}{x}\right)^2 +$$

$$\binom{4}{3}(x)^1\left(\frac{2}{x}\right)^3 + \binom{4}{4}(x)^0\left(\frac{2}{x}\right)^4$$

$$\left(x+\frac{2}{x}\right)^4 = \left(1\times x^4\times 1\right) + \left(4\times x^3\times\frac{2}{x}\right) + \left(6\times x^2\times\frac{4}{x^2}\right) + \left(4\times x\times\frac{8}{x^3}\right) + \left(1\times 1\times\frac{16}{x^4}\right)$$

$$\left(x+\frac{2}{x}\right)^4 = x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$$

IYGB – MP1 PAPER L – QUESTION 3

TIDY AND CREATE $\tan y$

$$\Rightarrow 2\sin y + 5\cos y = 2\cos y$$

$$\Rightarrow 2\sin y = -3\cos y$$

$$\Rightarrow \frac{2\sin y}{\cos y} = \frac{-3\cos y}{\cos y}$$

$$\Rightarrow 2\tan y = -3$$

$$\Rightarrow \tan y = -\frac{3}{2}$$

$$\underline{\arctan\left(-\tfrac{3}{2}\right) = -56.3^\circ}$$

$$\Rightarrow y = -56.3^\circ \pm 180n \qquad n = 0, 1, 2, 3, \ldots$$

$$\therefore \quad y_1 = 123.7^\circ$$

$$y_2 = 303.7^\circ$$

IYGB - MP1 PAPER L - QUESTION 4

LET $y = f(x) = x^3 - 4x + 1$

THEN
$$f(x+h) = (x+h)^3 - 4(x+h) + 1$$
$$= (x+h)(x+h)^2 - 4x - 4h + 1$$
$$= (x+h)(x^2 + 2xh + h^2) - 4x - 4h + 1$$
$$= \begin{matrix} x^3 + 2x^2h + xh^2 \\ x^2h + 2xh^2 + h^3 \end{matrix} - 4x - 4h + 1$$
$$= x^3 + 3x^2h + 3xh^2 + h^3 - 4x - 4h + 1$$

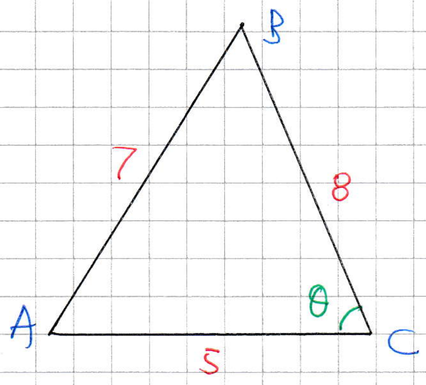
BY THE FORMAL DEFINITION OF THE DERIVATIVE WE HAVE

$$\frac{dy}{dx} = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$$
$$= \lim_{h \to 0}\left[\frac{(x^3 + 3x^2h + 3xh^2 + h^3 - 4x - 4h + 1) - (x^3 - 4x + 1)}{h}\right]$$
$$= \lim_{h \to 0}\left[\frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{- 4x} - 4h \cancel{+1} \cancel{- x^3} \cancel{+ 4x} \cancel{- 1}}{h}\right]$$
$$= \lim_{h \to 0}\left[\frac{3x^2h + 3xh^2 + h^3 - 4h}{h}\right]$$
$$= \lim_{h \to 0}\left[3x^2 + 3xh + h^2 - 4\right]$$
$$= 3x^2 - 4$$

AS REQUIRED

IYGB - MP1 PAPER L - QUESTION 5

BY THE COSINE RULE "BACKWARDS"



$$\Rightarrow |AB|^2 = |AC|^2 + |BC|^2 - 2|AC||BC|\cos\theta$$

$$\Rightarrow 7^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos\theta$$

$$\Rightarrow 80\cos\theta = 25 + 64 - 49$$

$$\Rightarrow 80\cos\theta = 40$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\Rightarrow \theta = 60° \quad //$$

USING THE TRIGONOMETRIC FORM FOR THE AREA OF THE TRIANGLE

$$\text{AREA} = \frac{1}{2}|AC||BC|\sin\theta$$

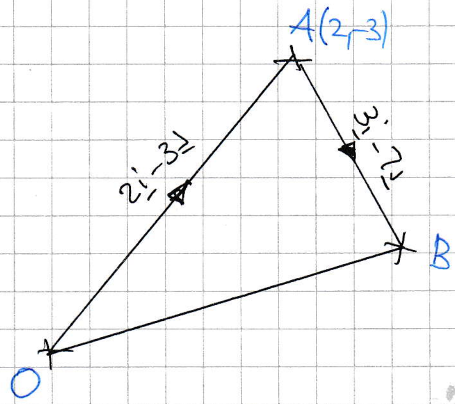
$$\text{AREA} = \frac{1}{2} \times 5 \times 8 \times \sin 60°$$

$$\text{AREA} = 20 \times \frac{\sqrt{3}}{2}$$

$$\text{AREA} = 10\sqrt{3} \quad //$$

IYGB – MP1 PAPER L – QUESTION 6

LOOKING AT A DIAGRAM



$\Rightarrow \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$\Rightarrow \overrightarrow{OB} = (2\underset{\sim}{i} - 3\underset{\sim}{j}) + (3\underset{\sim}{i} - 7\underset{\sim}{j})$

$\Rightarrow \overrightarrow{OB} = 5\underset{\sim}{i} - 10\underset{\sim}{j}$

$\therefore B(5, -10)$

DISTANCE OF B FROM O IS GIVEN BY

$$|\overrightarrow{OB}| = |5\underset{\sim}{i} - 10\underset{\sim}{j}| = \sqrt{5^2 + (-10)^2} = \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}$$

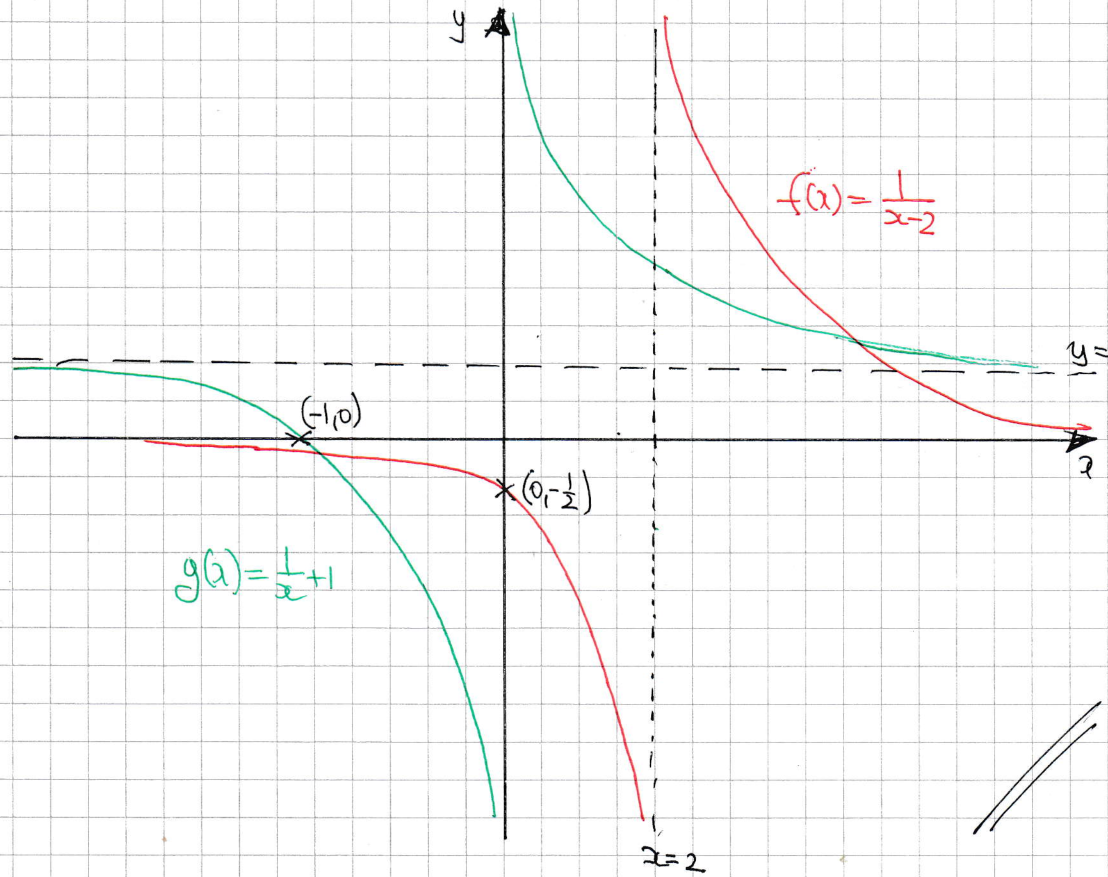
IYGB - MP1 PAPER L - QUESTION 7

a) LET $h(x) = \frac{1}{x}$

- $f(x) = h(x-2) = \frac{1}{x-2}$ — I.E TRANSLATION, TO THE "RIGHT", BY 2 UNITS
  I.E TRANSLATION BY THE VECTOR $\binom{2}{0}$

- $g(x) = h(x)+1 = \frac{1}{x}+1$ — I.E TRANSLATION, "UPWARDS", BY 1 UNIT
  I.E TRANSLATION BY THE VECTOR $\binom{0}{1}$

b) SKETCHING USING THE TRANSFORMATION DESCRIPTIONS FROM PART (a)



c) SOLVING THEIR EQUATIONS SIMULTANEOUSLY

$$\left.\begin{aligned} f(x) &= y = \frac{1}{x-2} \\ g(x) &= y = \frac{1}{x}+1 \end{aligned}\right\} \Rightarrow \frac{1}{x-2} = \frac{1}{x}+1$$

IYGB - MP1 PAPER L - QUESTION 7

$$\Rightarrow \frac{1}{x-2} = \frac{1}{x} + 1$$

MULTIPLY THROUGH BY x

$$\Rightarrow \frac{x}{x-2} = 1 + x$$

MULTIPLY THROUGH BY $(x-2)$

$$\Rightarrow x = 1(x-2) + x(x-2)$$

$$\Rightarrow x = x - 2 + x^2 - 2x$$

$$\Rightarrow 0 = x^2 - 2x - 2$$

BY COMPLETING THE SQUARE

$$\Rightarrow (x-1)^2 - 1 - 2 = 0$$

$$\Rightarrow (x-1)^2 = 3$$

$$\Rightarrow x - 1 = \left\langle \begin{array}{l} \sqrt{3} \\ -\sqrt{3} \end{array} \right.$$

$$\Rightarrow x = \left\langle \begin{array}{l} 1+\sqrt{3} \\ 1-\sqrt{3} \end{array} \right.$$

$$y = \frac{1}{x-2} = \left\langle \begin{array}{l} \frac{1}{1+\sqrt{3}-2} = \frac{1}{\sqrt{3}-1} = \frac{1}{2}(1+\sqrt{3}) \\ \frac{1}{1-\sqrt{3}-2} = \frac{1}{-\sqrt{3}-1} = \frac{1}{2}(1-\sqrt{3}) \end{array} \right.$$

$$\therefore \left[1+\sqrt{3}, \tfrac{1}{2}(1+\sqrt{3})\right] \ \& \ \left[1-\sqrt{3}, \tfrac{1}{2}(1-\sqrt{3})\right]$$

-1-

IYGB - MP1 PAPER L - QUESTION 8

a) BY THE FACTOR THEOREM

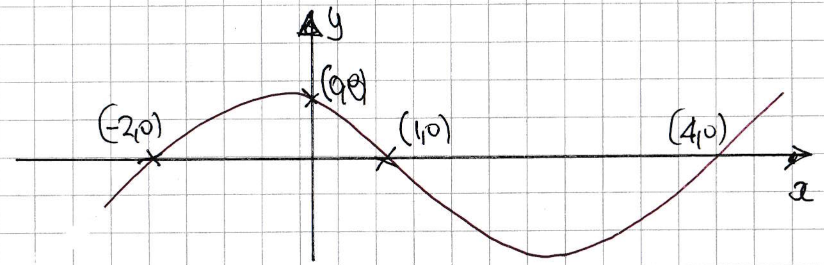
$$f(1) = 1^3 - 3\times1^2 - 6\times1 + 8$$
$$= 1 - 3 - 6 + 8$$
$$= 0$$

INDEED A FACTOR

b) BY LONG DIVISION OR MANIPULATION

$$x^3 - 3x^2 - 6x + 8 = x^2(x-1) - 2x(x-1) - 8(x-1)$$
$$= (x-1)(x^2 - 2x - 8)$$
$$= (x-1)(x+2)(x-4)$$

c)



$+x^3 \Rightarrow$

$x=0 \Rightarrow y=8 \quad (0,8)$

$y=0 \Rightarrow x = \begin{cases} -2 \\ 1 \end{cases} \quad (-2,0),\ (4,0),\ (-2,0)$

d) SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} y = x^4 + x^3 - 4x^2 - 10 \\ y = x^4 - x^3 + 2x^2 + 12x - 26 \end{array}\right\} \Longrightarrow$$

$$\Longrightarrow \cancel{x^4} + x^3 - 4x^2 - 10 = \cancel{x^4} - x^3 + 2x^2 + 12x - 26$$
$$\Longrightarrow 2x^3 - 6x^2 - 12x + 16 = 0$$
$$\Longrightarrow x^3 - 3x^2 - 6x + 8 = 0$$

USING PART (b) & $y = x^4 + x^3 - 4x^2 - 10$

$$x = \begin{cases} 1 \\ -2 \\ 4 \end{cases} \qquad y = \begin{cases} -12 \\ -18 \\ 246 \end{cases}$$

$\therefore P(-2,-18) \quad Q(1,-12) \quad R(4,246)$ //

IYGB - MP1 PAPER L - QUESTION 9

MANIPULATE AS FOLLOWS

$$\Rightarrow \ln x^2 + \frac{3}{\ln x} = 7$$

$$\Rightarrow 2\ln x + \frac{3}{\ln x} = 7$$

$$\Rightarrow 2(\ln x)^2 + 3 = 7\ln x$$

$$\Rightarrow 2(\ln x)^2 - 7\ln x + 3 = 0$$

FACTORIZE THE QUADRATIC

$$\Rightarrow (2\ln x - 1)(\ln x - 3)$$

$$\Rightarrow \ln x = \begin{cases} \frac{1}{2} \\ 3 \end{cases}$$

$$\Rightarrow x = \begin{cases} e^{\frac{1}{2}} = \sqrt{e} \\ e^3 \end{cases}$$

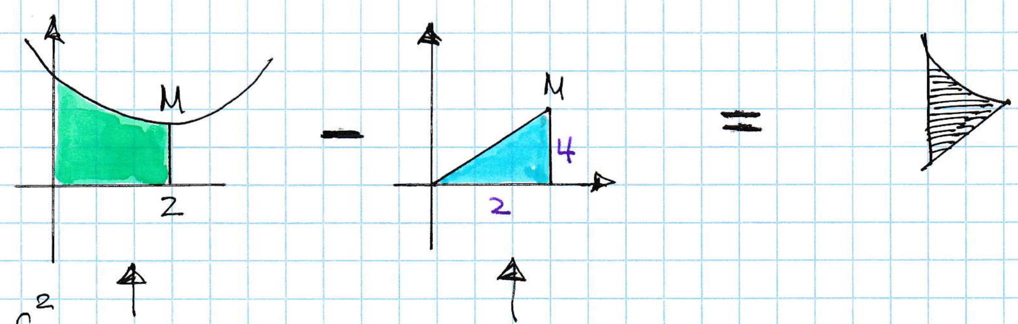
IYGB - MP1 PAPER L - QUESTION 10

START BY FINDING THE CO-ORDINATES OF M

$$\Rightarrow y = x^2 - 4x + 8$$
$$\Rightarrow y = (x-2)^2 - 4 + 8$$
$$\Rightarrow y = (x-2)^2 + 4$$

$$\therefore M(2,4)$$

LOOKING AT THE PICTORIAL EQUATION BELOW



$$\int_0^2 x^2 - 4x + 8 \, dx \qquad\qquad \tfrac{1}{2} \times 2 \times 4 = 4$$

$$= \left[\tfrac{1}{3}x^3 - 2x^2 + 8x\right]_0^2$$

$$= \left(\tfrac{8}{3} - 8 + 16\right) - (0)$$

$$= \frac{32}{3}$$

REQUIRED AREA $= \frac{32}{3} - 4 = \frac{20}{3}$

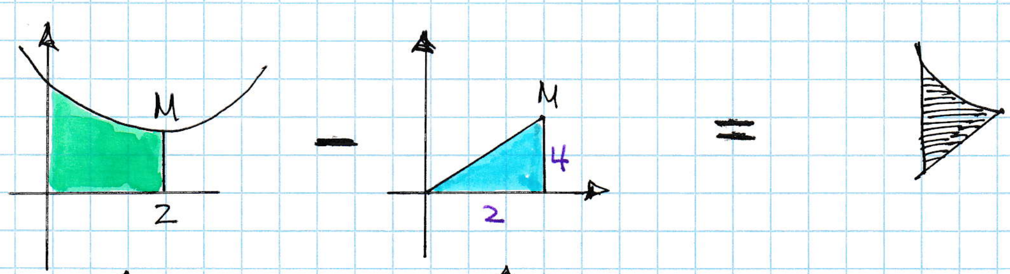
IYGB - MP1 PAPER L - QUESTION 10

START BY FINDING THE CO-ORDINATES OF M

$$\Rightarrow y = x^2 - 4x + 8$$
$$\Rightarrow y = (x-2)^2 - 4 + 8$$
$$\Rightarrow y = (x-2)^2 + 4$$

$\therefore M(2,4)$

LOOKING AT THE PICTORIAL EQUATION BELOW



$$\int_0^2 x^2 - 4x + 8 \, dx \qquad\qquad \frac{1}{2} \times 2 \times 4 = 4$$

$$= \left[\frac{1}{3}x^3 - 2x^2 + 8x\right]_0^2$$

$$= \left(\frac{8}{3} - 8 + 16\right) - (0)$$

$$= \frac{32}{3}$$

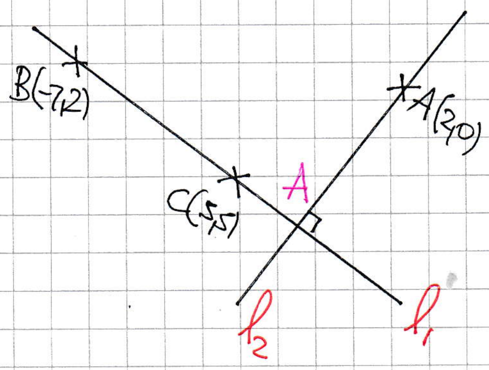
REQUIRED AREA $= \frac{32}{3} - 4 = \frac{20}{3}$

IYGB - MP1 PAPER L - QUESTION 11

START BY A DIAGRAM

- $\text{grad } BC = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{-7-5}$

$= \frac{-3}{-12} = \frac{1}{4}$



- $\text{grad } l_1 = \frac{1}{4}$
- $\text{grad } l_2 = -4$

FINALLY THE EQUATION OF l_1, $m = \frac{1}{4}$, $C(5,5)$ & l_2, $m = -4$, $A(2,0)$

- l_1: $y - y_0 = m(x - x_0)$

$y - 5 = \frac{1}{4}(x-5)$

$4y - 20 = x - 5$

$4y = x + 15$

- l_2: $y - y_0 = m(x - x_0)$

$y - 0 = -4(x-2)$

$y = -4x + 8$

SOLVING SIMULTANEOUSLY

$\Rightarrow 4(-4x+8) = x + 15$

$\Rightarrow -16x + 32 = x + 15$

$\Rightarrow -17x = -17$

$\Rightarrow x = 1$

& $y = -4 \times 1 + 8$

$y = 4$

$\therefore D(1,4)$

IYGB – MP1 PAPER L – QUESTION 12

CONSIDER THE EXPANSION OF $(\sqrt{k}-1)^2$

$$\Rightarrow (\sqrt{k}-1)^2 \geqslant 0$$

$$\Rightarrow (\sqrt{k})^2 - 2\times1\times\sqrt{k} + 1^2 \geqslant 0$$

$$\Rightarrow k - 2\sqrt{k} + 1 \geqslant 0$$

$$\Rightarrow k + 1 \geqslant 2\sqrt{k}$$

AS $\sqrt{k} > 0$ WE MAY DIVIDE IT

$$\Rightarrow \frac{k+1}{\sqrt{k}} \geqslant 2$$

// AS REQUIRED

ALTERNATIVE BY DIFFERENTIATION

FIRSTLY LET US NOTE THAT AS k GETS LARGER, THE WHOLE EXPRESSION GETS LARGER WITHOUT BOUND, SO ANY STATIONARY POINT WILL BE AN ABSOLUTE MINIMUM

$$\text{e.g.} \quad \lim_{k\to\infty}\left(\frac{k+1}{\sqrt{k}}\right) = \lim_{k\to\infty}\left(\sqrt{k} + \frac{1}{\sqrt{k}}\right)$$

$$y = \frac{k+1}{\sqrt{k}} = \frac{k}{k^{\frac{1}{2}}} + \frac{1}{k^{\frac{1}{2}}} = k^{\frac{1}{2}} + k^{-\frac{1}{2}}$$

$$\frac{dy}{dk} = \frac{1}{2}k^{-\frac{1}{2}} - \frac{1}{2}k^{-\frac{3}{2}}$$

SOLVING FOR ZERO, TO LOOK FOR MINIMUM

$$0 = \frac{1}{2}k^{-\frac{1}{2}} - \frac{1}{2}k^{-\frac{3}{2}}$$

IYGB - MP1 PAPER L - QUESTION 12

$$\Rightarrow \frac{1}{2}k^{-\frac{1}{2}} = \frac{1}{2}k^{-\frac{3}{2}}$$

$$\Rightarrow k^{-\frac{1}{2}} = k^{-\frac{3}{2}}$$

$$\Rightarrow \frac{1}{k^{\frac{1}{2}}} = \frac{1}{k^{3/2}}$$

$$\Rightarrow \frac{k^{3/2}}{k^{1/2}} = 1$$

AS $k>0$, WE MAY DIVIDE

$$\Rightarrow k = 1$$

$$\therefore \left(\frac{k+1}{\sqrt{k}}\right)_{MIN} = \frac{1+1}{\sqrt{1}} = \frac{2}{1} = 2$$

AS REQUIRED

BEST METHOD IS PROOF BY CONTRADICTION

SUPPOSE THAT $\frac{k+1}{\sqrt{k}} < 2$

$$\Rightarrow \left(\frac{k+1}{\sqrt{k}}\right)^2 < 4$$

$$\Rightarrow \frac{(k+1)^2}{k} < 4$$

$$\Rightarrow (k+1)^2 < 4k \qquad (k>0)$$

$$\Rightarrow k^2+2k+1 < 4k$$

$$\Rightarrow k^2-2k+1 < 0$$

$$\Rightarrow (k-1)^2 < 0$$

WHICH IS A CONTRADICTION

$$\therefore \frac{k+1}{\sqrt{k}} \geqslant 2$$

IYGB – MP1 PAPER L – QUESTION 13

START BY FINDING THE CO-ORDINATES OF Q

$$y = x^2 - 14x + 48$$
$$y = (x-6)(x-8)$$

$\therefore$ $P(6,0)$ $\quad$ $Q(8,0)$

FIND THE GRADIENT AT Q

$$\frac{dy}{dx} = 2x - 14$$
$$\left.\frac{dy}{dx}\right|_{x=8} = 2\times 8 - 14 = 2$$

EQUATION OF TANGENT (L_1)

$$y - y_0 = m(x - x_0)$$
$$y - 0 = 2(x-8)$$
$$y = 2x - 16$$

AS L_2 IS PERPENDICULAR TO L_2 ITS GRADIENT IS $-\frac{1}{2}$

$$\frac{dy}{dx} = -\frac{1}{2} \implies 2x - 14 = -\frac{1}{2}$$
$$\implies 4x - 28 = -1$$
$$\implies 4x = 27$$
$$\implies x = \frac{27}{4} \leftarrow \text{POINT } R$$

$$y = \left(\frac{27}{4}\right)^2 - 14\times\frac{27}{4} + 48$$
$$y = -\frac{15}{16}$$

$\therefore R\left(\frac{27}{4}, -\frac{15}{16}\right)$

FIND THE EQUATION OF L_2

$$y - y_0 = m(x - x_0)$$
$$y + \frac{15}{16} = -\frac{1}{2}\left(x - \frac{27}{4}\right)$$

IYGB - MP1 PAPER L - QUESTION 13

SOLVING SIMULTANEOUSLY

$$y + \frac{15}{16} = -\frac{1}{2}\left(x - \frac{27}{4}\right) \quad \& \quad y = 2x - 16$$

$$\left(2x - 16\right) + \frac{15}{16} = -\frac{1}{2}\left(x - \frac{27}{4}\right) \quad \times 16$$

$$32x - 256 + 15 = -8\left(x - \frac{27}{4}\right)$$

$$32x - 241 = -8x + 54$$

$$40x = 295$$

$$x = \frac{59}{8}$$

$$\& \quad y = 2\left(\frac{59}{8}\right) - 16$$

$$y = -\frac{5}{4}$$

$$\therefore S\left(\frac{59}{8}, -\frac{5}{4}\right)$$

IYGB – MP1 PAPER L – QUESTION 14

a) COMPLETE THE SQUARE IN x & y

$\Rightarrow x^2+y^2=8y$

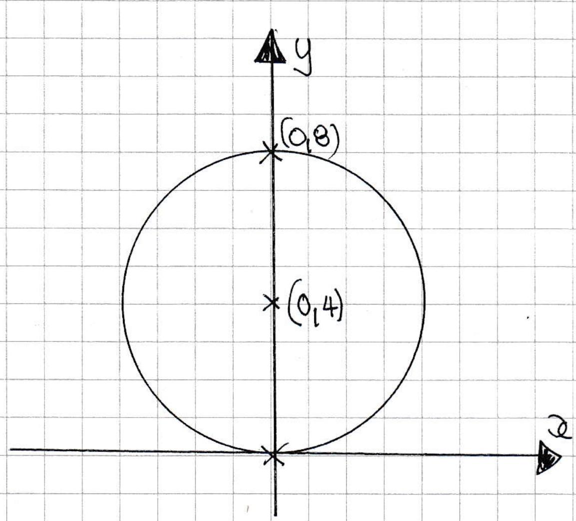
$\Rightarrow x^2+y^2-8y=0$

$\Rightarrow x^2+(y-4)^2-16=0$

$\Rightarrow x^2+(y-4)^2=16$

IE CENTRE AT $(0,4)$ & RADIUS 4

b) SKETCHING THE CIRCLE



c) SOLVING SIMULTANEOUSLY

$$\left.\begin{array}{l} x^2+y^2=8y \\ x+y=k \end{array}\right\} \Rightarrow \boxed{x=k-y}$$

$\Rightarrow (k-y)^2+y^2=8y$

$\Rightarrow k^2-2ky+y^2+y^2=8y$

$\Rightarrow 2y^2-2ky-8y+k^2=0$

$\Rightarrow 2y^2-(2k+8)y+k^2=0$

IYGB - MP1 PAPER L - QUESTION 14

IF TANGENT WE MUST HAVE REPEATED ROOTS

$$b^2-4ac=0 \implies [-(2k+8)]^2-4\times2\times k^2=0$$

$$\implies (2k+8)^2-8k^2=0$$

$$\implies 4k^2+32k+64-8k^2=0$$

$$\implies 0=4k^2-32k-64$$

$$\implies \boxed{k^2-8k-16=0}$$

$$\implies (k-4)^2-16-16=0$$

$$\implies (k-4)^2=32$$

$$\implies k-4=\pm\sqrt{32}$$

$$\implies k = \begin{cases} 4+4\sqrt{2} \\ 4-\sqrt{2} \end{cases}$$

OR QUADRATIC FORMULA

$$k=\frac{8\pm\sqrt{64-4\times1(-16)}}{2\times1}$$

$$k=\frac{8\pm\sqrt{64+64}}{2}$$

$$k=\frac{8\pm\sqrt{128}}{2}$$

$$k=\frac{8\pm8\sqrt{2}}{2}$$

$$k=4\pm4\sqrt{2}$$