CM

A Level Maths Question Countdown

6 days until the 1st exam

Information

- Each of the ten sheets will contain five pure questions and two applied questions.

Pure questions

- Two of the pure questions will be ‘standard’.
- Two of the pure questions will be ‘problems’.
- The last pure question will involve modelling.

Applied questions

- One of the questions will focus on statistics.
- One of the questions will focus on mechanics.
- On alternate days, the statistics question will look at the large data set. Note that these questions may be brief as opposed to full length exam questions.

Notes to self

Pure questions – standard

1 Consider the equation

$$\frac{x^2 - 4x - 4}{x+1} = k$$

where k is a constant.

Find the set of values of k for which the equation has no real roots.

2 A student was set the following problem.

> Using the small angle approximations, use first principles to find
>
> $$\frac{\mathrm{d}}{\mathrm{d}x}(\cos x), \quad \text{where } x \text{ is measured in radians}$$

The student's attempt at the problem is written below.

$\frac{\mathrm{d}}{\mathrm{d}x}(\cos x) = \lim_{h\to 0}\left[\frac{\cos(x+h) - \cos x}{h}\right]$	by using the definition of the derivative
$= \lim_{h\to 0}\left[\frac{\cos x \cos h + \sin x \sin h - \cos x}{h}\right]$	by expanding $\cos(x + h)$
$= \lim_{h\to 0}\left[\frac{(\cos x)(h) + (\sin x)(h) - \cos x}{h}\right]$	by using the small angle approximations
$= \lim_{h\to 0}\left[\cos x + \sin x - \frac{\cos x}{h}\right]$	by partitioning the denominator
$= \cos x + \sin x$	by taking the limit

The student's solution contains three errors.

One of these errors is in the final line where they have taken the limit incorrectly.

(a) Identify the two **other** errors made by the student.

(b) Find a correct solution to the problem.

Pure questions – problems

3

$$y = 5x - x\ln x, \quad x > 0$$

(a) Complete the table below with the value of y corresponding to x = 2e and x = 4e, giving your answer to 3 decimal places.

x	e	2e	3e	4e	5e
y	10.873		23.660		32.491

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an approximate value for

$$\int_{e}^{5e} (5x - x\ln x)\ \mathrm{d}x$$

giving your answer to two decimal places.

You should show each stage in your working.

(c) **Hence** find an estimate for

$$\int_{e}^{5e} (5x - x\ln x + 2)\ \mathrm{d}x$$

giving your answer to two decimal places.

4 (i) Two non-zero, non-parallel vectors, **a** and **b**, are such that

$$m\mathbf{a} + n\mathbf{b} = p\mathbf{a} + q\mathbf{b}$$

Prove that $m = p$ and $n = q$.

(ii) Relative to a fixed origin O, the points A, B and C have position vectors $2\mathbf{i} + s\mathbf{j} - \mathbf{k}$, $7\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $(s + 1)\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ respectively, where s is a constant.

(a) Find, in terms of s, $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

Given that $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right|$,

(b) find the value of s.

Pure questions – modelling

5 At time t minutes, the position of a robot is modelled by the parametric equations

$$x = 1 - 2\sin(t), \quad y = 2\cos t, \quad 0 \leqslant t \leqslant 3\pi$$

(a) Find the coordinates of the robot's initial and final position according to the model.

(b) (i) Show that the robot moves in a circle.

(ii) Write down the centre and radius of this circle.

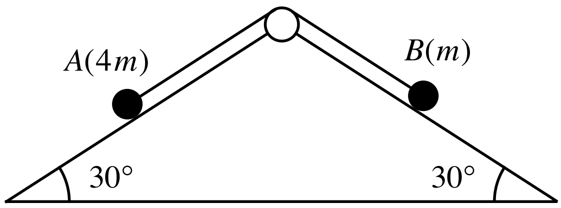
(c) Determine whether the robot moves clockwise or anticlockwise around the circle.

The robot is placed back at its initial position.
It is now programmed to complete four revolutions of the same circle and in same direction.

(d) Refine the domain of t in the parametric equations so that they model this new path of the robot.

Applied questions – mechanics

6



A fixed wedge has two faces, each inclined at 30° to the horizontal. Two particles, A and B, of masses $4m$ and m respectively are attached to the ends of a light inextensible string. Each particle lies on one of the plane faces of the wedge. The string passes over a small smooth pulley that is fixed to the top of the wedge. Each of the faces of the wedge are rough and the coefficient of friction between each particle and the corresponding face is $\frac{\sqrt{3}}{10}$. Particle A is held at rest with the string taut. The string lies in the same vertical plane as the lines of greatest slope of each face of the wedge.

Particle A is released from rest and start to move. Particle A moves downwards and particle B moves upwards towards the pulley.

(a) Showing your working clearly, find the tension in the string and the acceleration of the particles.

(b) Find the magnitude and direction of the resultant force exerted by the string on the pulley.

[Extension: see overleaf]

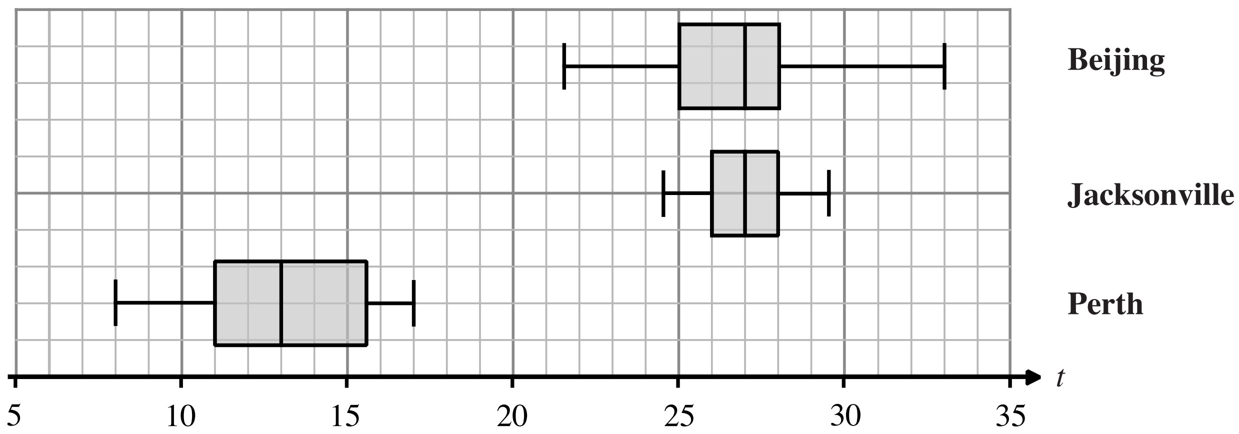
6 **Extension - unstructured and thus tricky:**

After 5 seconds, the string breaks. In the subsequent motion, B does not reach the pulley.

Given that B is at a height of 6 m above the ground, find the time taken for B to reach the ground at the edge of the wedge from when it is at its highest point above the ground.

Applied questions – statistics

7



The box plots above show the daily mean temperature distributions, t °C, for Beijing, Jacksonville and Perth in July 2015 using the large data set.

(a) (i) Using your knowledge of the large data set, and with reference to the location of these two places, explain why you expect Beijing and Jacksonville to have similar temperature distributions.

(ii) State **one** way in which the box plots support that Beijing and Jacksonville have a similar temperature distribution.

(iii) The data for Beijing has a larger range than the data for Jacksonville.

State **one** disadvantage of using the range to compare data distributions.

A data point is considered an outlier if it is more than 1.5 times the interquartile range below the lower quartile or more than 1.5 times the interquartile range above the upper quartile.

(b) For each location, determine whether the data contains outliers according to this measure.

Adelaide is a city in South Australia.

(c) Using the data above, and your knowledge of the location of the overseas regions in the large data set, suggest a value for the average daily mean temperature in Adelaide in July 2015. Justify your answer.

Extension - how do you expect the box plots to change if they are created for temperature distributions in December as opposed to July? Hint: use knowledge about the location of the regions and thus their seasons.