IYGB – SYNOPTIC PAPER F – QUESTION 1

$$f(x) = f(x+2)$$

$$\Rightarrow x^2 - 7x + \cancel{6} = (x+2)^2 - 7(x+2) + \cancel{6}$$

$$\Rightarrow \cancel{x^2} - \cancel{7x} = \cancel{x^2} + 4x + 4 - \cancel{7x} - 14$$

$$\Rightarrow 10 = 4x$$

$$\Rightarrow x = \frac{5}{2}$$

IYGB – SYNOPTIC PAPER F – QUESTION 2

a) START BY LINEARIZING THE EQUATION

$$\Rightarrow H = kt^n$$
$$\Rightarrow \log H = \log(kt^n)$$
$$\Rightarrow \log H = \log k + \log t^n$$
$$\Rightarrow \log H = \log k + n\log t$$
$$\Rightarrow \underbrace{\log H}_{Y} = \underbrace{n}_{m}\underbrace{(\log t)}_{X} + \underbrace{\log k}_{C}$$

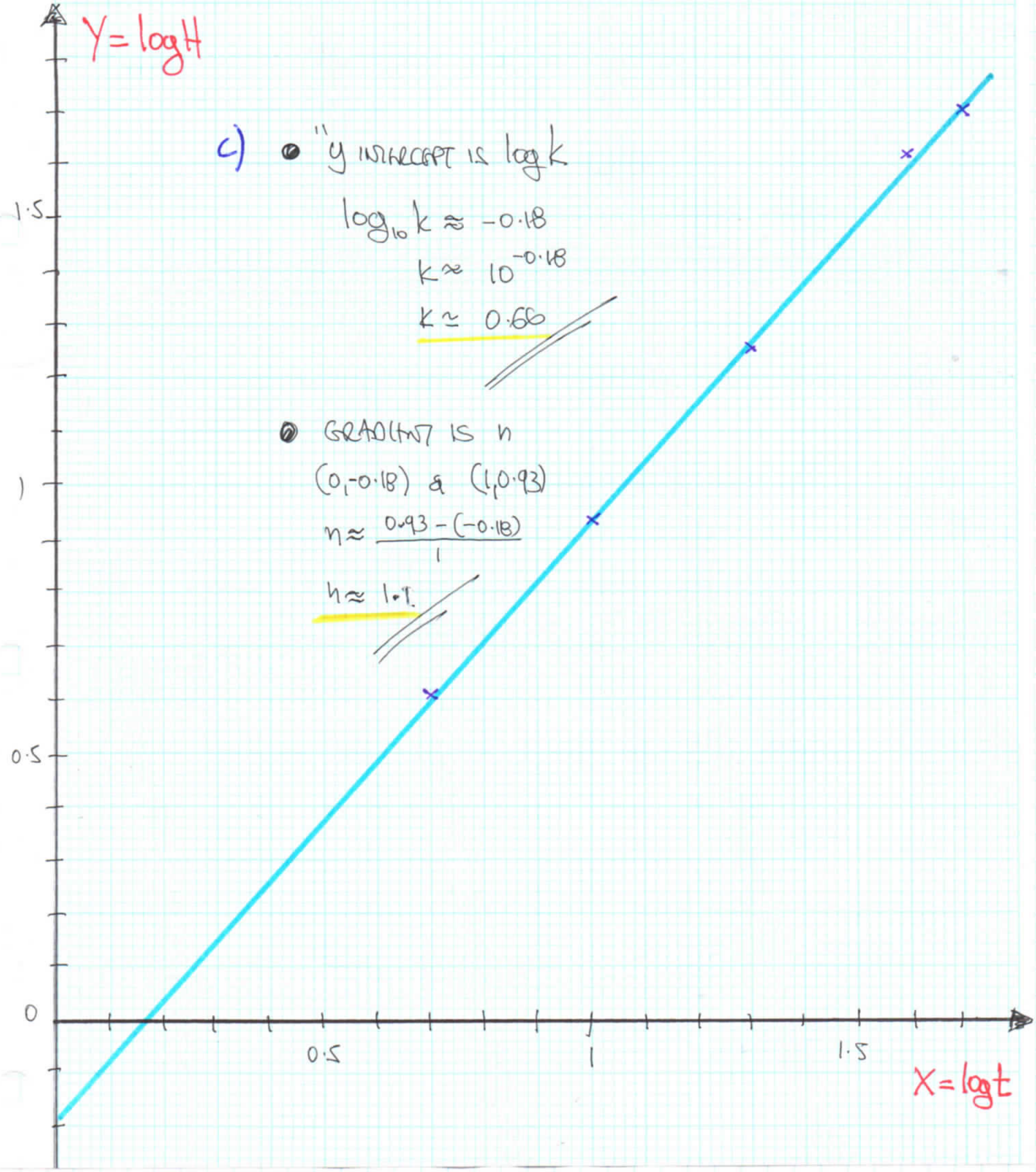
b) MODIFYING THE TABLE OF VALUES

t	5	10	20	40	50
H	4.1	8.5	18.0	42.0	50.0

$X = \log t$	0.70	1	1.30	1.60	1.70
$Y = \log H$	0.61	0.93	1.26	1.62	1.70

PLOTTING THESE VALUES ACCURATELY

IYGB - SYNOPTIC PAPER F - QUESTION 2



c) • "y intercept is log k"

$\log_{10} k \approx -0.18$

$k \approx 10^{-0.18}$

$k \simeq 0.66$

• GRADIENT IS n

$(0,-0.18)$ & $(1,0.93)$

$n \approx \dfrac{0.93-(-0.18)}{1}$

$n \approx 1.1$

IYGB - SYNOPTIC PAPER F - QUESTION 3

COMPLETING THE SQUARE

$$y = x^2 + 8x + 12$$

$$y = (x+4)^2 - 4^2 + 12$$

$$y = (x+4)^2 - 16 + 12$$

$$y = (x+4)^2 - 4$$

NOW WE HAVE

x^2 REPLACE x BY $x+4$ $(x+4)^2$ SUBTRACT 4 FROM THE FUNCTION TO GIVE $(x+4)^2 - 4$

TRANSLATION, BY 4 UNITS TO THE "LEFT"

TRANSLATION BY 4 UNITS "DOWNWARDS"

$\therefore$ TRANSLATION BY THE VECTOR $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$ //

IYGB - SYNOPTIC PAPER F - QUESTION 4

$$\boxed{x_{n+1} = \frac{k - 5x_n}{x_n}}$$

- $x_1 = 1$
- $x_2 = \frac{k - 5x_1}{x_1} = \frac{k - 5\times 1}{1} = k - 5$
- $x_3 = \frac{k - 5x_2}{x_2} = \frac{k - 5(k-5)}{k-5} = \frac{k - 5k + 25}{k-5} = \frac{25 - 4k}{k-5}$

NOW WE ARE GIVEN THAT $x_3 > 6$ & $k > 5$ SO $k - 5 > 0$

$\Rightarrow x_3 > 6$

$\Rightarrow \frac{25-4k}{k-5} > 6$

$\Rightarrow 25 - 4k > 6(k-5)$

$\Rightarrow 25 - 4k > 6k - 30$

$\Rightarrow -10k > -55$

$\Rightarrow k < \frac{55}{10} = \frac{11}{2}$

$\therefore 5 < k < \frac{11}{2}$

IYGB - SYNOPTIC PAPER F - QUESTION 5

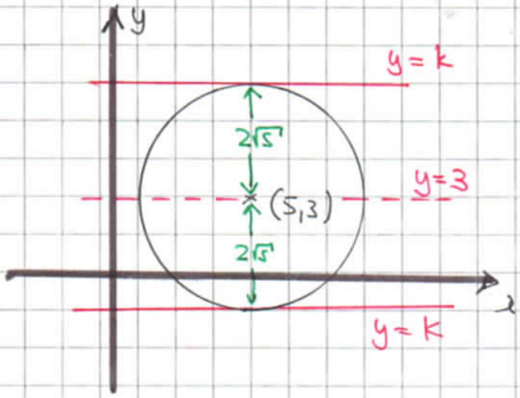
a) REWRITE THE EQUATION OF THE CIRCLE IN THE "STANDARD FORM"

$$x^2+y^2-10x-6y+14=0$$
$$x^2-10x+y^2-6y+14=0$$
$$(x-5)^2-25+(y-3)^2-9+14=0$$
$$(x-5)^2+(y-3)^2=20$$

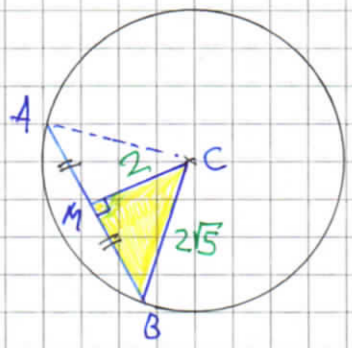
CENTRE AT $(5,3)$, RADIUS $=\sqrt{20}=2\sqrt{5}$

LOOKING AT THE DIAGRAM OPPOSITE

$$k=3\pm2\sqrt{5}$$



b) LOOKING AT THE DIAGRAM BELOW & USING PYTHAGORAS' THEOREM



$$\Rightarrow |MB|^2+|MC|^2=|CB|^2$$
$$\Rightarrow |MB|^2+2^2=(2\sqrt{5})^2$$
$$\Rightarrow |MB|^2+4=20$$
$$\Rightarrow |MB|^2=16$$
$$\Rightarrow |MB|=4$$
$$\therefore |AB|=8$$

c) SOLVING THE TWO EQUATIONS SIMULTANEOUSLY

$$\left.\begin{array}{l} x-2y-9=0 \\ (x-5)^2+(y-3)^2=20 \end{array}\right\} \Rightarrow x=2y+9$$

$$\Rightarrow [(2y+9)-5]^2+(y-3)^2=20$$

IYGB - SYNOPTIC PAPER F - QUESTION 5

$\Rightarrow (2y+4)^2 + (y-3)^2 = 20$

$\Rightarrow 4y^2 + 16y + 16 + y^2 - 6y + 9 = 20$

$\Rightarrow 5y^2 + 10y + 5 = 0$

$\Rightarrow y^2 + 2y + 1 = 0$

$\Rightarrow (y+1)^2 = 0$ (EXPECTED A REPEATED ROOT)

$\Rightarrow y = -1$

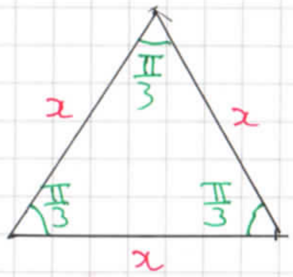
& USING $x = 2y + 9$

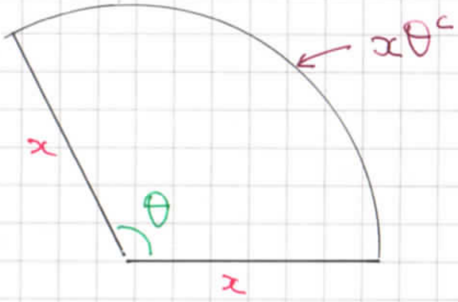
$x = 7$

$\therefore D(7,-1)$

IYGB - SYNOPTIC PAPER F - QUESTION 6

a)





CONSTRAINT ON THE LENGTH OF THE WIRE

$$\Rightarrow \quad P = 60$$

$$\Rightarrow \quad 3x + 2x + x\theta = 60$$

$$\Rightarrow \quad x\theta + 5x = 60$$

$$\Rightarrow \quad x\theta = 60 - 5x$$

AS REQUIRED

b) TOTAL AREA IS GIVEN BY

$$\frac{1}{2}x^2 \sin\frac{\pi}{3} + \frac{1}{2}x^2\theta^c \leftarrow \frac{1}{2}r^2\theta^c$$

$$\Rightarrow \quad A = \frac{1}{2}x^2\frac{\sqrt{3}}{2} + \frac{1}{2}x(x\theta)$$

$$\Rightarrow \quad A = \frac{1}{4}\sqrt{3}x^2 + \frac{1}{2}x(60 - 5x)$$

$$\Rightarrow \quad A = \frac{1}{4}\sqrt{3}x^2 + 30x - \frac{5}{2}x^2$$

$$\Rightarrow \quad A = \frac{1}{4}\sqrt{3}x^2 - \frac{5}{2}x^2 + 30x$$

$$\Rightarrow \quad A = \frac{1}{4}(\sqrt{3} - 10)x^2 + 30x$$

AS REQUIRED

c) DIFFERENTIATE & SOLVE FOR ZERO

$$\Rightarrow \quad \frac{dA}{dx} = \frac{1}{2}(\sqrt{3} - 10)x + 30$$

$$\Rightarrow \quad 0 = \frac{1}{2}(\sqrt{3} - 10)x + 30$$

$$\Rightarrow \quad 0 = (\sqrt{3} - 10)x + 60$$

$$\Rightarrow \quad -60 = (\sqrt{3} - 10)x$$

$$\Rightarrow \quad x = \frac{-60}{\sqrt{3} - 10}$$

$$\Rightarrow \quad x = 7.26$$

IYGB - SYNOPTIC PAPER F - QUESTION 6

d) $\Rightarrow A = \frac{1}{4}(\sqrt{3}-10)x^2 + 30x$

$$\Rightarrow A_{MAX} = \frac{1}{4}(\sqrt{3}-10)(7.26..)^2 + 30(7.26...)^2$$

$$\Rightarrow A_{MAX} = 109 \text{ cm}^2$$

(3sf)

AND FINALLY JUSTIFYING

$$\Rightarrow \frac{dA}{dx} = \frac{1}{2}(\sqrt{3}-10)x + 30$$

$$\Rightarrow \frac{d^2A}{dx^2} = \frac{1}{2}(\sqrt{3}-10)$$

$$\Rightarrow \left.\frac{d^2A}{dx^2}\right|_{x=7.26} = -4.13... < 0$$

INDEED A MAXIMUM

IYGB - SYNOPTIC PAPER F - QUESTION 7

$$\boxed{x^2 + (k-1)x + (k+2) = 0 \quad , \; k \in \mathbb{R}}$$

REPEATED ROOTS $\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (k-1)^2 - 4 \times 1 \times (k+2) = 0$

$\Rightarrow (k-1)^2 - 4(k+2) = 0$

$\Rightarrow k^2 - 2k + 1 - 4k - 8 = 0$

$\Rightarrow k^2 - 6k - 7 = 0$

$\Rightarrow (k+1)(k-7) = 0$

$\Rightarrow k = \begin{cases} -1 \\ 7 \end{cases}$

THERE ARE NOW TWO CASES TO CONSIDER

- IF $k = -1$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

- IF $k = 7$

$x^2 + 6x + 9 = 0$

$(x+3)^2 = 0$

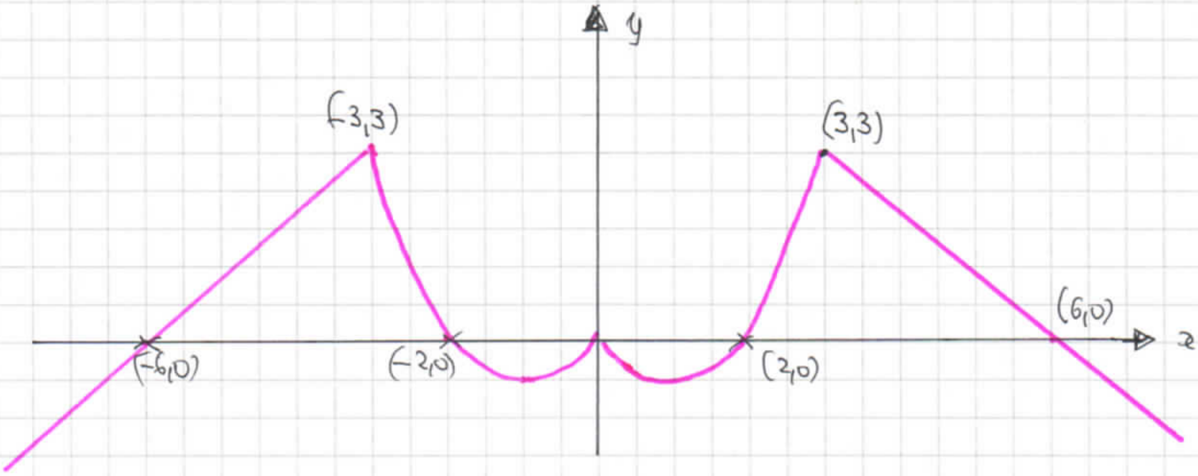
$x = -3$

IYGB - SYNOPTIC PAPER F - QUESTION 8

a) EVEN IMPLIES SYMMETRY ABOUT THE y AXIS - SKETCH FOR $x \geq 0$

$$f_1(0) = 0^2 - 2\times 0 = 0$$
$$f_1(3) = 3^2 - 2\times 3 = 3$$
$$f_2(3) = 6 - 3 = 3$$

HENCE WE HAVE NOTING THAT $x^2 - 2x = x(x-2)$



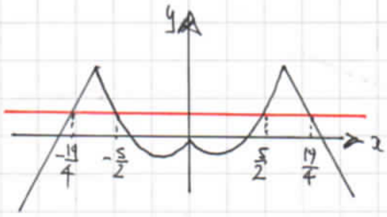
b) SOLVING $f(x) = \frac{5}{4}$ $\quad 0 < x < 3$ & $x > 3$

- $x^2 - 2x = \frac{5}{4}$

$$4x^2 - 8x = 5$$
$$4x^2 - 8x - 5 = 0$$
$$(2x+1)(2x-5) = 0$$
$$x = \begin{cases} 5/2 \\ \cancel{-\frac{1}{2}} \end{cases}$$

- $6 - x = \frac{5}{4}$

$$24 - 4x = 5$$
$$19 = 4x$$
$$x = \frac{19}{4}$$



BUT $f(x)$ IS EVEN

$\therefore\ x = \pm\frac{5}{2},\ \pm\frac{19}{4}$

IYGB - SYNOPTIC PAPER F - QUESTION 9

METHOD A

$$\ln(2\sqrt{e}) - \frac{1}{3}\ln\left(\frac{8}{e^2}\right) - \ln\left(\frac{1}{3}e\right) = \frac{1}{3}\left[3\ln(2e^{\frac{1}{2}}) - \ln\left(\frac{8}{e^2}\right) - 3\ln\left(\frac{e}{3}\right)\right]$$

$$= \frac{1}{3}\left[\ln\left(8e^{\frac{3}{2}}\right) - \ln\left(\frac{8}{e^2}\right) - \ln\left(\frac{e^3}{27}\right)\right]$$

$$= \frac{1}{3}\left[\ln\left(8e^{\frac{3}{2}}\right) + \ln\left(\frac{e^2}{8}\right) + \ln\left(\frac{27}{e^3}\right)\right]$$

$$= \frac{1}{3}\ln\left[\cancel{8}e^{\frac{3}{2}} \times \frac{e^2}{\cancel{8}} \times \frac{27}{e^3}\right]$$

$$= \frac{1}{3}\ln\left[27e^{\frac{1}{2}}\right]$$

$$= \frac{1}{3}\left[\ln 27 + \ln e^{\frac{1}{2}}\right]$$

$$= \frac{1}{3}\left[\ln 3^3 + \frac{1}{2}\ln e\right]$$

$$= \frac{1}{3}\left[3\ln 3 + \frac{1}{2}\times 1\right]$$

$$= \ln 3 + \frac{1}{6}$$

$a = 6 \quad b = 3$

METHOD B

$$\ln(2\sqrt{e}) - \frac{1}{3}\ln\left(\frac{8}{e^2}\right) - \ln\left(\frac{1}{3}e\right) = \ln 2 + \ln\sqrt{e} - \frac{1}{3}\left[\ln 8 - \ln e^2\right] - \left[\ln\frac{1}{3} + \ln e\right]$$

$$= \ln 2 + \ln e^{\frac{1}{2}} - \frac{1}{3}\left[\ln 2^3 - 2\ln e\right] - \left[\ln\frac{1}{3} + 1\right]$$

$$= \ln 2 + \frac{1}{2}\ln e - \frac{1}{3}\left[3\ln 2 - 2\times 1\right] - \left[\ln\frac{1}{3} + 1\right]$$

$$= \ln 2 + \frac{1}{2}\times 1 - \frac{1}{3}\left[3\ln 2 - 2\right] - \left[\ln\frac{1}{3} + 1\right]$$

$$= \cancel{\ln 2} + \frac{1}{2} - \cancel{\ln 2} + \frac{2}{3} - \ln\frac{1}{3} - 1$$

$$= \frac{1}{6} - \ln\frac{1}{3}$$

$$= \frac{1}{6} + \ln 3$$

AS BEFORE

IYGB - SYNOPTIC PAPER F - QUESTION 10

START WITH THE STATIONARY POINTS

$$f(x) = 3x^4 - 8x^3 - 6x^2 + 24x - 8$$
$$f'(x) = 12x^3 - 24x^2 - 12x + 24$$

SOLVING FOR ZERO

$$\Rightarrow 12x^3 - 24x^2 - 12x + 24 = 0$$
$$\Rightarrow x^3 - 2x^2 - x + 2 = 0$$
$$\Rightarrow x^2(x-2) - (x-2) = 0$$
$$\Rightarrow (x-2)(x^2-1) = 0$$
$$\Rightarrow (x-2)(x-1)(x+1) = 0$$

$$x = \begin{cases} 2 \\ 1 \\ -1 \end{cases} \qquad y = \begin{cases} 0 \\ 5 \\ -27 \end{cases} \qquad \therefore \boxed{\begin{array}{l}(2,0)\\(1,5)\\(-1,-27)\end{array}}$$

NOW AS THE FUNCTION IS STATIONARY ON THE x AXIS IT MUST HAVE A REPEATED ROOT AT $x=2$ (NO INFLEXION AS THERE ARE TWO MORE STATIONARY VALUES).

HENCE DIVIDE BY $(x-2)^2$

$$\begin{array}{r|l}
 & 3x^2 + 4x - 2 \\
x^2 - 4x + 4 & 3x^4 - 8x^3 - 6x^2 + 24x - 8 \\
 & -3x^4 + 12x^3 - 12x^2 \\
\hline
 & 4x^3 - 18x^2 + 24x - 8 \\
 & -4x^3 + 16x^2 - 16x \\
\hline
 & -2x^2 + 8x - 8 \\
 & 2x^2 - 8x + 8 \\
\hline
\end{array}$$

$$\therefore f(x) = (x-2)^2(3x^2+4x-2)$$

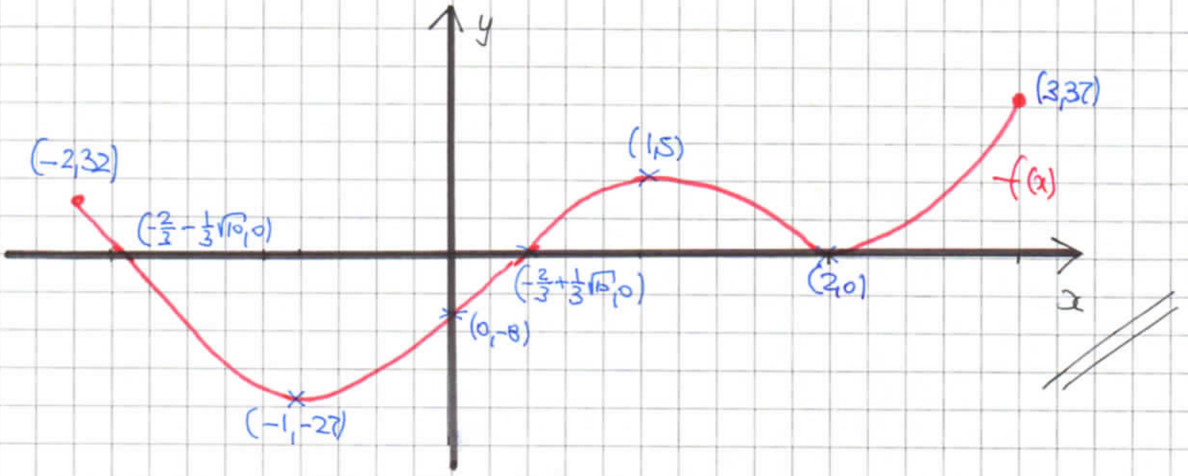
$$b^2 - 4ac = 16 - 4 \times 3 \times (-2) = 40$$

IYGB - SYNOPTIC PAPER F - QUESTION 10

GETTING THE "ZEROS" OF THE QUADRATIC

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{40}}{2 \times 3} = \frac{-4 \pm 2\sqrt{10}}{6} = -\frac{2}{3} \pm \frac{1}{3}\sqrt{10}$$

HENCE WE OBTAIN A FINAL SKETCH OF THE QUARTIC FUNCTION



AND FINALLY THE RANGE LOOKING AT THE ABOVE GRAPH IS

$$-27 \leqslant f(x) \leqslant 37$$

IYGB - SYNOPTIC PAPER F - QUESTION 11

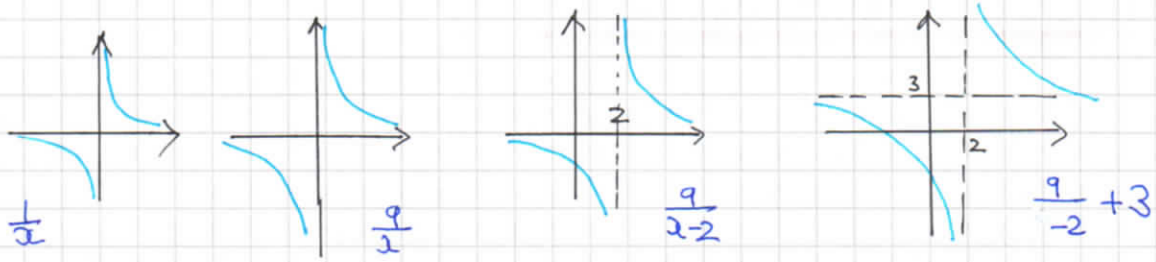
a) EASIER TO REWRITE & USE TRANSFORMATIONS

$$f(x) = \frac{3x+3}{x-2} = \frac{3(x-2)+9}{x-2} = \frac{3(x-2)}{x-2} + \frac{9}{x-2} = 3 + \frac{9}{x-2}$$

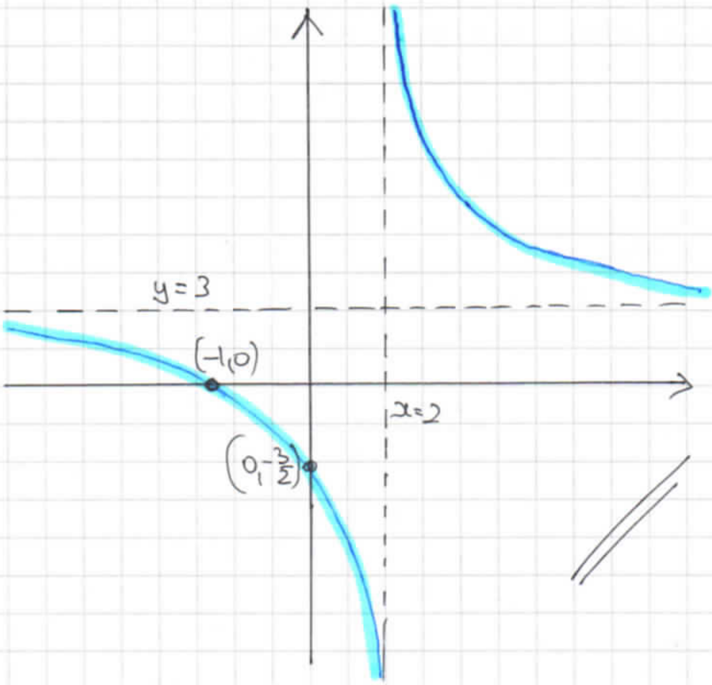
$$\frac{1}{x} \longmapsto 9\left(\frac{1}{x}\right) \longmapsto \frac{9}{x-2} \longmapsto \frac{9}{x-2} + 3$$

$$9\left(\frac{1}{x}\right) = \frac{9}{x}$$

VERTICAL STRETCH BY S.F 9 — TRANSLATION, 2 UNITS, "RIGHT" — TRANSLATION, 3 UNIT "UPWARDS"



HENCE THE SKETCH CAN NOW BE OBTAINED



$x=0, \ y=-\frac{3}{2} \quad (0,-\frac{3}{2})$

$y=0, \ x=-1 \quad (-1,0)$

b) SOLVING $f(x) = 2$

$$\frac{3x+3}{x-2} = 2$$

$$3x+3 = 2x-4$$

$$x = -7$$

IYGB - SYNOPTIC PAPER F - QUESTION 11

OR USING THE ALTERNATIVE FORM FOUND

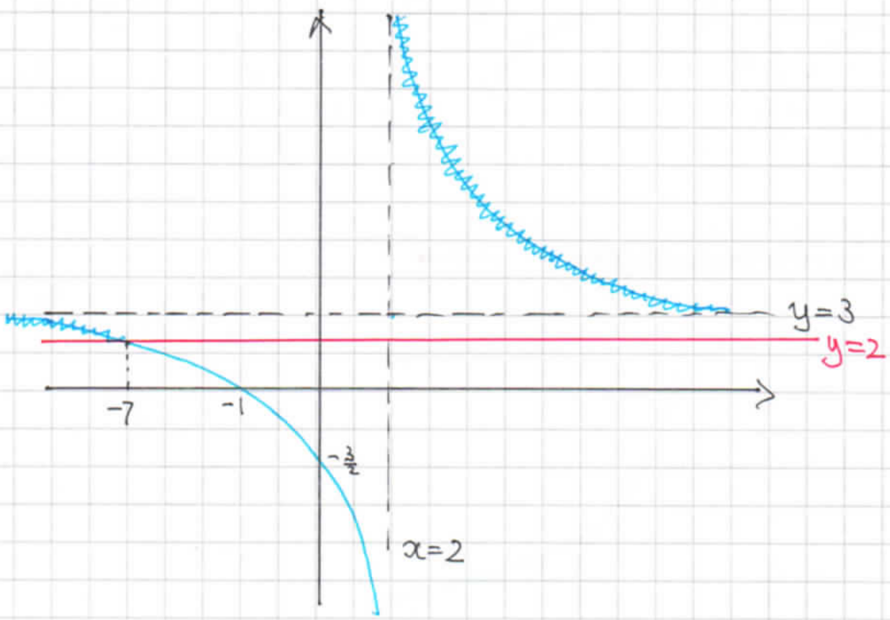
$$\Rightarrow 3+\frac{9}{x-2}=2$$

$$\Rightarrow \frac{9}{x-2}=-1$$

$$\Rightarrow 9=-x+2$$

$$\Rightarrow x=-7$$

c) LOOKING AT THE SKETCH BELOW



$$x\leq -7 \quad \text{OR} \quad x>2$$

IYGB - SYNOPTIC PAPER F - QUESTION 12

a) OBTAIN THE FIRST DERIVATIVE

$$y = x - 2\ln(x^2+4)$$

$$\frac{dy}{dx} = 1 - 2 \times \frac{1}{x^2+4} \times 2x$$

$$\frac{dy}{dx} = 1 - \frac{4x}{x^2+4}$$

DIFFERENTIATE AGAIN BY THE QUOTIENT RULE

$$\frac{d^2y}{dx^2} = 0 - \frac{(x^2+4)\times 4 - 4x(2x)}{(x^2+4)^2}$$

$$\frac{d^2y}{dx^2} = -\frac{4x^2+16-8x^2}{(x^2+4)^2}$$

$$\frac{d^2y}{dx^2} = -\frac{16-4x^2}{(x^2+4)^2}$$

$$\frac{d^2y}{dx^2} = \frac{4x^2-16}{(x^2+4)^2} = \frac{4(x^2-4)}{(x^2+4)^2}$$

AS REQUIRED

b) SOLVING $\frac{dy}{dx} = 0$

$$1 - \frac{4x}{x^2+4} = 0$$

$$x^2+4-4x = 0$$

$$x^2-4x+4 = 0$$

$$(x-2)^2 = 0$$

$$x = 2$$

$$y = 2 - 2\ln(2^2+4)$$

$$y = 2 - 2\ln 8$$

$$y = 2 - 6\ln 2$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=2} = \frac{4(2^2-4)}{8^2} = 0$$

$\therefore$ $(2, 2-6\ln 2)$ IS A POINT OF INFLEXION

IYGB - SYNOPTIC PAPER F - QUESTION 13

a) SETTING UP TWO EQUATIONS

- $S_n = \frac{n}{2}\left[a + L\right]$

$S_{25} = 1050$

$\frac{25}{2}(a+L) = 1050$

$25(a+72) = 2100$

$a + 72 = \frac{2100}{25}$

$a + 72 = \frac{4200}{50}$

$a + 72 = \frac{8400}{100}$

$a + 72 = 84$

$a = 12$ //

- $u_n = a + (n-1)d$

$u_{25} = 72$

$a + 24d = 72$

$12 + 24d = 72$

$24d = 60$

$d = \frac{260}{24}$

$d = \frac{5}{2}$ //

b) LET US NOTE THAT $\sum_{n=1}^{25} u_n = 1050$ (GIVEN IN PART a)

$$T_k = \sum_{n=1}^{k} u_n$$

$\Rightarrow 117\left[1050 - T_k\right] = 233\,T_k$

$\Rightarrow 117 \times 1050 - 117\,T_k = 233\,T_k$

$\Rightarrow 117 \times 1050 = 350\,T_k$

$\Rightarrow T_k = \frac{117 \times \cancel{1050}^{\,3}}{\cancel{350}_{\,1}}$

$\Rightarrow T_k = 351$

$\Rightarrow \sum_{n=1}^{k} u_n = 351$

IYGB - SYNOPTIC PAPER F - QUESTION 13

USING $a=12$ & $d=5/2$

$$\Rightarrow \frac{k}{2}\left[2\times 12+(k-1)\times\frac{5}{2}\right]=351$$

$$\Rightarrow \frac{k}{2}\left[24+\frac{5}{2}(k-1)\right]=351$$

$$\Rightarrow k\left[12+\frac{5}{4}(k-1)\right]=351$$

$$\Rightarrow 4k\left[12+\frac{5}{4}(k-1)\right]=1404 \qquad \times 4$$

$$\Rightarrow k\left[48+5(k-1)\right]=1404$$

$$\Rightarrow k\left[5k+43\right]=1404$$

NOW BY TRIAL & IMPROVEMENT, NOTING THAT $k<24$

IF $k=10 \Rightarrow 10\times 93<1404$

IF $k=15 \Rightarrow 15\times 118>1404$

IF $k=13 \Rightarrow 13\times 108=1404$

$\therefore k=13$

ALTERNATIVE

$$\frac{k}{2}\left[2\times 12+(k-1)\times\frac{5}{2}\right]=351$$

$$\frac{k}{2}\left[24+\frac{5}{2}k-\frac{5}{2}\right]=351$$

$$12k+\frac{5}{4}k^2-\frac{5}{4}k=351$$

$$48k+5k^2-5k=1404$$

$$5k^2+43k-1404=0$$

$$k=\frac{-43\pm\sqrt{43^2-4\times5\times(-1404)}}{2\times5}=\begin{cases}\frac{-43+173}{10}=13\\[4pt] \frac{-43-173}{10}=\cancel{-21.6}\end{cases}$$

IYGB - SYNOPTIC PAPER F - QUESTION 14

$\sqrt{2}x + \sqrt{3}y = 5$ $\qquad$ $(5\sqrt{3}-\sqrt{2})x + (5\sqrt{2}-\sqrt{3})y = 10\sqrt{6}$

START WITH THE SECOND EQUATION

$$\Rightarrow (5\sqrt{3}-\sqrt{2})x + (5\sqrt{2}-\sqrt{3})y = 10\sqrt{6}$$
$$\Rightarrow 5\sqrt{3}x - \sqrt{2}x + 5\sqrt{2}y - \sqrt{3}y = 10\sqrt{6}$$
$$\Rightarrow 5\sqrt{3}x + 5\sqrt{2}y = 10\sqrt{6} + \sqrt{2}x + \sqrt{3}y$$
$$\Rightarrow 5\sqrt{3}x + 5\sqrt{2}y = 10\sqrt{6} + 5$$
$$\Rightarrow \sqrt{3}x + \sqrt{2}y = 2\sqrt{6} + 1$$

NEXT ELIMINATE AS FOLLOWS

$$\left.\begin{aligned} \sqrt{2}x + \sqrt{3}y &= 5 \\ \sqrt{3}x + \sqrt{2}y &= 2\sqrt{6}+1 \end{aligned}\right) \begin{aligned} &\times\sqrt{3} \\ &\times\sqrt{2} \end{aligned} \Longrightarrow \begin{aligned} \sqrt{6}x + 3y &= 5\sqrt{3} \\ \sqrt{6}x + 2y &= 2\sqrt{12} + \sqrt{2} \end{aligned}$$

SUBTRACTING

$$\Rightarrow y = 5\sqrt{3} - (2\sqrt{12} + \sqrt{2})$$
$$\Rightarrow y = 5\sqrt{3} - 4\sqrt{3} - \sqrt{2}$$
$$\Rightarrow y = \sqrt{3} - \sqrt{2}$$

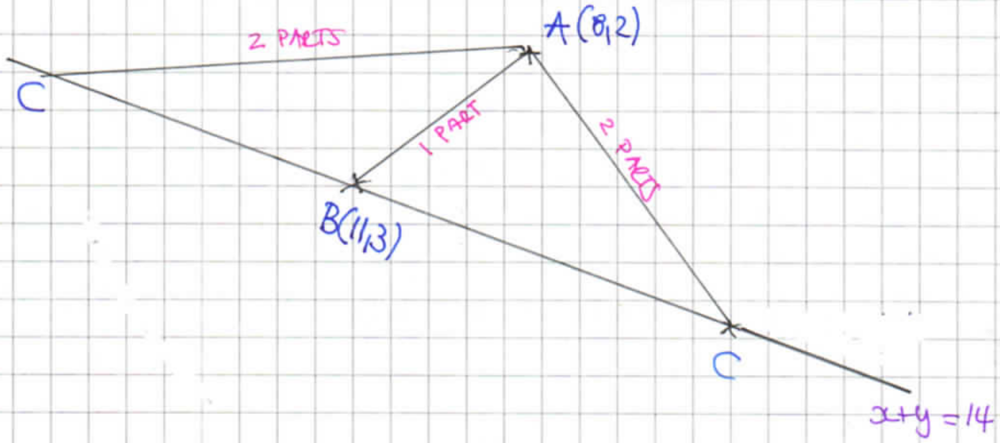
AND TO FIND x

$$\sqrt{2}x + \sqrt{3}y = 5$$
$$\sqrt{2}x + \sqrt{3}(\sqrt{3}-\sqrt{2}) = 5$$
$$\sqrt{2}x + 3 - \sqrt{6} = 5$$
$$\left.\begin{aligned} \sqrt{2}x &= 2 + \sqrt{6} \\ 2x &= 2\sqrt{2} + \sqrt{12} \end{aligned}\right) \times\sqrt{2}$$
$$2x = 2\sqrt{2} + 2\sqrt{3}$$

$$\therefore x = \sqrt{3} + \sqrt{2}$$

IYGB – SYNOPTIC PAPER F – QUESTION 15

LOOKING AT THE DIAGRAM BELOW



DETERMINE THE DISTANCE

$$|AB| = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{(2-3)^2+(8-11)^2} = \sqrt{10}$$

LET $C(x,y)$

$$|AC| = \sqrt{(8-x)^2+(2-y)^2}$$

NOW $|AC| = 2|AB|$

$$\Rightarrow \sqrt{(8-x)^2+(2-y)^2} = 2\sqrt{10}$$

$$\Rightarrow (8-x)^2 + (2-y)^2 = 40$$

$$\Rightarrow (x-8)^2 + (y-2)^2 = 40$$

$(8-x)^2 = 64-16x+x^2$
$(x-8)^2 = x^2-16x+64$

BUT THE POINT C LIES ON $x+y=14$

$$\Rightarrow [(14-y)-8]^2 + (y-2)^2 = 40$$

$$\Rightarrow (6-y)^2 + (y-2)^2 = 40$$

$$\Rightarrow \begin{pmatrix} 36-12y+y^2 \\ 4-4y+y^2 \end{pmatrix} = 40$$

$$\Rightarrow 2y^2-16y+40 = 40$$

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$\Longrightarrow \quad 2y^2 - 16y = 0$

$\Longrightarrow \quad 2y(y-8)$

$\Longrightarrow \quad y = \begin{cases} 0 \\ 8 \end{cases}$

& USING $\quad x = 14 - y$

$\Longrightarrow \quad x = \begin{cases} 14 \\ 6 \end{cases}$

$\therefore \quad C(14,0) \quad \text{OR} \quad C(6,8)$

IYGB - SYNOPTIC PAPER F - QUESTION 16

a) STARTING FROM THE R.H.S

$$\text{R.H.S} = \frac{2\tan x}{1+\tan^2 x} = \frac{2\tan x}{\sec^2 x} = 2\tan x \times \cos^2 x$$

$$= 2 \times \frac{\sin x}{\cancel{\cos x}} \times \cos^{\cancel{2}} x = 2\sin x\cos x = 2\sin 2x$$

$$= \text{L.H.S.}$$

AS REQUIRED

b) THE PARTIAL FRACTIONS NEXT

$$\frac{8}{(3t+1)(t+3)} \equiv \frac{A}{3t+1} + \frac{B}{t+3}$$

$$\boxed{8 \equiv A(t+3) + B(3t+1)}$$

- IF $t=-3$

$8 = -8B$

$B = -1$

- IF $t = -\frac{1}{3}$

$8 = \frac{8}{3}A$

$A = 3$

$$\therefore \frac{8}{(3t+1)(t+3)} = \frac{3}{3t+1} - \frac{1}{t+3}$$

c) USING THE SUBSTITUTION GIVEN & THE PREVIOUS PARTS

$$\Rightarrow t = \tan x$$

$$\Rightarrow \frac{dt}{dx} = \sec^2 x$$

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$$\Rightarrow dx = \frac{dt}{\sec^2 x}$$

$$\Rightarrow dx = \frac{dt}{\tan^2 x + 1}$$

$$\Rightarrow \boxed{dx = \frac{dt}{1+t^2}}$$

- WHEN $x=0$, $t=0$
- WHEN $x=\frac{\pi}{4}$, $t=1$

TRANSFORMING THE INTEGRAL

$$\int_0^{\frac{\pi}{4}} \frac{8}{3+5\sin 2x}\,dx = \int_0^{\frac{\pi}{4}} \frac{8}{3+5\left(\frac{2\tan x}{1+\tan^2 x}\right)}\,dx \quad \leftarrow \text{BY PART (a)}$$

$$= \int_0^1 \frac{8}{3+5\left(\frac{2t}{1+t^2}\right)}\left(\frac{dt}{1+t^2}\right) \quad \leftarrow \text{BY THE SUBSTITUTION}$$

$$= \int_0^1 \frac{8}{3(1+t^2)+10t}\,dt = \int_0^1 \frac{8}{3t^2+10t+3}\,dt$$

$$= \int_0^1 \frac{8}{(3t+1)(t+3)}\,dt = \int_0^1 \frac{3}{3t+1} - \frac{1}{t+3}\,dt \quad \leftarrow \text{BY (b)}$$

$$= \Big[\ln|3t+1| - \ln|t+3|\Big]_0^1 = \cancel{(\ln 4 - \ln 4)} - (\cancel{\ln 1} - \ln 3)$$

$$= \ln 3$$

AS REQUIRED

IYGB - SYNOPTIC PAPER F - QUESTION 17

a) FIND THE GRADIENT FUNCTION IN TERMS OF t

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3at^2}{3a} = t^2$$

SO NORMAL GRADIENT AT A GENERAL POINT WILL BE $-\frac{1}{t^2}$

EQUATION OF NORMAL AT $(3at, at^3)$

$$\Rightarrow y - y_0 = m(x - x_0)$$
$$\Rightarrow y - at^3 = -\frac{1}{t^2}(x - 3at)$$
$$\Rightarrow t^2y - at^5 = -x + 3at$$
$$\Rightarrow yt^2 + x = at^5 + 3at$$

AS REQUIRED

b) USING THE TWO POINTS GIVEN INTO THE GENERAL NORMAL

$$\left.\begin{array}{l}(7,3) \Rightarrow 3t^2+7 = 3at + at^5 \\ (-1,5) \Rightarrow 5t^2 - 1 = 3at + at^5\end{array}\right\} \Rightarrow 5t^2 - 1 = 3t^2 + 7$$
$$\Rightarrow 2t^2 = 8$$
$$\Rightarrow t^2 = 4$$
$$\Rightarrow t = \pm 2$$

NOW IF $t = 2$

$$3 \times 2^2 + 7 = 6a + 32a$$
$$19 = 38a$$
$$a = \frac{1}{2}$$

AND IF $t = -2$

$$3(-2)^2 + 7 = -6a - 32a$$
$$19 = -38a$$
$$\cancel{a = -\frac{1}{2}} \quad a > 0$$

$\therefore$ $a = \frac{1}{2}$, $t = 2$ YIELDS

$$P(3at, at^3) = P(3,4)$$

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ALTERNATIVE BY FINDING THE EQUATION OF THE NORMAL

$$(7,3) \ \& \ (-1,5) \implies m = \frac{5-3}{-1-7} = \frac{2}{-8} = -\frac{1}{4}$$

NORMAL GRADIENT $= -\frac{1}{t^2} = -\frac{1}{4}$

$$\therefore t^2 = 4$$
$$t = \left\langle \begin{matrix} 2 \\ -2 \end{matrix} \right.$$

• IF $t=2$

$$4y + x = 6a + 32a$$
$$4y + x = 38a$$

$(7,3)$: $\quad 12 + 7 = 38a$
$$19 = 38a$$
$$a = \frac{1}{2}$$

$\{$OR $(-1,5)$: $\quad 20 - 1 = 38a\}$

• IF $t=-2$

$$4y + x = -6a - 32a$$
$$4y + x = -38$$

$(7,3)$: $\quad 12 + 7 = -38a$
$$\cancel{a = -\frac{1}{2}} \quad a > 0$$

AS BEFORE $a = \frac{1}{2}$, $t = 2$ YIELDS $P(3,4)$ //

IYGB - SYNOPTIC PAPER F - QUESTION 18

USING THE IDENTITY FOR $\tan(A-B)$

$$\tan(A-B) \equiv \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\Rightarrow \tan(\overset{A}{\arctan 3x} - \overset{B}{\arctan 2}) + \tan(\overset{A}{\arctan 3} - \overset{B}{\arctan 2x}) = \frac{3}{8}$$

$$\Rightarrow \frac{3x-2}{1+(3x)(2)} + \frac{3-2x}{1+3(2x)} = \frac{3}{8}$$

$$\Rightarrow \frac{3x-2}{1+6x} + \frac{3-2x}{1+6x} = \frac{3}{8}$$

$$\Rightarrow \frac{x+1}{1+6x} = \frac{3}{8}$$

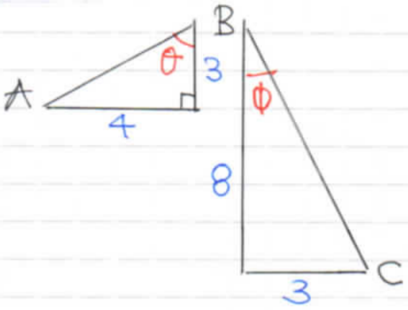
$$\Rightarrow 8x + 8 = 3 + 18x$$

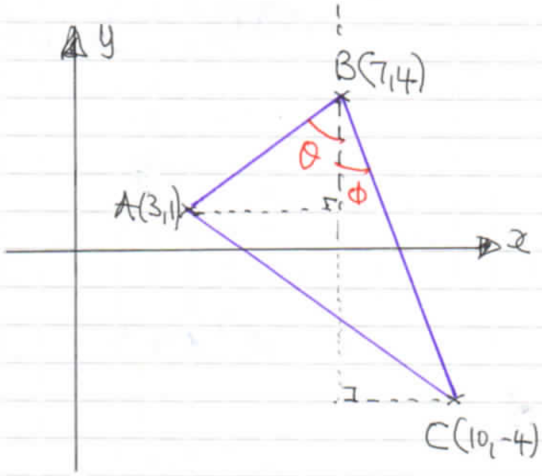
$$\Rightarrow 5 = 10x$$

$$\Rightarrow x = \frac{1}{2}$$

IYGB - SYNOPTIC PAPER F - QUESTION 19

START WITH A DIAGRAM ; $A(3,1)$, $B(7,4)$, $C(10,-4)$





$\tan\theta = \frac{4}{3}$ $\qquad \tan\phi = \frac{3}{8}$

HENCE BY USING THE COMPOUND ANGLE IDENTITY FOR $\tan(\theta+\phi)$

$$\Rightarrow \tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$

$$= \frac{\frac{4}{3} + \frac{3}{8}}{1 - \frac{4}{3}\times\frac{3}{8}}$$

$$= \frac{\frac{4}{3} + \frac{3}{8}}{\frac{1}{2}}$$

$$= 2\left(\frac{4}{3} + \frac{3}{8}\right)$$

$$= 2\left(\frac{32+9}{24}\right)$$

$$= 2 \times \frac{41}{24}$$

$$= \frac{41}{12}$$

IYGB - SYNOPTIC PAPER F - QUESTION 20

a) I) LOOKING AT THE DIAGRAM & BY RELATING VARIABLES

$$\Rightarrow \frac{dV}{dt} = +k(2r-y) \qquad k>0$$

$$\Rightarrow \frac{dV}{dy} \times \frac{dy}{dt} = k(2r-y)$$

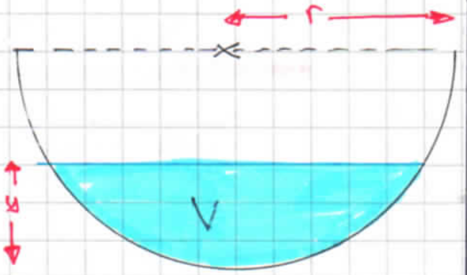
$$\Rightarrow \pi(2ry-y^2)\frac{dy}{dt} = k(2r-y)$$

$$\Rightarrow \pi y(2r-y)\frac{dy}{dt} = k(2r-y)$$

$$\Rightarrow \pi y \frac{dy}{dt} = k \qquad (2r-y \neq 0)$$

$$\Rightarrow \frac{dy}{dt} = \frac{k}{\pi y}$$

// AS REQUIRED



$$V = \frac{1}{3}\pi(3ry^2 - y^3)$$

$$\frac{dV}{dy} = \frac{1}{3}\pi(6ry - 3y^2)$$

$$\frac{dV}{dy} = \pi(2ry - y^2)$$

NOTE r = CONSTANT

II) SOLVING THE O.D.E BY SEPARATION OF VARIABLES

$$\Rightarrow y\,dy = \frac{k}{\pi}\,dt$$

$$\Rightarrow \int_{y=0}^{r} y\,dy = \int_{t=0}^{t} \frac{k}{\pi}\,dt$$

$$\Rightarrow \left[\frac{1}{2}y^2\right]_{y=0}^{y=r} = \left[\frac{k}{\pi}t\right]_0^t$$

$$\Rightarrow \frac{1}{2}r^2 - 0 = \frac{k}{\pi}t - 0$$

$$\Rightarrow \frac{r^2}{2} = \frac{kt}{\pi}$$

$$\Rightarrow t = \frac{\pi r^2}{2k}$$

// AS REQUIRED

IYGB - SYNOPTIC PAPER F - QUESTION 20

b) REMODEL THE O.D.E

WATER GOING IN AT CONSTANT RATE

$$\frac{dV}{dt} = k(2r-y) = 2kr - ky$$ ← WATER LEAKING OUT PROPORTIONAL TO y

HENCE THE REQUIRED O.D.E FOR LEAKING ONLY IS

$$\frac{dV}{dt} = -ky \quad , \quad k>0 \quad \text{SUBJECT TO } t=0 \quad y=r$$

BY RELATING VARIABLES AS BEFORE

$$\Rightarrow \frac{dV}{dy}\frac{dy}{dt} = -ky$$

$$\Rightarrow \pi\cancel{y}(2r-y)\frac{dy}{dt} = -k\cancel{y}$$

$$\Rightarrow (2r-y)\,dy = -\frac{k}{\pi}\,dt$$

INTEGRATING SUBJECT TO $t=0$, $y=r$ & REQUIRING t AT $y=0$

$$\Rightarrow \int_{y=r}^{y=0} 2r-y \; dy = \int_{t=0}^{t} -\frac{k}{\pi}\,dt$$

$$\Rightarrow \left[2ry - \frac{1}{2}y^2\right]_{y=r}^{y=0} = \left[-\frac{k}{\pi}t\right]_0^t$$

$$\Rightarrow 0 - \left(2r^2 - \frac{1}{2}r^2\right) = -\frac{k}{\pi}t - 0$$

$$\Rightarrow 2r^2 - \frac{1}{2}r^2 = \frac{k}{\pi}t$$

$$\Rightarrow \frac{3}{2}r^2 = \frac{k}{\pi}t$$

$$\Rightarrow t = \frac{3\pi r^2}{2k}$$

$$\Rightarrow t = 3\left(\frac{\pi r^2}{2k}\right)$$

IT TAKES $\frac{\pi r^2}{2k}$ TO FILL UP

IT TAKES $3\left(\frac{\pi r^2}{2k}\right)$ TO EMPTY

IE 3 TIMES AS LONG