Worksheet 3 Solutions

Question 1 Solution.

Rationalising the denominator gives:

$$\begin{aligned}\frac{2+\sqrt{\pi}}{1-\sqrt{\pi}} &= \frac{2+\sqrt{\pi}}{1-\sqrt{\pi}} \times \frac{1+\sqrt{\pi}}{1+\sqrt{\pi}} \\ &= \frac{(2+\sqrt{\pi})(1+\sqrt{\pi})}{(1-\sqrt{\pi})(1+\sqrt{\pi})} \\ &= \frac{2+3\sqrt{\pi}+\pi}{1-\pi} \\ &= \frac{2+\pi}{1-\pi} + \frac{3\sqrt{\pi}}{1-\pi}\end{aligned}$$

so $\boxed{a = 2 \text{ and } b = 3}$. [Identification of a and b is not necessary, but good practice.]

Question 2 Solution.

(a) Making the bases the same, we have

$$\begin{aligned}8^{3y+2x} = 16^y &\Leftrightarrow (2^3)^{3y+2x} = (2^4)^y \\ &\Leftrightarrow 2^{9y+6x} = 2^{4y} \\ &\Leftrightarrow 9y + 6x = 4y \\ &\Leftrightarrow y = -\frac{6}{5}x\end{aligned}$$

(b) It is probably going to be easier to substitute for y in the first equation, so let's make y the subject of $x + y = 4 \Rightarrow y = 4 - x$. Now substituting this into the other equation, we have

$$(x-4)^2 + 3(4-x)^2 = 4$$

Then

$$(x-4)^2 = 1 \Rightarrow x^2 - 8x + 15 = 0$$

$$\Rightarrow (x-3)(x-5) = 0$$

So $x = 3$ or $x = 5$. Substituting back, we find that $y = 4 - 3 = 1$ when $x = 3$ and $y = 4 - 5 = -1$.

So the solutions are $\boxed{x = 3,\ y = 1 \text{ or } x = 5,\ y = -1}$.

Question 3 Solution.

(a) Start by re-arranging:

$$\begin{aligned}&\sin^2(2\theta) - 3\cos(2\theta)\sin(2\theta) = 0\\ &\Rightarrow \sin(2\theta)(\sin(2\theta) - 3\cos(2\theta)) = 0\\ &\Rightarrow \sin(2\theta) = 0 \ \ (1) \ \ \text{OR} \ \ \sin(2\theta) - 3\cos(2\theta) = 0 \ \ (2)\end{aligned}$$

Equation (1) implies that $2\theta = 0,\ 180,\ 360,\ 540,\ 720,\ 900$. Our original range is $0 \leq \theta \leq 360$, which means our range for 2θ is $0 \leq 2\theta \leq 720$, so the values of 2θ we're interested in are $2\theta = 0,\ 180,\ 360,\ 540,\ 720 \Rightarrow \theta = 0,\ 90,\ 180,\ 270,\ 360$.

Now let's work with Equation (2):

$$\begin{aligned}&\sin(2\theta) - 3\cos(2\theta) = 0\\ &\Rightarrow \sin(2\theta) = 3\cos(2\theta)\\ &\Rightarrow \frac{\sin(2\theta)}{\cos(2\theta)} = 3\\ &\Rightarrow \tan(2\theta) = 3\\ &\Rightarrow 2\theta = 71.565..., 180 + 71.565..., 360 + 71.565..., 540 + 71.565...\\ &\Rightarrow 2\theta = 71.565..., 251.565..., 431.565..., 611.565...\\ &\Rightarrow \theta = 35.78..., 125.78..., 215.782..., 305.78...\end{aligned}$$

[Again, we know which values of 2θ to choose as our range for 2θ is $0 \leq 2\theta \leq 720$ and any other values of tan are out of range.]

Putting all the values of θ together, we have that the solutions to the equation in the range $0 \leq \theta \leq 360$ are: $\boxed{\theta = 0,\ 35.8,\ 90,\ 125.8,\ 180,\ 215.8,\ 305.8,\ 360}$

(b) [NB: You can also work in the backwards direction, which is actually mildly easier.]

$$\begin{aligned}\tan^2 x \sin^2 x &\equiv \left(\frac{\sin^2 x}{\cos^2 x}\right)\sin^2 x\\ &\equiv \frac{\sin^4 x}{\cos^2 x}\\ &\equiv \frac{\sin^2 x \times \sin^2 x}{\cos^2 x}\\ &\equiv \frac{\sin^2 x(1 - \cos^2 x)}{\cos^2 x}\\ &\equiv \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x \cos^2 x}{\cos^2 x}\\ &\equiv \tan^2 x - \sin^2 x\end{aligned}$$

(c) Take $x = 0$, then the LHS $= 0$, but the RHS $= -1$ and $0 \neq -1$.

Question 4 Solution.

(a) (i) Consider block A. Firstly, $R = 5g$. Then an equation of motion for A is: $T - 5kg = 5a$

Consider block B. An equation of motion for B is $8g - T = 8a$.

Adding both equations together we get

$$-5kg + 8g = 13a \Rightarrow a = \frac{8g - 5kg}{13}$$

which is the magnitude of the acceleration of the blocks since the positive direction chosen in both equations of motion was such that a is positive.

(ii) The restriction is because the magnitude of the acceleration cannot be negative.

(b) Letting $k = 0.4$ and substituting our acceleration back into one of the equation of motions gives

$$T = 2g + 5\left(\frac{8g - 2g}{13}\right) = \frac{56}{13}g$$

(c) By Pythagoras', the magnitude of the resultant force on the pulley can be given by

$$F = \sqrt{2\left(\frac{56}{13}g\right)^2} = \frac{56}{13}g\sqrt{2}$$

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