1. $\mathbf{T} = \begin{pmatrix} -1 & 0 & -2 \\ 1 & -1 & -1 \\ -2 & -3 & 1 \end{pmatrix}$, $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 4 \end{pmatrix}$.

(a) Determine the 3×3 matrix $\mathbf{B}$ for which

$$\mathbf{BT} = \mathbf{A}.$$

(8)

(b) Find the 3×3 matrix $\mathbf{C}$ such that

$$\mathbf{C} = \mathbf{TAT}^{-1}.$$

(3)

The image of vector $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ when transformed by $\mathbf{T}$ is the vector $\begin{pmatrix} 5 \\ -4 \\ -1 \end{pmatrix}$.

(c) Find the values of p, q and r.

(3)
(Total 14 marks)

2. Write down the 2×2 matrix that represents

(a) an enlargement with centre (0, 0) and scale factor 8,

(1)

(b) a reflection in the x-axis.

(1)

Hence, or otherwise,

(c) find the matrix $\mathbf{T}$ that represents an enlargement with centre (0, 0) and scale factor 8, followed by a reflection in the x-axis.

$\mathbf{A} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix}$, where k and c are constants.

(2)

(d) Find **AB**.

(3)

Given that **AB** represents the same transformation as **T**,

(e) find the value of k and the value of c.

(2)
(Total 9 marks)

3. $\mathbf{A} = \begin{pmatrix} a & -2 \\ -1 & 4 \end{pmatrix}$, where a is a constant.

(a) Find the value of a for which the matrix **A** is singular.

$$\mathbf{B} = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$$

(2)

(b) Find $\mathbf{B}^{-1}$.

(3)

The transformation represented by **B** maps the point P onto the point Q.

Given that Q has coordinates $(k - 6, 3k + 12)$, where k is a constant,

(c) show that P lies on the line with equation $y = x + 3$.

(3)
(Total 8 marks)

4. $\mathbf{R} = \begin{pmatrix} a & 2 \\ a & b \end{pmatrix}$, where a and b are constants and $a > 0$.

(a) Find $\mathbf{R}^2$ in terms of a and b.

(3)

Given that $\mathbf{R}^2$ represents an enlargement with centre (0, 0) and scale factor 15,

(b) find the value of a and the value of b.

(5)
(Total 8 marks)

5. $$\mathbf{M} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the geometrical transformation represented by the matrix $\mathbf{M}$.

(2)

The transformation represented by $\mathbf{M}$ maps the point A with coordinates (p, q) onto the point B with coordinates $(3\sqrt{2}, 4\sqrt{2})$.

(b) Find the value of p and the value of q.

(4)

(c) Find, in its simplest surd form, the length OA, where O is the origin.

(2)

(d) Find $\mathbf{M}^2$.

(2)

The point B is mapped onto the point C by the transformation represented by $\mathbf{M}^2$.

(e) Find the coordinates of C.

(2)
(Total 12 marks)