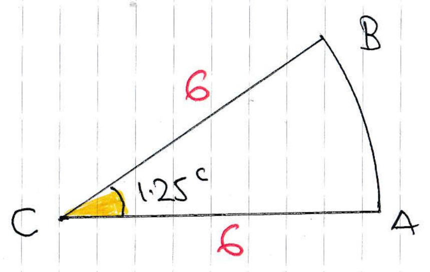
IYGB - MP2 PAPER K - QUESTION 1

a)



- USING "$A = \frac{1}{2}r^2\theta^c$"

$\text{AREA} = \frac{1}{2} \times 6^2 \times 1.25 =$

$\text{AREA} = 22.5 \text{ cm}^2$

- USING "$L = r\theta^c$"

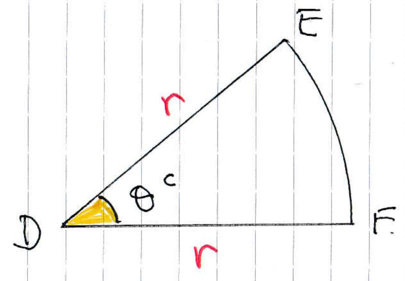
$L = 6 \times 1.25$

$L = 7.5 \text{ cm}$

- REQUIRED PERIMETER $= 6 + 6 + 7.5$

$= 19.5 \text{ cm}$

b)



$$\boxed{\begin{aligned} \tfrac{1}{2}r^2\theta &= 22.5 \\ 2r + r\theta &= 19.5 + 1.5 \end{aligned}}$$

- TIDY THE EQUATIONS & SOLVE

$$\left.\begin{aligned} r^2\theta &= 45 \\ 2r + r\theta &= 21 \end{aligned}\right\} \xrightarrow{\times r} \left.\begin{aligned} r^2\theta &= 45 \\ 2r^2 + r^2\theta &= 21r \end{aligned}\right\} \Rightarrow 2r^2 + 45 = 21r$$

- SOLVING THE QUADRATIC

$\Rightarrow 2r^2 - 21r + 45 = 0$

$\Rightarrow (2r - 15)(r - 3) = 0$

$\Rightarrow r = \begin{cases} 3 \\ \frac{15}{2} \end{cases}$ $\qquad \theta = \frac{45}{r^2} = \begin{cases} \frac{45}{9} = 5 \\ \frac{45}{(15/2)^2} = 0.8 \end{cases}$

- THUS WE OBTAIN

$(r, \theta^c) = (3, 5)$ OR $(\frac{15}{2}, 0.8)$

IYGB – MP2 PAPER K – QUESTION 2

a) FORMING A TABLE OF VALUES

x	0	0.2	0.4	0.6	0.8	1.0	1.2
$y = \sin^2 x$	0	0.0395	0.1516	0.3188	0.5146	0.7081	0.8687

USING THE TRAPEZIUM RULE

$$\int_0^{1.2} \sin^2 x \, dx \approx \text{"}\frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2 \times \text{REST}\right]\text{"}$$

$$\approx \frac{0.2}{2}\left[0 + 0.8687 + 2(0.0395 + 0.1516 + 0.3188 + 0.5146 + 0.7081)\right]$$

$$\approx 0.433$$

b)
$$\int_0^{1.2} \cos 2x \, dx = \int_0^{1.2} 1 - 2\sin^2 x \, dx$$

$$= \int_0^{1.2} 1 \, dx - 2\int_0^{1.2} \sin^2 x \, dx$$

$$\left[x\right]_0^{1.2} - 2 \times 0.433$$

$$\approx (1.2 - 0) - 0.866$$

$$\approx 0.334$$

IYGB - MP2 PAPER K - QUESTION 2

c) $\int_0^{1.2} \cos^4 x - \sin^4 x \, dx = \int (\cos^2 x)^2 - (\sin^2 x)^2 \, dx$

$= \int_0^{1.2} (\cos^2 x - \sin^2 x)\cancel{(\cos^2 x + \sin^2 x)} \, dx$

$\cos^4 x - \sin^4 x = (\cos^2 x)^2 - (\sin^2 x)^2 = \ldots$ difference of squares

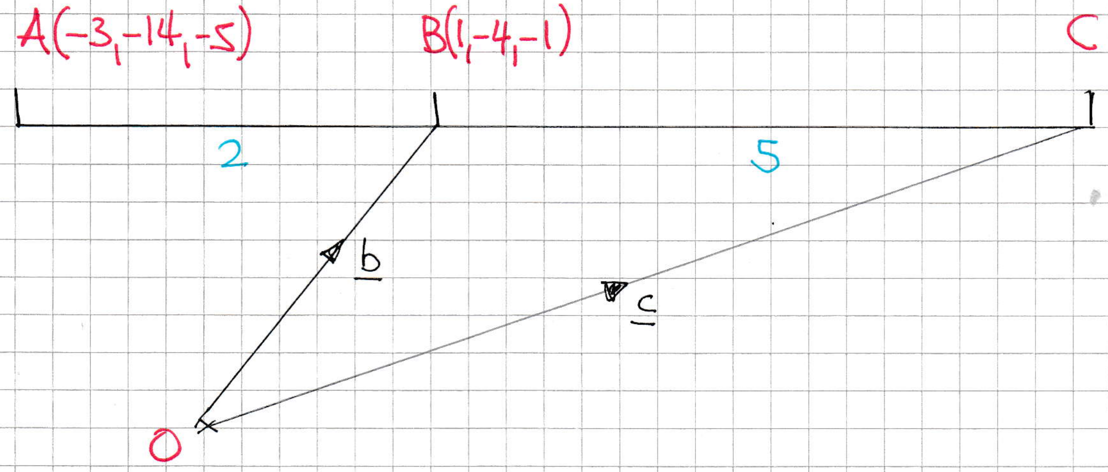
$= \int \cos^2 x - \sin^2 x \, dx$

$= \int \cos 2x \, dx$

$= 0.334$

IYGB – MP2 PAPER K – QUESTION 3

PUT THE INFORMATION IN A DIAGRAM



$$\Rightarrow \overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$$

$$\Rightarrow \overrightarrow{OC} = \overrightarrow{OB} + \frac{5}{2}\overrightarrow{AB}$$

$$\Rightarrow \underline{c} = \underline{b} + \frac{5}{2}(\underline{b} - \underline{a})$$

$$\Rightarrow \underline{c} = \underline{b} + \frac{5}{2}\underline{b} - \frac{5}{2}\underline{a}$$

$$\Rightarrow \underline{c} = \frac{7}{2}\underline{b} - \frac{5}{2}\underline{a}$$

$$\Rightarrow \underline{c} = \frac{1}{2}(7\underline{b} - 5\underline{a})$$

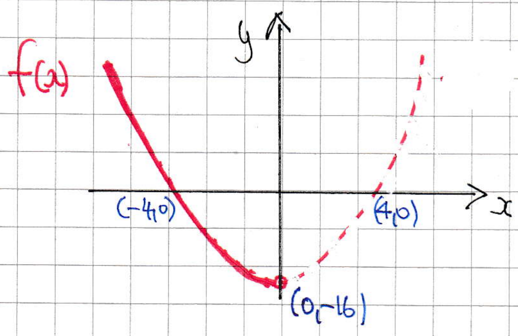
$$\Rightarrow \underline{c} = \frac{1}{2}\left[7(1,-4,-1) - 5(-3,-14,-5)\right]$$

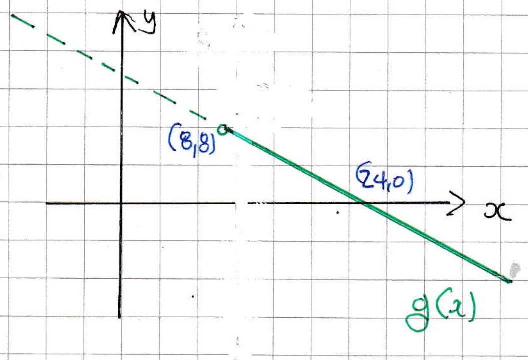
$$\Rightarrow \underline{c} = \frac{1}{2}(22, 42, 18)$$

$$\Rightarrow \underline{c} = (11, 21, 9)$$

IYGB – MP2 PAPER K – QUESTION 4

a)I) SKETCHING THE TWO FUNCTIONS





RANGE OF $f(x)$

$f(x) \geq -16$

RANGE OF $g(x)$

$g(x) < 8$

a)II)

$\xrightarrow{x>8} \boxed{g} \xrightarrow{g(x)} \qquad \xrightarrow{x\leq 0} \boxed{f} \longrightarrow$

- $f(g(x)) = f\left(12 - \frac{1}{2}x\right) = \left(12 - \frac{1}{2}x\right)^2 - 16$

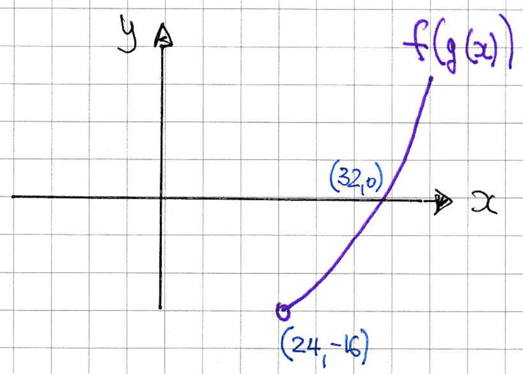
  $= \frac{1}{4}(x-24)^2 - 16$

- THE DOMAIN MUST SATISFY $x > 8$ AND $g(x) < 0$

  $12 - \frac{1}{2}x < 0$

  $-\frac{1}{2}x < -12$

  $x > 24$



- THE RANGE CAN BE FOUND BY INSPECTION OR BY LOOKING AT THE GRAPH OPPOSITE

  $f(g(x)) > -16$

IYGB – MP2 PAPER K – QUESTION 4

c) SOLVING THE EQUATION

$\Longrightarrow f(g(x)) = f(2x-22)$

$\Longrightarrow \frac{1}{4}(x-24)^2 - 16 = 12 - \frac{1}{2}(2x-22)$

(FOUND IN PART b II)

$\times 4$

$\Longrightarrow (x-24)^2 - 64 = 48 - 2(2x-22)$

$\Longrightarrow x^2 - 48x + 576 - 64 = 48 - 4x + 44$

$\Longrightarrow x^2 - 44x + 420 = 0$

BY THE QUADRATIC FORMULA OR FACTORIZATION

$\Longrightarrow (x-30)(x-14) = 0$

$\Longrightarrow x = \begin{cases} \cancel{14} \\ 30 \end{cases}$ $\quad x > 24$

IYGB – MP2 PAPER K – QUESTION 5

a) STARTING WITH THE COMPOUND ANGLE IDENTITIES

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

SUBTRACTING THE EQUATIONS (IDENTITIES) ABOVE

$$\Longrightarrow \underbrace{\cos(A+B)}_{P} - \underbrace{\cos(A-B)}_{Q} = -2\sin A\sin B$$

$$\therefore \left.\begin{array}{l} A+B=P \\ A-B=Q \end{array}\right\} \Longrightarrow$$ ADDING GIVE $2A = P+Q$

$$\Longrightarrow \boxed{A = \frac{P+Q}{2}}$$

$$\Longrightarrow \boxed{B = \frac{P-Q}{2}}$$ (BY SUBTRACTING)

$$\therefore \cos P - \cos Q = -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

b) STARTING WITH THE DEFINITION OF A DERIVATIVE

$$\frac{dy}{dx} = f'(x) = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$ WITH $f(x) = \cos x$

$$f'(x) = \lim_{h\to 0}\left[\frac{\overset{P}{\cos(x+h)} - \overset{Q}{\cos x}}{h}\right]$$

$$f'(x) = \lim_{h\to 0}\left[\frac{-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h}\right]$$

$$f'(x) = \lim_{h\to 0}\left[\frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}\right]$$

$$f'(x) = \lim_{h\to 0}\left[-2\sin\left(\frac{2x+h}{2}\right)\times\frac{\sin\frac{h}{2}}{h}\right]$$

IYGB – MP2 PAPER K – QUESTION 5

FOR SMALL θ, $\sin\theta \approx \theta$, θ IN RADIANS

FOR SMALL h, $\sin\frac{h}{2} \approx \frac{h}{2}$, h IN RADIANS

$$\Longrightarrow \quad f'(x) = \lim_{h\to 0}\left[-2\sin\left(\frac{2x+h}{2}\right) \times \frac{h/2}{h}\right]$$

$$\Longrightarrow \quad f'(x) = \lim_{h\to 0}\left[-\sin\left(\frac{2x+h}{2}\right)\right]$$

$$\Longrightarrow \quad \frac{d}{dx}(\cos x) = -\sin\left(\frac{2x}{2}\right) = -\sin x$$

AS REQUIRED

IYGB - MP2 PAPER K - QUESTION 6

a) EXPAND BINOMIALLY UP TO x^3

$$\Rightarrow f(x) = \left(\tfrac{1}{2}-x\right)^{-3} = \left(\tfrac{1}{2}\right)^{-3}\left[1-2x\right]^{-3} = 8(1-2x)^{-3}$$

$$\Rightarrow f(x) = 8\left[1 + \frac{-3}{1}(-2x)^1 + \frac{-3(-4)}{1\times 2}(-2x)^2 + \frac{-3(-4)(-5)}{1\times 2\times 3}(-2x)^3 + O(x^4)\right]$$

$$\Rightarrow f(x) = 8\left[1 + 6x + 24x^2 + 80x^3 + O(x^4)\right]$$

$$\Rightarrow f(x) = 8 + 48x + 192x^2 + 640x^3 + O(x^4)$$

b) PROCEED AS FOLLOWS

$$\Rightarrow g(x) = \frac{a+bx}{\left(\frac{1}{2}-x\right)^3} = (a+bx)\left(\tfrac{1}{2}-x\right)^{-3}$$

$$\Rightarrow g(x) = (a+bx)\left[8 + 48x + 192x^2 + 640x^3 + O(x^4)\right]$$

$$\Rightarrow g(x) = 8a + 48ax + 192ax^2 + 640ax^3 + O(x^4)$$
$$8bx + 48bx^2 + 192bx^3 + O(x^4)$$

$$\Rightarrow g(x) = 8a + (48a+8b)x + \underbrace{(192a+48b)}_{42}x^2 + \underbrace{(640a+192b)}_{136}x^3 + O(x^4)$$

FINALLY WE HAVE

$$\left.\begin{array}{l} 192a + 48b = 42 \\ 640a + 192b = 136 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 32a + 8b = 7 \\ 80a + 24b = 17 \end{array}\right\} \begin{array}{l} \times(-3) \\ \times 1 \end{array}$$

$$\left.\begin{array}{l} -96a - 24b = -21 \\ 80a + 24b = 17 \end{array}\right\} \text{ ADDING } \quad -16a = -4$$
$$a = \tfrac{1}{4}$$

AND $32a + 8b = 7$

$$8 + 8b = 7$$
$$b = -\tfrac{1}{8}$$

IYGB - MP2 PAPER K - QUESTION 7

a) MANIPULATE AS SUGGESTED

$\Rightarrow y = \arcsin x$

$\Rightarrow \sin y = x$

$\Rightarrow x = \sin y$

TAKE "ARCCOS" ON BOTH SIDES

$\Rightarrow \arccos x = \arccos(\sin y)$

$\Rightarrow \arccos x = \arccos\left(\cos\left(\frac{\pi}{2} - y\right)\right)$ ← $\sin A \equiv \cos\left(\frac{\pi}{2} - A\right)$
$\cos A \equiv \sin\left(\frac{\pi}{2} - A\right)$

$\Rightarrow \arccos x = \frac{\pi}{2} - y$

$\Rightarrow \arccos x = \frac{\pi}{2} - \arcsin x$ ← AS $y = \arcsin x$

$\Rightarrow \arccos x + \arcsin x = \frac{\pi}{2}$ // AS REQUIRED

b) USING PART (a), LET $y = x - 1$

$\Rightarrow 3\arcsin(x-1) = 2\arccos(x-1)$

$\Rightarrow 3\arcsin y = 2\arccos y$

$\Rightarrow 3\arcsin y = 2\left[\frac{\pi}{2} - \arcsin y\right]$

$\Rightarrow 3\arcsin y = \pi - 2\arcsin y$

$\Rightarrow 5\arcsin y = \pi$

$\Rightarrow \arcsin y = \frac{\pi}{5}$

$\Rightarrow y = \sin\frac{\pi}{5}$

$\Rightarrow x - 1 = \sin\frac{\pi}{5}$

$\Rightarrow x = 1 + \sin\frac{\pi}{5} \simeq 1.58778\ldots$

IYGB - MP2 PAPER K - QUESTION 8

$$\int_0^{\frac{1}{2}} \frac{2x^3-5x^2+5}{(x^2-3x+2)(x^2-2x+1)}\,dx = \int_0^{\frac{1}{2}} \frac{2x^3-5x^2+5}{(x-2)(x-1)(x-1)(x-1)}\,dx$$

$$= \int \frac{2x^3-5x^2+5}{(x-2)(x-1)^3}\,dx$$

- PROCEED BY PARTIAL FRACTIONS

$$\frac{2x^3-5x^2+5}{(x-2)(x-1)^3} \equiv \frac{A}{x-2} + \frac{B}{(x-1)^3} + \frac{C}{(x-1)^2} + \frac{D}{x-1}$$

$$\boxed{2x^3-5x^2+5 \equiv A(x-1)^3 + B(x-2) + C(x-1)(x-2) + D(x-2)(x-1)^2}$$

- IF $x=1$ $\quad 2=-B$

$B=-2$

- IF $x=2$ $\quad 1=A$

$A=1$

- IF $x=0$

$5=-A-2B+2C-2D$

$5=-1+4+2C-2D$

$2=2C-2D$

$C-D=1$

- IF $x=3$

$14=8A+B+2C+4D$

$14=8-2+2C+4D$

$8=2C+4D$

$C+2D=4$

SUBTRACTING YIELDS

$3D=3$

$D=1$

& $C=2$

IYGB - MP2 PAPER K - QUESTION 8

- RETURNING TO THE INTEGRAL WITH THE FRACTION SPLIT

$$\ldots = \int_0^{\frac{1}{2}} \frac{1}{x-2} - 2(x-1)^{-3} + 2(x-1)^{-2} + \frac{1}{x-1} \, dx$$

$$= \Big[\ln|x-2| + (x-1)^{-2} - 2(x-1)^{-1} + \ln|x-1| \Big]_0^{\frac{1}{2}}$$

$$= \Big[\ln|x-2| + \ln|x-1| + \frac{1}{(x-1)^2} - \frac{2}{x-1} \Big]_0^{\frac{1}{2}}$$

$$= \left(\ln\frac{3}{2} + \ln\frac{1}{2} + \frac{1}{\frac{1}{4}} - \frac{2}{-\frac{1}{2}} \right) - \left(\ln 2 + \cancel{\ln 1} + 1 + 2 \right)$$

$$= \ln\frac{3}{2} + \ln\frac{1}{2} - \ln 2 + 4 + 4 - 1 - 2$$

$$= 5 + \ln\frac{\frac{3}{4}}{2}$$

$$= 5 + \ln\frac{3}{8}$$

IYGB - MP2 PAPER K - QUESTION 9

THE ARITHMETIC SERIES IS

$$\underbrace{350 + 340 + 330 + \cdots\cdots + (\ ?\ )}_{n \text{ terms}} = 6200$$

WHERE n IS A POSITIVE INTEGER

HERE WE HAVE $a = 350$, $d = -10$ & $S_n = 6200$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$6200 = \frac{n}{2}\left[2 \times 350 + (n-1)(-10)\right]$$
$$6200 = \frac{n}{2}\left[700 - 10n + 10\right]$$
$$6200 = \frac{n}{2}\left[710 - 10n\right]$$
$$620 = \frac{n}{2}(71 - n)$$
$$1240 = n(71 - n)$$
$$1240 = 71n - n^2$$
$$n^2 - 71n + 1240 = 0$$

BY THE QUADRATIC FORMULA OR FACTORIZATION

$$n = \frac{71 \pm \sqrt{(-71)^2 - 4 \times 1 \times 1240}}{2 \times 1} = \frac{71 \pm 9}{2} = \begin{cases} 40 \\ 31 \end{cases}$$

TO CHECK WHICH SOLUTION IS VALID USE $u_n = a + (n-1)d$

- $n = 31$

$$u_{31} = 350 + (31-1)(-10)$$
$$u_{31} = 350 - 300$$
$$u_{31} = 50$$

- $n = 40$

$$u_{40} = 350 + (40-1)(-10)$$
$$u_{40} = 350 - 390$$
$$u_{40} = \cancel{-40}$$

$$\therefore n = 31$$

IYGB – MP2 PAPER K – QUESTION 10

a) FORMING A DIFFERENTIAL EQUATION

$$\Rightarrow \frac{dA}{dt} = -cA^2 \quad [c = \text{PROPORTIONALITY CONSTANT}]$$

$$\Rightarrow \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt} = -cA^2$$

$$\Rightarrow (8\pi r)\left(\frac{1}{4\pi r^2}\right)\frac{dV}{dt} = -c(4\pi r^2)^2$$

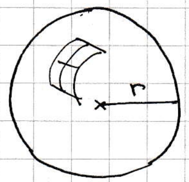
$$\Rightarrow \frac{2}{r}\frac{dV}{dt} = -16\pi^2 c r^4$$

$$\Rightarrow \frac{dV}{dt} = -8\pi^2 c r^5$$

$$\Rightarrow \frac{dV}{dt} = -8\pi^2 c\left(\frac{3}{4\pi}\right)^{\frac{5}{3}} V^{\frac{5}{3}}$$

$$\Rightarrow \frac{dV}{dt} = -kV^{\frac{5}{3}}$$

// AS REQUIRED



$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$\frac{3V}{4\pi} = r^3$

$\left(\frac{3V}{4\pi}\right)^{\frac{5}{3}} = (r^3)^{\frac{5}{3}}$

$\left(\frac{3}{4\pi}\right)^{\frac{5}{3}} V^{\frac{5}{3}} = r^5$

ALTERNATIVE MANIPULATION

$$\Rightarrow \frac{dA}{dt} = -cA^2$$

$$\Rightarrow \frac{dA}{dV} \times \frac{dV}{dt} = -cA^2$$

$$\Rightarrow \frac{2}{3}(36\pi)^{\frac{1}{3}} V^{-\frac{1}{3}} \times \frac{dV}{dt} = -c\left[(36\pi)^{\frac{1}{3}} V^{\frac{2}{3}}\right]^2$$

$$\Rightarrow \frac{2}{3}(36\pi)^{\frac{1}{3}} V^{-\frac{1}{3}} \frac{dV}{dt} = -c(36\pi)^{\frac{2}{3}} V^{\frac{4}{3}}$$

$$\Rightarrow \frac{2(36\pi)^{\frac{1}{3}}}{3V^{1/3}} \frac{dV}{dt} = -c(36\pi)^{2/3} V^{\frac{4}{3}}$$

$$\Rightarrow \frac{dV}{dt} = \boxed{\frac{-3c(36\pi)^{2/3}}{2(36\pi)^{1/3}}} V^{\frac{5}{3}}$$

$$\Rightarrow \frac{dV}{dt} = -kV^{\frac{5}{3}}$$

$\left.\begin{array}{l} V = \frac{4}{3}\pi r^3 \\ A = 4\pi r^2 \end{array}\right\} \Rightarrow$

$\left.\begin{array}{l} V^2 = \frac{16}{9}\pi^2 r^6 \\ A^3 = 64\pi^3 r^6 \end{array}\right\} \Rightarrow$

$\frac{A^3}{V^2} = \frac{64\pi^3 r^6}{\frac{16}{9}\pi^2 r^6}$

$\frac{A^3}{V^2} = 36\pi$

$A^3 = 36\pi V^2$

$A = (36\pi)^{\frac{1}{3}} V^{\frac{2}{3}}$

$\frac{dA}{dV} = \frac{2}{3}(36\pi)^{\frac{1}{3}} V^{-\frac{1}{3}}$

IYGB - MP2 PAPER K - QUESTION 10

b) SEPARATING VARIABLES

$$\Rightarrow \frac{dV}{dt} = -kV^{\frac{5}{3}}$$

$$\Rightarrow \int \frac{1}{V^{\frac{5}{3}}}\,dV = \int -k\,dt$$

$$\Rightarrow \int V^{-\frac{5}{3}}\,dV = \int -k\,dt$$

$$\Rightarrow -\frac{3}{2}V^{-\frac{2}{3}} = -kt + A$$

$$\Rightarrow V^{-\frac{2}{3}} = Bt + D$$

$$\Rightarrow \boxed{\frac{1}{V^{\frac{2}{3}}} = Bt + D}$$

WHEN $t=0$ $V=1000$ (GIVEN)

$$\frac{1}{100} = B \times 0 + D$$

$$D = \frac{1}{100}$$

$$\therefore \boxed{\frac{1}{V^{\frac{2}{3}}} = \frac{1}{100} + Bt}$$

WHEN $t=20$, $V=729$

$$\frac{1}{81} = \frac{1}{100} + 20B$$

$$20B = \frac{19}{8100}$$

$$B = \frac{19}{162000}$$

$$\boxed{\frac{1}{V^{\frac{2}{3}}} = \frac{1}{100} + \frac{19}{162000}t}$$

FINALLY WHEN $V=512$

$$\Rightarrow \frac{1}{512^{\frac{2}{3}}} = \frac{1}{100} + \frac{19}{162000}t$$

$$\Rightarrow \frac{1}{64} = \frac{1}{100} + \frac{19}{162000}t$$

$$\Rightarrow \frac{9}{1600} = \frac{19}{162000}t$$

$$\Rightarrow t = \frac{3645}{76}$$

$$\Rightarrow t \simeq 48$$

IYGB - MP2 PAPER K - QUESTION 11

USING THE FACT, USUALLY GIVEN IN EXAMS THAT

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$y = \underbrace{2x\arcsin 2x}_{\text{PRODUCT}} + (1-4x^2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \underbrace{2 \times \arcsin 2x + 2x \times \frac{1}{\sqrt{1-(2x)^2}} \times 2}_{\text{PRODUCT RULE}} + \frac{1}{2}(1-4x^2)^{-\frac{1}{2}}(-8x)$$

$$\frac{dy}{dx} = 2\arcsin 2x + \cancel{\frac{4x}{\sqrt{1-4x^2}}} - \cancel{\frac{4x}{\sqrt{1-4x^2}}}$$

$$\boxed{\frac{dy}{dx} = 2\arcsin 2x}$$

DIFFERENTIATING AGAIN

$$\frac{d^2y}{dx^2} = 2 \times \frac{1}{\sqrt{1-(2x)^2}} \times 2$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{4}{\sqrt{1-4x^2}}}$$

FROM HERE THERE ARE TWO VARIANTS

<VARIANT A> by manipulations

$$\Rightarrow \sqrt{1-4x^2}\,\frac{d^2y}{dx^2} = 4$$

USING $y = 2x\arcsin 2x + \sqrt{1-4x^2}$

$$\Rightarrow (y - 2x\arcsin 2x)\frac{d^2y}{dx^2} = 4$$

USING $\frac{dy}{dx} = 2\arcsin 2x$

$$\Rightarrow \left(y - x\frac{dy}{dx}\right)\frac{d^2y}{dx^2} = 4$$

IYGB - MP2 PAPER K - QUESTION 11

NOW DIFFERENTIATE BOTH SIDES WITH RESPECT TO x

$$\Rightarrow \frac{d}{dx}\left[\underbrace{\left(y - x\frac{dy}{dx}\right)\frac{d^2y}{dx^2}}_{\text{PRODUCT}}\right] = \frac{d}{dx}[4]$$

PRODUCT WITHIN PRODUCT

$$\Rightarrow \left[\cancel{\frac{dy}{dx}} - 1 \times \cancel{\frac{dy}{dx}} - x\frac{d^2y}{dx^2}\right]\frac{d^2y}{dx^2} + \left[\left(y - x\frac{dy}{dx}\right)\frac{d^3y}{dx^3}\right] = 0$$

$$\Rightarrow -x\left(\frac{d^2y}{dx^2}\right)^2 = \left(y - x\frac{dy}{dx}\right)\frac{d^3y}{dx^3} = 0$$

$$\Rightarrow \left(y - x\frac{dy}{dx}\right)\frac{d^3y}{dx^3} = x\left(\frac{d^2y}{dx^2}\right)^2$$

<VARIANT B> by verification of both sides

OBTAIN THIRD DERIVATIVE FROM $\frac{d^2y}{dx^2} = 4(1-4x^2)^{-\frac{1}{2}}$

$$\frac{d^3y}{dx^3} = 4\left(-\tfrac{1}{2}\right)(1-4x^2)^{-\frac{3}{2}}(-8x) = 16x(1-4x^2)^{-\frac{3}{2}}$$

NOW VERIFY BOTH SIDES

- $L.H.S = \Big[\underbrace{\cancel{2x\arcsin 2x} + (1-4x^2)^{\frac{1}{2}}}_{y} - \cancel{\underbrace{x}_{x} \times \underbrace{2\arcsin 2x}_{\frac{dy}{dx}}}\Big] \times \underbrace{16x(1-4x^2)^{-\frac{3}{2}}}_{\frac{d^3y}{dx^3}}$

$$= (1-4x^2)^{\frac{1}{2}} \times 16x(1-4x^2)^{-\frac{3}{2}} = 16x(1-4x^2)^{-1} = \frac{16x}{1-4x^2}$$

- $RHS = x\Big[\underbrace{4(1-4x^2)^{-\frac{1}{2}}}_{\left(\frac{d^2y}{dx^2}\right)^2}\Big]^2 = x \times 16(1-4x^2)^{-1} = \frac{16}{1-4x^2}$

AS LHS = RHS

$$\left(y - x\frac{dy}{dx}\right)\frac{d^3y}{dx^3} = x\left(\frac{d^2y}{dx^2}\right)^2$$

AS REQUIRED