IYGB - MP1 PAPER Y - QUESTION 1

a) PROCEED AS FOLLOWS

$$-1 \leqslant \sin(2x+k)^\circ \leqslant 1$$
$$-4 \leqslant 4\sin(2x+k)^\circ \leqslant 4$$
$$-4 \leqslant -4\sin(2x+k)^\circ \leqslant 4$$
$$-1 \leqslant 3-4\sin(2x+k) \leqslant 7$$

$\therefore A = -1$

$B = 7$

b) USING THE POINT $(15,5)$ WE OBTAIN

$\Rightarrow 5 = 3 - 4\sin(2\times 15 + k)^\circ$

$\Rightarrow 2 = -4\sin(k+30)^\circ$

$\Rightarrow \sin(k+30)^\circ = -\frac{1}{2}$

$\arcsin\left(-\frac{1}{2}\right) = -30^\circ$

$\Rightarrow \begin{cases} k+30 = -30 \pm 360n \\ k+30 = 210 \pm 360n \end{cases}$

$\Rightarrow \begin{cases} k = -60 \pm 360n \\ k = 180 \pm 360n \end{cases}$

$k = -60$ $\qquad -90 < k < 90$

c) SOLVING THE EQUATION $f(x) = -1$, $0 \leqslant x \leqslant 360$

$\Rightarrow 3 - 4\sin(2x-60) = -1$

$\Rightarrow 4 = 4\sin(2x-60)$

$\Rightarrow \sin(2x-60) = 1$

$\arcsin 1 = 90$

$\begin{cases} 2x-60 = 90 \pm 360n \\ \cancel{2x-60 = 90 \pm 360n} \end{cases}$ $\quad n = 0,1,2,3,\ldots$

$\Rightarrow 2x = 150 \pm 360n$

$\Rightarrow x = 75 \pm 180n$

$\therefore x_1 = 75$

$x_2 = 255$

IYGB - MP1 PAPER Y - QUESTION 2

a) EXPANDING USING THE BINOMIAL FORMULA

$$(1+cx)^6 = 1 + \frac{6}{1}(cx)^1 + \frac{6\times5}{1\times2}(cx)^2 + \frac{6\times5\times4}{1\times2\times3}(cx)^3 + \dots$$

$$= 1 + 6cx + 15c^2x^2 + 20c^3x^3 + \dots$$

b) PROCEED AS FOLLOWS

$$\Rightarrow \left(a+\frac{b}{x}\right)(1+cx)^6 \equiv 74 - \frac{4}{x} - 576x + \dots$$

$$\Rightarrow \left(a+\frac{b}{x}\right)\left(1+6cx+15c^2x^2+20c^3x^3+\dots\right) \equiv 74 - \frac{4}{x} - 576x + \dots$$

$$\Rightarrow \begin{array}{l} a + 6acx + 15ac^2x^2 + 20ac^3x^3 + \dots \\ \frac{b}{x} + 6bc + 15bc^2x + 20bc^3x^2 + \dots \end{array} \equiv 74 - \frac{4}{x} - 576x + \dots$$

$$\Rightarrow \frac{b}{x} + (a+6bc) + (6ac+15bc^2)x + \dots \equiv 74 - \frac{4}{x} - 576x$$

$$\therefore\ b = -4 \qquad \left(\frac{b}{x} = -\frac{4}{x}\right)$$

ALSO WE HAVE

- $a + 6bc = 74$

$\Rightarrow a - 24c = 74$

$\Rightarrow a = 74 + 24c$

- $6ac + 15bc^2 = -576$

$\Rightarrow 6ac - 60c^2 = -576$

$\Rightarrow ac - 10c^2 = -96$

$(74+24c)c - 10c^2 = -96$

$\Rightarrow 74c + 24c^2 - 10c^2 = -96$

$\Rightarrow 14c^2 + 74c + 96 = 0$

$\Rightarrow 7c^2 + 37c + 48 = 0$

$\Rightarrow (7c+16)(c+3) = 0$

$\Rightarrow c = \begin{cases} -3 \\ -\frac{16}{7} \end{cases}$

MP1 PAPER Y - QUESTION 3

a) **THE TRANSFORMATIONS ARE SHOWN IN THE "CHAIN" BELOW**

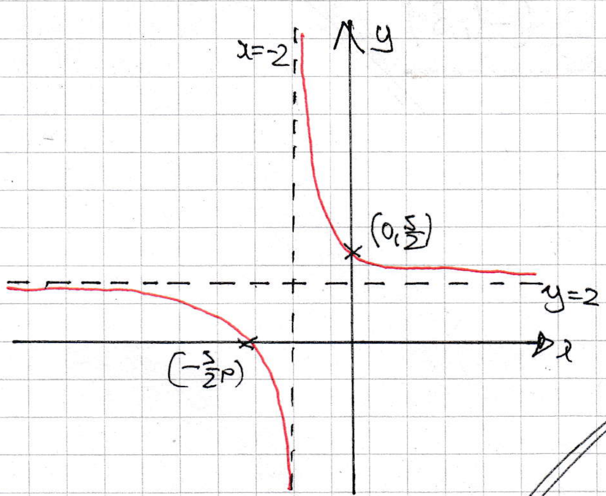
$$f(x) = \frac{1}{x} \longmapsto \frac{1}{(x+2)} \longmapsto \frac{1}{x+2} + 2 = g(x)$$

TRANSLATION, 2 UNITS, TO THE "LEFT" $f(x+2)$

TRANSLATION, 2 UNITS "UPWARDS"

$\therefore$ TRANSLATION BY THE VECTOR $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ //

b)



WHEN $x=0$

$y = \frac{1}{0+2} + 2 = \frac{1}{2} + 2 = \frac{5}{2}$

$\therefore (0, \frac{5}{2})$

WHEN $y=0$

$0 = \frac{1}{x+2} + 2$

$0 = 1 + 2(x+2)$

$0 = 1 + 2x + 4$

$2x = -5$

$x = -\frac{5}{2}$ $\therefore (-\frac{5}{2}, 0)$

//

c) **SOLVING SIMULTANEOUSLY**

$$\left.\begin{array}{l} y = \frac{1}{x+2} + 2 \\ 3y + x = 8 \end{array}\right\} \Rightarrow 3\left(\frac{1}{x+2} + 2\right) + x = 8$$

$$\Rightarrow \frac{3}{x+2} + 6 + x = 8$$

$$\Rightarrow \frac{3}{x+2} + x - 2 = 0$$

MULTIPLY THROUGH BY $(x+2)$

$$\Rightarrow 3 + x(x+2) - 2(x+2) = 0$$

$$\Rightarrow 3 + x^2 + 2x - 2x - 4 = 0$$

IYGB - MP1 PAPER Y - QUESTION 3

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \begin{cases} 1 \\ -1 \end{cases}$$

$$y = \frac{1}{1+2} + 2 = \frac{1}{3} + 2 = \frac{7}{3}$$

$$y = \frac{1}{-1+2} + 2 = 1 + 2 = 3$$

$$\therefore \left(1, \frac{7}{3}\right) \ \& \ (-1, 3)$$

IYGB – MP1 PAPER Y QUESTION 4

SOLVE THE TWO EQUATIONS TO "FIND" INTERSECTIONS

$$\left.\begin{array}{l} y = mx \\ x^2+y^2+2x-4y+1=0 \end{array}\right\} \Rightarrow x^2+(mx)^2+2x-4(mx)+1=0$$

$$\Rightarrow x^2+m^2x^2+2x-4mx+1=0$$

$$\Rightarrow (1+m^2)x^2+(2-4m)x+1=0$$

NOW IF THE LINE IS A TANGENT THIS QUADRATIC MUST HAVE REPEATED (TOUCHING POINT)

$$b^2-4ac=0 \Rightarrow (2-4m)^2-4(1+m^2)\times 1=0$$

$$\Rightarrow 4(1-2m)^2-4(1+m^2)=0$$

$$\Rightarrow (1-2m)^2-(1+m^2)=0$$

$$\Rightarrow 1-4m+4m^2-1-m^2=0$$

$$\Rightarrow 3m^2-4m=0$$

$$\Rightarrow m(3m-4)=0$$

$$m = \begin{cases} 0 \\ \frac{4}{3} \end{cases}$$

IF $m=0$, $y=0$

$x^2+2x+1=0$

$(x+1)^2=0$

$x=-1$

& $y=0$

$(-1,0)$

IF $m=\frac{4}{3}$, $y=\frac{4}{3}x$

$\left[1+\left(\frac{4}{3}\right)^2\right]x^2+\left[2-4\left(\frac{4}{3}\right)\right]x+1=0$

$\frac{25}{9}x^2-\frac{10}{3}x+1=0$

$25x^2-30x+9=0$

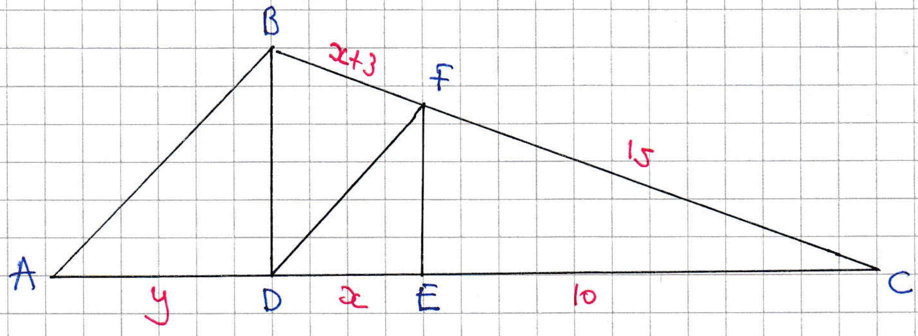
$(5x-3)^2=0$

$x=\frac{3}{5}$ & $y=\frac{4}{5}$

$\therefore \left(\frac{3}{5},\frac{4}{5}\right)$

IYGB - MP1 PAPER Y - QUESTION 5

a) START WITH A DRAWING



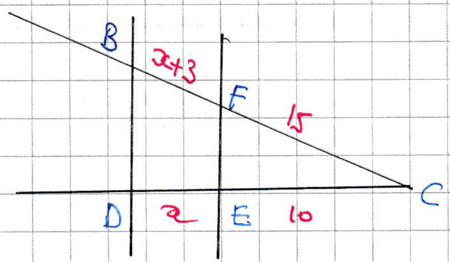
BY RATIOS

$$\frac{|BF|}{|FC|} = \frac{|DE|}{|EC|} \Rightarrow \frac{x+3}{15} = \frac{x}{10}$$

$$\Rightarrow 10x + 30 = 15x$$

$$\Rightarrow 30 = 5x$$

$$\Rightarrow x = 6$$



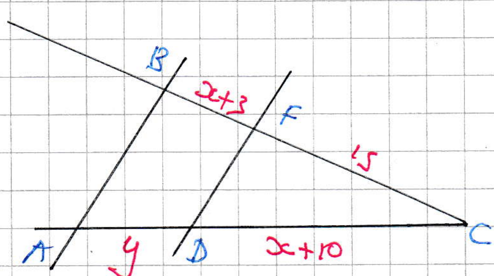
b) BY RATIOS AGAIN

$$\frac{|BF|}{|AD|} = \frac{|FC|}{|DC|} \Rightarrow \frac{x+3}{y} = \frac{15}{x+10}$$

$$\Rightarrow \frac{9}{y} = \frac{15}{16}$$

$$\Rightarrow 15y = 144$$

$$\Rightarrow y = 9.6$$ AS REQUIRED



c) FIRSTLY BY PYTHAGORAS

$$|FE|^2 + |EC|^2 = |FC|^2$$

$$|FE|^2 + 10^2 = 15^2$$

$$|FE|^2 = 125$$

$$|FE| = \sqrt{125}$$

IYGB - MP1 PAPER Y - QUESTION 5

NEXT BY RATIOS

$$\frac{|BD|}{|FE|} = \frac{|BC|}{|FC|} \implies \frac{|BD|}{5\sqrt{5}} = \frac{x+18}{15}$$

$$\implies \frac{|BD|}{5\sqrt{5}} = \frac{24}{15}$$

$$\implies |BD| = 8\sqrt{5}$$

FINALLY THE AREA

$$\text{AREA} = \frac{1}{2}|AC||BD|$$

$$= \frac{1}{2}(y+x+10)(8\sqrt{5})$$

$$= \frac{1}{2}(9.6+6+10)(8\sqrt{5})$$

$$= \frac{1}{2} \times 25.6 \times 8\sqrt{5}$$

$$= \frac{512\sqrt{5}}{5} \simeq 229 \quad (3 \text{ s.f.})$$

IYGB - MP1 PAPER Y - QUESTION 6

MANIPULATE AS FOLLOWS

$$x^2 + \frac{4000}{x} = \frac{x^3+4000}{x} = \frac{\left(2000^{\frac{1}{3}}\right)^3 + 4000}{2000^{\frac{1}{3}}}$$

$\sqrt[3]{2000}$

$$= \frac{2000+4000}{2000^{\frac{1}{3}}} = \frac{6000}{2000^{\frac{1}{3}}}$$

$$= \frac{6000 \times 2000^{\frac{2}{3}}}{2000^{\frac{1}{3}} \times 2000^{\frac{2}{3}}} = \frac{6000 \times 2000^{\frac{2}{3}}}{2000^1}$$

$$= 3 \times 2000^{\frac{2}{3}}$$

$$= 3 \times (2 \times 1000)^{\frac{2}{3}}$$

$$= 3 \times 2^{\frac{2}{3}} \times 1000^{\frac{2}{3}}$$

$$= 3 \times \left(\sqrt[3]{2}\right)^2 \times \left(\sqrt[3]{1000}\right)^2$$

$$= 3 \times \sqrt[3]{2^2} \times 10^2$$

$$= 3 \times 100 \times \sqrt[3]{4}$$

$$= 300 \times \sqrt[3]{4}$$

IYGB - MP1 PAPER Y - QUESTION 7

SOLVE THE LINEAR INEQUALITY FIRST

$\Rightarrow 5x+13 > 4(x+2)$

$\Rightarrow 5x+13 > 4x+8$

$\Rightarrow x > -5$

SO ONE OF THE CRITICAL VALUES NOW CAN BE ASSIGNED TO THE LINEAR INEQUALITY WHICH MEANS THAT $x=-\frac{17}{4}$ IS A CRITICAL VALUE OF THE QUADRATIC INEQUALITY

$$\Rightarrow (x-2)^2 - k(x-2)(x+3) < 0$$

SUB $x=-\frac{17}{4}$ & SOLVE AN EQUATION

$$\Rightarrow \left(-\frac{17}{4}-2\right)^2 - k\left(-\frac{17}{4}-2\right)\left(-\frac{17}{4}+3\right) = 0$$

$$\Rightarrow \left(-\frac{25}{4}\right)^2 - k\left(-\frac{25}{4}\right)\left(-\frac{5}{4}\right) = 0$$

$$\Rightarrow \frac{625}{16} - \frac{125}{16}k = 0$$

$$\Rightarrow 625 - 125k = 0$$

$$\Rightarrow 125k = 625$$

$$\Rightarrow k = 5$$

NEXT WE HAVE

$$\Rightarrow (x-2)^2 - 5(x-2)(x+3) < 0$$

$$\Rightarrow (x-2)\left[(x-2) - 5(x+3)\right] < 0$$

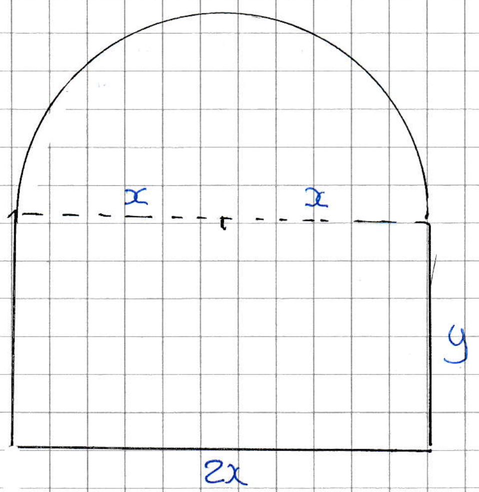
$$\Rightarrow (x-2)(-4x-17) < 0$$

$$C.V = \begin{cases} -17/4 \\ 2 \end{cases}$$

$\therefore m = 2$

IYGB - MP1 PAPER Y - QUESTION 8

a)



CONSTRAINT ON PERIMETER

$P = 6$

$2x + 2y + \frac{1}{2}(2\pi x) = 6$

$2x + 2y + \pi x = 6$

$2y = 6 - 2x - \pi x$ $\quad \times x$

$2xy = 6x - 2x^2 - \pi x^2$

$A = 2xy + \frac{1}{2}\pi x^2$

$A = (6x - 2x^2 - \pi x^2) + \frac{1}{2}\pi x^2$

$A = 6x - 2x^2 - \frac{1}{2}\pi x^2$

$A = 6x - \frac{1}{2}(4+\pi)x^2$ // AS REQUIRED

b) TO MAXIMIZE USE DIFFERENTIATION

$\frac{dA}{dx} = 6 - (4+\pi)x$

FOR MIN/MAX $\frac{dA}{dx} = 0$

$6 - (4+\pi)x = 0$

$6 = (4+\pi)x$

$x = \frac{6}{4+\pi} \approx 0.84\text{ m}$

IYGB - MP1 PAPER X - QUESTION 8

TO JUSTIFY THAT THIS VALUE OF x YIELDS A MAX

$$\frac{dA}{dx} = 6 - (4+\pi)x$$

$$\frac{d^2A}{dx^2} = -(4+\pi)$$

$$\left.\frac{d^2A}{dx^2}\right|_{x=\frac{6}{4+\pi}} = -(4+\pi) < 0$$

INDEED $x = \frac{6}{4+\pi}$ MAXIMIZES A

$A = 6x - \frac{1}{2}(4+\pi)x^2$

$$A_{MAX} = 6\left(\frac{6}{4+\pi}\right) - \frac{1}{2}(4+\pi)\left(\frac{6}{4+\pi}\right)^2$$

$$A_{MAX} = \frac{36}{4+\pi} - \frac{1}{2}\cancel{(4+\pi)} \times \frac{36}{(4+\pi)^{\cancel{2}}}$$

$$A_{MAX} = \frac{36}{4+\pi} - \frac{18}{4+\pi}$$

$$A_{MAX} = \frac{18}{4+\pi}$$

AS REQUIRED

IYGB - MP1 PAPER Y - QUESTION 9

LOOKING AT THE FIRST STAMP

$$V = Ae^{pt}$$

$t=0 \quad V = 16000 \quad$ (YEAR 2000)

$t=10 \quad V = 64000 \quad$ (YEAR 2010)

- $16000 = Ae^{p\times 0}$
  $16000 = A \times 1$
  $A = 16000$

- $64000 = 16000e^{10p}$
  $4 = e^{10p}$
  $p = \frac{1}{10}\ln 4$

LOOKING AT THE SECOND STAMP

$$U = Be^{2pt}$$

$t=0 \quad U = 2 \quad$ (YEAR 1990)

- $U = Be^{2pt}$
  $2 = Be^{0}$
  $B = 2$

NOW WE HAVE

$$V = 16000e^{\left(\frac{1}{10}\ln 4\right)t}$$

(t STARTS FROM 2000)

$$U = 2e^{\left(\frac{2}{10}\ln 4\right)t}$$

(t STARTS FROM 1990)

ADJUST THE TIME SO THEY BOTH START FROM 1990.

$$V = 16000e^{\left(\frac{2}{10}\ln 2\right)(t-10)} \quad (t \geqslant 10)$$

$$U = 2e^{\left(\frac{4}{10}\ln 2\right)t} \quad (t \geqslant 0)$$

SOLVING NOW YIELDS, IF $V = U$

$$\Rightarrow 16000e^{\left(\frac{2}{10}\ln 2\right)(t-10)} = 2e^{\left(\frac{4}{10}\ln 2\right)t}$$

IYGB - MP1 PAPER Y - QUESTION 9

$$\Rightarrow \quad 8000e^{\left(\frac{1}{5}\ln 2\right)(t-10)} = e^{\left(\frac{2}{5}\ln 2\right)t}$$

$$\Rightarrow \quad 8000\, e^{\left(\frac{1}{5}\ln 2\right)t - 2\ln 2} = e^{\left(\frac{2}{5}\ln 2\right)t}$$

$$\Rightarrow \quad 8000\, e^{\left(\frac{1}{5}\ln 2\right)t} \times e^{-2\ln 2} = e^{\left(\frac{2}{5}\ln 2\right)t}$$

$$\Rightarrow \quad 8000\left(e^{\ln 2}\right)^{\frac{1}{5}t} \times e^{\ln\frac{1}{4}} = \left(e^{\ln 2}\right)^{\frac{2}{5}t}$$

$$\Rightarrow \quad 8000 \times 2^{\frac{1}{5}t} \times \frac{1}{4} = 2^{\frac{2}{5}t}$$

$$\Rightarrow \quad 2000 \times 2^{\frac{1}{5}t} = 2^{\frac{2}{5}t}$$

DIVIDE BY $2^{\frac{1}{5}t}$

$$\Rightarrow \quad 2000 = 2^{\frac{1}{5}t}$$

TAKING LOGS (ANY BASE)

$$\Rightarrow \quad \ln 2000 = \ln\left(2^{\frac{1}{5}t}\right)$$

$$\Rightarrow \quad \ln 2000 = \frac{1}{5}t\ln 2$$

$$\Rightarrow \quad \left(\frac{1}{5}\ln 2\right)t = \ln 2000$$

$$\Rightarrow \quad t = \frac{5\ln 2000}{\ln 2} \simeq 54.82\ldots$$

$\therefore$ YEAR 2044

IYGB - MP1 PAPER Y - QUESTION 10

$$\sqrt{2}(a-1) + \sqrt{6}\,b = 2(1+\sqrt{3})$$
$$\sqrt{6}\,a - \sqrt{3}\,b = 2\sqrt{3}$$

SOLUTION BY ELIMINATION - MULTIPLY THE 2^{ND} EQUATION BY $\sqrt{2}$

$$\sqrt{2}(a-1) + \sqrt{6}\,b = 2(1+\sqrt{3})$$
$$\sqrt{12}\,a - \sqrt{6}\,b = 2\sqrt{6}$$

ADDING THE EQUATIONS

$$\Rightarrow \sqrt{2}(a-1) + \sqrt{12}\,a = 2(1+\sqrt{3}) + 2\sqrt{6}$$

$$\Rightarrow \sqrt{2}a - \sqrt{2} + 2\sqrt{3}a = 2 + 2\sqrt{3} + 2\sqrt{6}$$

$$\Rightarrow \sqrt{2}a + 2\sqrt{3}a = 2 + 2\sqrt{3} + 2\sqrt{6} + \sqrt{2}$$

$$\Rightarrow (\sqrt{2} + 2\sqrt{3})a = \sqrt{2} + 2\sqrt{3} + 2 + 2\sqrt{6}$$

$$\Rightarrow a = \frac{(\sqrt{2}+2\sqrt{3}) + 2 + 2\sqrt{6}}{\sqrt{2} + 2\sqrt{3}}$$

"SPLITTING THE FRACTION"

$$\Rightarrow a = 1 + \frac{2(1+\sqrt{6})}{2\sqrt{3} + \sqrt{2}}$$

$$\Rightarrow a = 1 + \frac{2(1+\sqrt{6})(2\sqrt{3}-\sqrt{2})}{(2\sqrt{3}+\sqrt{2})(2\sqrt{3}-\sqrt{2})}$$

$$\Rightarrow a = 1 + \frac{2(2\sqrt{3} - \sqrt{2} + 2\sqrt{18} - \sqrt{12})}{4\times3 - 2}$$

IYGB - MP1 PAPER Y - QUESTION 10

$$\Rightarrow a = 1 + \frac{2(\cancel{2\sqrt{3}} - \sqrt{2} + 2\times 3\sqrt{2} - \cancel{2\sqrt{3}})}{10}$$

$$\Rightarrow a = 1 + \frac{2\times 5\sqrt{2}}{10}$$

$$\Rightarrow a = 1 + \sqrt{2}$$

FINALLY TO FIND b

$$\Rightarrow \sqrt{6}a - \sqrt{3}b = 2\sqrt{3}$$

$$\Rightarrow \sqrt{6}(1+\sqrt{2}) - \sqrt{3}b = 2\sqrt{3}$$

$$\Rightarrow \sqrt{6} + \sqrt{12} - \sqrt{3}b = 2\sqrt{3}$$

$$\Rightarrow \sqrt{6} + \cancel{2\sqrt{3}} - \sqrt{3}b = \cancel{2\sqrt{3}}$$

$$\Rightarrow \sqrt{3}b = \sqrt{6}$$

$$\Rightarrow \cancel{\sqrt{3}}b = \cancel{\sqrt{3}}\sqrt{2}$$

$$\Rightarrow b = \sqrt{2}$$