CM

A Level Maths Question Countdown
10 days until the 1st exam

Information

- Each of the ten sheets will contain five pure questions and two applied questions.

Pure questions

- Two of the pure questions will be 'standard'.
- Two of the pure questions will be 'problems'.
- The last pure question will involve modelling.

Applied questions

- One of the questions will focus on statistics.
- One of the questions will focus on mechanics.
- On alternate days, the statistics question will look at the large data set. Note that these questions may be brief as opposed to full length exam questions.

Notes to self	

Pure questions – standard

1 $f(x) = 4x^3 + 3x^2 - 9x + 2$

(a) Show that $(x + 2)$ is a factor of $f(x)$.

(b) Hence express $f(x)$ as a product of three linear factors.

(c) Showing your working clearly, simplify

$$\frac{2x^2 - 3x + 1}{4x^3 + 3x^2 - 9x + 2} + \frac{2}{36x - 9}$$

giving your answer as a single fraction in its simplest form.

2 The curve C is defined parametrically by

$$x = 3\ln(1 - 2t), \quad y = 5e^t, \quad t < \frac{1}{2}$$

Find the equation of the tangent to C at the point where $x = 0$.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

Pure questions – problems

3 Showing your working clearly, show that

$$\int_2^{\frac{7}{2}} \frac{12}{4t^2 - 9}\, dt = \ln\frac{14}{5}$$

4 The first term in a geometric series is 5 and the common ratio of the series is r.

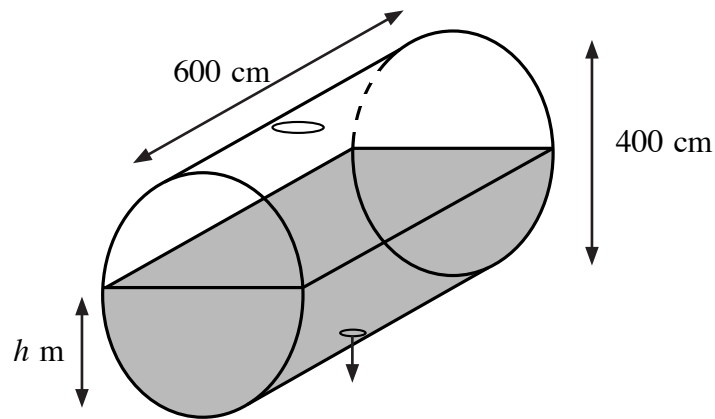(a) **Prove** that the sum of the first N terms of this series, S_N, is given by

$$S_N = \frac{5(1 - r^N)}{1 - r}$$

Given that $S_\infty = 6$,

(b) explain why $|r| < 1$,

(c) and find the value of r.

5

The diagram above shows a cylindrical tank of diameter 400 cm and length 600 cm which contains some liquid. The tank is vented at the top and contains an outlet at the bottom through which the liquid can drain. At time $t = 0$, the outlet is opened and liquid begins to drain.

The depth of the liquid in the tank h cm at time t s is modelled by the equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{k}{\sqrt{400 - h}}, \quad t \geqslant 0$$

where k is a constant.

Given that, when $h = 300$, the decrease in the depth of liquid in the tank is 42 cm^2 per second,
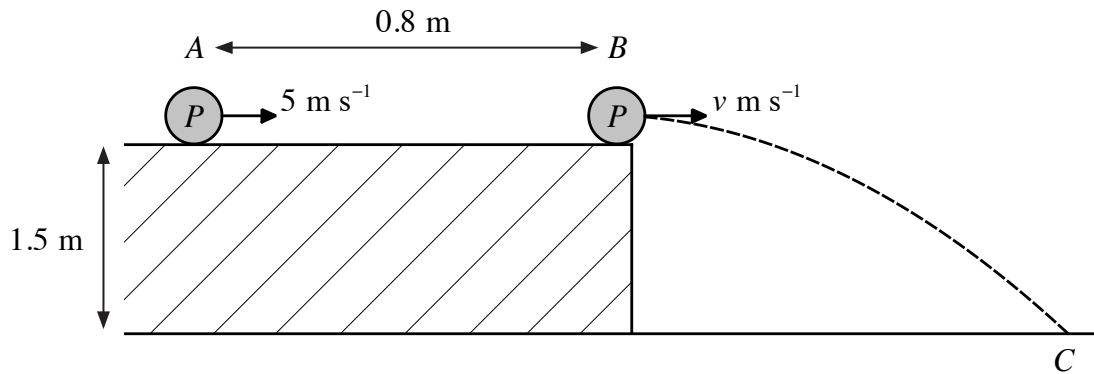
(a) find the value of k.

Using this value of k and assuming the tank is initially completely filled with liquid,

(b) solve the differential equation to obtain a particular solution to the problem,

(c) and hence determine the time taken for the tank to empty.

Applied questions – mechanics

6

A particle P is moving on a horizontal table whose surface is 1.5 m above the ground. The points A and B lie on the table with $AB = 0.8$ m and B at the edge of the table.

Particle P passes through A at a speed of 5 m s^{-1} and leaves the table at B with speed v m s^{-1}. In the subsequent motion after leaving the table, P moves freely under the influence of gravity until it reaches the ground at C.

In a model for this situation, the horizontal table is assumed to be a smooth surface and it is assumed that at the instant when P passes through B, its velocity is parallel to the table. The acceleration due to gravity is taken to be 9.8 m s^{-2}.

Given that the acceleration of P when it is on the table is 1.2 m s^{-2},

(a) show that $v = 5.2$.

(b) Determine the magnitude and direction of the velocity of P at the instant when it reaches C.

(c) Calculate the total time taken for P to move from A to C.

The model is refined so that the table is now modelled as a rough surface.

(d) Without further calculation, explain how this will affect your answer to part (a).

Applied questions – statistics

7 Tom is investigating the variation of daily total sunshine, s hours, in Leuchars. Using one of the data sheets for Leuchars in the large data set, he obtains the following summary statistics for s

$$\sum s = 813.2 \qquad \sum s^2 = 6184.22 \qquad n = 167$$

(a) Using your knowledge of the large data set, explain why Tom must have cleaned the data before producing the summary statistics above.

(b) Use the summary statistics to calculate the mean and standard deviation of the summarised data.

(c) Use your knowledge of the large data set to give two reasons why the answers to part (b) may not be a good representation of the overall daily total sunshine in Leuchars in 2019.

Tom assumes that s is a normally distributed random variable.

(d) Using your answers to part (b), find $P(s > 5)$.
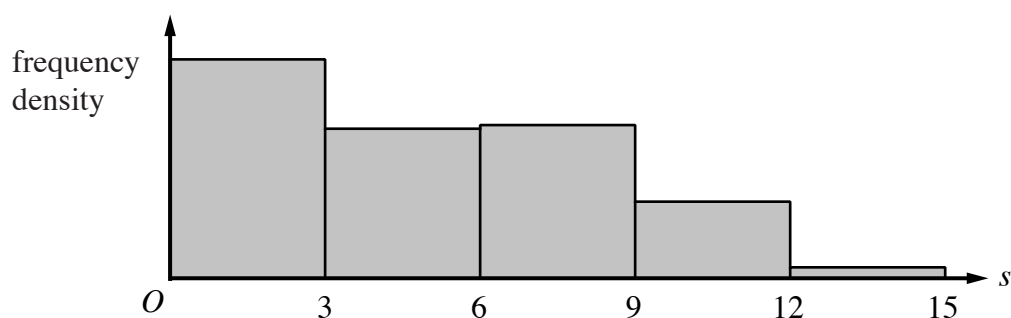
The diagram above shows a histogram of the summarised data.

(e) With reference to this histogram, comment on Tom's assumption that s is normally distributed.