1. (a) Expand and simplify $(7 + \sqrt{5})(3 - \sqrt{5})$.

    (3)

    (b) Express $\dfrac{7 + \sqrt{5}}{3 + \sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

    (3)
    (Total 6 marks)

2. Given that

    $$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

    (a) express $f(x)$ in the form $(x - a)^2 + b$, where a and b are integers.

    (3)

    The curve C with equation $y = f(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

    (b) Sketch the graph of C, showing the coordinates of P and Q.

    (4)

    The line $y = 41$ meets C at the point R.

    (c) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers.

    (5)
    (Total 12 marks)

3. Solve the equation $2^{1-x} = 4^x$.

    (Total 3 marks)

4. Find the value of

    (a) $81^{\frac{1}{2}}$,

    (1)

    (b) $81^{\frac{3}{4}}$,

    (2)

    (c) $81^{-\frac{3}{4}}$.

    (1)
    (Total 4 marks)

5. $$f(x) = x^2 + 4kx + (3 + 11k), \quad \text{where } k \text{ is a constant.}$$

(a) Express f(x) in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k.

(3)

Given that the equation f(x) = 0 has no real roots,

(b) find the set of possible values of k.

(4)

Given that $k = 1$,

(c) sketch the graph of y = f(x), showing the coordinates of any point at which the graph crosses a coordinate axis.

(3)
(Total 10 marks)

6. The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(Total 4 marks)

7. The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

(b) Hence find the set of possible values of k.

(4)
(Total 7 marks)

8.

$$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

(a) Find the values of the constants a and b.

(2)

(b) In the space provided below, sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(3)

(c) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (b).

(2)

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(d) Find the set of possible values of k, giving your answer in surd form.

(4)
(Total 11 marks)