IYGB – MP1 PAPER W – QUESTION 1

IF $z_1 = -i$ IS A SOLUTION THEN $z+i$ MUST BE A FACTOR

$$\Longrightarrow (z+i)(z+A+Bi) \equiv z^2 - 2z + 1 - 2i$$

$$\Longrightarrow \begin{array}{l} z^2 + Az + Biz \\ \qquad\qquad + iz + Ai - B \end{array} \equiv z^2 - 2z + 1 - 2i$$

$$\Longrightarrow z^2 + Az + (B+1)iz + Ai - B \equiv z^2 - 2z + 1 - 2i$$

AS A & B ARE REAL, $A = -2$
$B = -1$

$\therefore z_2 = 2+i$

ALTERNATIVE BY CONSIDERING POLYNOMIAL ROOTS

$$z^2 - 2z + 1 - 2i = 0$$

$$\alpha + \beta = -\frac{b}{a}$$

$$-i + \beta = -\frac{-2}{1}$$

$$-i + \beta = 2$$

$$\beta = 2 + i$$

IYGB – FP1 PAPER W – QUESTION 2

WORKING IN PARAMETRIC

$$V = \pi \int_{y_1}^{y_2} [x(y)]^2 \, dy = \pi \int_{\theta_1}^{\theta_2} [x(\theta)]^2 \frac{dy}{d\theta} \, d\theta$$

TRANSFORMING THE INTEGRAL

$y = 1$ $\qquad$ $y = 3$

$\sqrt{3}\tan\theta = 1$ $\qquad$ $\sqrt{3}\tan\theta = 3$

$\tan\theta = \frac{1}{\sqrt{3}}$ $\qquad$ $\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{6}$ $\qquad$ $\theta = \frac{\pi}{3}$

HENCE WE HAVE

$$V = \pi \int_{\pi/6}^{\pi/3} \underbrace{(2\cos^2\theta)^2}_{x^2} \underbrace{(\sqrt{3}\sec^2\theta)}_{\frac{dy}{d\theta}} \, d\theta = 4\sqrt{3}\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^4\theta \sec^2\theta \, d\theta$$

$$= 4\sqrt{3}\pi \int_{\pi/6}^{\pi/3} \cos^2\theta \, d\theta = 4\sqrt{3}\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{2} + \frac{1}{2}\cos 2\theta \, d\theta$$

$$= 4\sqrt{3}\pi \left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta \right]_{\pi/6}^{\pi/3}$$

$$= 4\sqrt{3}\pi \left[\left(\frac{\pi}{6} + \cancel{\frac{1}{4}\frac{\sqrt{3}}{2}} \right) - \left(\frac{\pi}{12} + \cancel{\frac{1}{4}\frac{\sqrt{3}}{2}} \right) \right]$$

$$= 4\sqrt{3}\pi \times \frac{\pi}{12}$$

$$= \frac{\sqrt{3}\pi^2}{3} = \frac{\sqrt{3}\pi^2\sqrt{3}}{3\sqrt{3}} = \frac{3\pi^2}{3\sqrt{3}}$$

$$= \frac{\pi^2}{\sqrt{3}}$$

AS REQUIRED

IYGB – FP1 PAPER W – QUESTION 3

METHOD A

$$\underline{w} = \underline{u} + \underline{v}$$

DOTTING THE EQUATION BY $\underline{u}$

$$\underline{w} \cdot \underline{u} = \underline{u} \cdot \underline{u} + \cancel{\underline{v} \cdot \underline{u}}$$

PERPENDICULAR

LET $\underline{u} = \lambda(1,1,1)$, $\lambda \neq 0$

$$\Longrightarrow \begin{pmatrix} 2 \\ 8 \\ -1 \end{pmatrix} \cdot \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\Longrightarrow \lambda(2+8-1) = \lambda^2(1+1+1)$$

$$\Longrightarrow 9\lambda = 3\lambda^2$$

$$\Longrightarrow 3 = \lambda \qquad (\lambda \neq 0)$$

FINALLY AS $\underline{w} = \underline{u} + \underline{v}$

$$\begin{pmatrix} 2 \\ 8 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} + \underline{v}$$

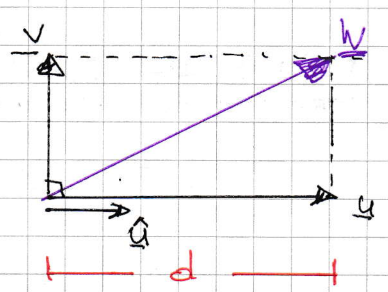
$$\therefore \underline{v} = \begin{pmatrix} -1 \\ 5 \\ -4 \end{pmatrix} \quad \& \quad \underline{u} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$$

IYGB - FP1 PAPER W - QUESTION 3

METHOD B

START WITH A DIAGRAM WITH $\underline{u}$ & $\underline{v}$ PERPENDICULAR, AND $\underline{w}$ THEIR "RESULTANT"

PROJECT $\underline{w}$ ONTO THE DIRECTION OF $\underline{u}$



$$d = \underline{w} \cdot \hat{\underline{u}}$$

THUS WE HAVE

$$\Rightarrow \underline{u} = d\,\hat{\underline{u}}$$

$$\Rightarrow \underline{u} = (\underline{w} \cdot \hat{\underline{u}})\,\hat{\underline{u}}$$

$$\Rightarrow \underline{u} = \left[(2,8,-1) \cdot \frac{(1,1,1)}{\sqrt{3}}\right] \frac{(1,1,1)}{\sqrt{3}}$$

$$\Rightarrow \underline{u} = \frac{1}{3}\left[(2,8,-1) \cdot (1,1,1)\right](1,1,1)$$

$$\Rightarrow \underline{u} = \frac{1}{3}(2+8-1)(1,1,1)$$

$$\Rightarrow \underline{u} = 3(1,1,1)$$

$$\Rightarrow \underline{u} = (3,3,3)$$

& FINALLY SINCE $\underline{w} = \underline{u} + \underline{v}$

$$(2,8,-1) = (3,3,3) + \underline{v}$$

$$\underline{v} = (-1,5,-4)$$

IYGB - FP1 PAPER W - QUESTION 3

METHOD C

LET $\underline{v} = (x,y,z)$, $\underline{u} = \lambda(1,1,1)$, $\lambda \neq 0$

$$\underline{v} \perp \underline{u} \implies (x,y,z)\cdot\lambda(1,1,1) = 0$$
$$\implies \lambda(x+y+z) = 0$$
$$\implies \underline{x+y+z=0}$$

NOW WE HAVE $\underline{w} = \underline{u} + \underline{v}$

$$\implies (2,8,-1) = (\lambda,\lambda,\lambda) + (x,y,z)$$

$$\implies \left\{\begin{array}{l} x+\lambda = 2 \\ y+\lambda = 8 \\ z+\lambda = -1 \end{array}\right\}$$

ADDING THESE EQUATIONS

$$x+y+z+3\lambda = 9$$
$$0 + 3\lambda = 9$$
$$\lambda = 3$$

$$\therefore \quad \underline{u} = (3,3,3) \quad \& \quad \underline{v} = (x,y,z) = (-1,5,-4)$$

IYGB - FP1 PAPER W - QUESTION 4

CHECK THE BASE CASE, $n=2$

$$\frac{d^2}{dx^2}(\sin 3x) = \frac{d}{dx}(3\cos 3x) = -9\sin 3x$$

$$(-1)^{\frac{2}{2}} \times 3^2 \times \sin 3x = (-1)^1 \times 9 \times \sin 3x = -9\sin 3x$$

IE THE RESULT HOLDS FOR $n=2$

SUPPOSE THAT THE RESULT HOLDS FOR $n=k=2m$, $m \in \mathbb{N}$

$$\frac{d^k}{dx^k}(\sin 3x) = (-1)^{\frac{k}{2}} \times 3^k \times \sin 3x$$

$$\frac{d^{k+1}}{dx^{k+1}}(\sin 3x) = \frac{d}{dx}\left[(-1)^{\frac{k}{2}} \times 3^k \times \sin 3x\right] = (-1)^{\frac{k}{2}} \times 3^k \times 3\cos 3x$$

$$\frac{d^{k+2}}{dx^{k+2}}(\sin 3x) = \frac{d}{dx}\left[(-1)^{\frac{k}{2}} \times 3^k \times 3\cos 3x\right] = (-1)^{\frac{k}{2}} \times 3^k \times (-9\sin 3x)$$

$$\frac{d^{k+2}}{dx^{k+2}} = (-1)^{\frac{k}{2}}(-1) \times 3^k \times 3^2 \times \sin 3x$$

$$\frac{d^{k+2}}{dx^{k+2}}(\sin 3x) = (-1)^{\frac{k}{2}+1} \times 3^{k+2} \times \sin 3x$$

$$\frac{d^{k+2}}{dx^{k+2}}(\sin 3x) = (-1)^{\frac{1}{2}(k+2)} \times 3^{k+2} \times \sin 3x$$

IF THE RESULT HOLDS FOR $n=k=2m$, THEN IT MUST HOLD FOR $n=k+2=2(m+1)$.

AS THE RESULT HOLD FOR $n=2$, THEN IT MUST HOLD FOR ALL EVEN INTEGERS

IYGB - FP1 PAPER W - QUESTION 5

a) LET A VECTOR PERPENDICULAR TO BOTH VECTORS BE (x,y,z)

$$\left.\begin{array}{l}(x,y,z)\cdot(-3,4,-7)=0\\(x,y,z)\cdot(2,-2,3)=0\end{array}\right\} \Longrightarrow \begin{array}{l}-3x+4y-7z=0\\2x-2y+3z=0\end{array}$$

LET $z=1$

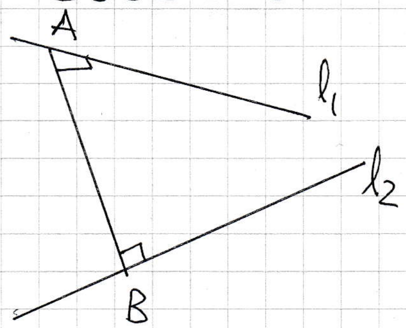
$$\left.\begin{array}{l}-3x+4y-7=0\\2x-2y+3=0\end{array}\right\} \Longrightarrow \begin{array}{l}-3x+4y=7\\4x-4y=-6\end{array}$$

$$x=1$$

$$y=\frac{5}{2}$$

$\therefore$ A "NORMAL" VECTOR WILL BE $(1,\frac{5}{2},1)$ OR $(2,5,2)$

b) LOOKING AT THE DIAGRAM



$\underline{r}_1=(0,-13,1)+\lambda(-3,4,-7)$

$\underline{r}_2=(5,25,0)+\mu(2,-2,3)$

$\underline{r}_1=(-3\lambda,4\lambda-13,-7\lambda+1)$

$\underline{r}_2=(2\mu+5,25-2\mu,3\mu)$

- LET $\lambda=a$ AT A

  LET $\mu=b$ AT B

- $\underline{a}=(-3a,4a-13,7a+1)$

  $\underline{b}=(2b+5,25-2b,3b)$

- $\overrightarrow{AB}=\underline{b}-\underline{a}$

$$=(2b+5,25-2b,3b)-(3a,4a-13,-7a+1)$$

$$=(2b+3a+5,-2b-4a+38,3b+7a-1)$$

IYGB – FP1 PAPER W – QUESTION 5

NOW $\vec{AB}$ MUST BE PARALLEL TO THE NORMAL VECTOR

$\vec{AB} = k(2,5,2)$ FOR $k \neq 0$

$$\left.\begin{aligned} 3a+2b+5 &= 2k \\ -4a-2b+38 &= 5k \\ 7a+3b-1 &= 2k \end{aligned}\right\} \Rightarrow \begin{aligned} 15a+10b+25 &= 10k \quad ① \\ -8a-4b+76 &= 10k \quad ② \\ 35a+15b-5 &= 10k \quad ③ \end{aligned}$$

$$\left.\begin{aligned} \underset{①}{15a+10b+25} &= \underset{②}{-8a-4b+76} \\ \underset{③}{35a+15b-5} &= \underset{②}{-8a-4b+76} \end{aligned}\right\} \Rightarrow \begin{aligned} 23a+14b &= 51 \quad (\times 19) \\ 43a+19b &= 81 \quad (\times 14) \end{aligned}$$

$$\left.\begin{aligned} 437a+266b &= 969 \\ 602a+266b &= 1134 \end{aligned}\right\} \Rightarrow \begin{aligned} 165a &= 165 \\ \underline{a} &\underline{= 1} \end{aligned}$$

& $23\times1 + 14b = 51$

$14b = 28$

$\underline{b = 2}$

$\therefore A(-3,-9,-6)$ & $B(9,21,6)$

IYGB - FP1 PAPER W - QUESTION 6

a) BY COMPARING ELEMENTS IN THE MATRIX EQUATION

$$\underline{A}^2 + k\underline{I} = h\underline{A}$$

$$\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} + k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = h\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 23 & 16 \\ 56 & 39 \end{pmatrix} + \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = \begin{pmatrix} 3h & 2h \\ 7h & 5h \end{pmatrix}$$

LOOKING AT a_{12}

$$16 + 0 = 2h$$

$$h = 8$$

LOOKING AT a_{11}

$$23 + k = 3h$$

$$23 + k = 24$$

$$k = 1$$

b) USING THE EQUATION OF PART

$$\underline{A}^2 + \underline{I} = 8\underline{A}$$

$$\underline{A}^2\underline{A}^{-1} + \underline{I}\underline{A}^{-1} = 8\underline{A}\underline{A}^{-1}$$

$$\underline{A} + \underline{A}^{-1} = 8\underline{I}$$

$$\underline{A}^{-1} = 8\underline{I} - \underline{A}$$

$$\underline{A}^{-1} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$$

$$\underline{A}^{-1} = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$$

or

$$\underline{A}^{-1}\underline{A}^2 + \underline{A}^{-1}\underline{I} = 8\underline{A}^{-1}\underline{A}$$

$$\underline{A} + \underline{A}^{-1} = 8\underline{I}$$

IYGB - FP1 PAPER W - QUESTION 7

a) FIRSTLY LET US NOTE $z \neq -1$, BY INSPECTION

$$\Rightarrow \quad z + \frac{1}{z} = -1$$

$$\Rightarrow \quad z^2 + 1 = -z$$

$$\Rightarrow \quad z^2 + z + 1 = 0$$

$$\Rightarrow \quad (z-1)(z^2+z+1) = 0$$

$$\Rightarrow \quad \begin{matrix} z^3 + z^2 + z \\ \quad -z^2 - z - 1 \end{matrix} = 0$$

$$\Rightarrow \quad z^3 - 1 = 0$$

$$\Rightarrow \quad z^3 = 1$$

b) PROCEED AS FOLLOWS

$$z^8 + z^4 + 4 = z^6 z^2 + z^3 z + 4$$

$$= (z^3)^2 z^2 + (z^3) z + 4$$

BUT $z^3 = 1$

$$= z^2 + z + 4$$

$$= (z^2 + z + 1) + 3 \qquad \text{FROM PART (a)}$$

$$= 0 + 3$$

$$= 3$$

$$\therefore \quad z^8 + z^4 + 4 = 3$$

$$z^8 + z^4 = -1$$

IYGB - FP1 PAPER W - QUESTION 8

METHOD A

REGROUP THE TERMS

$$99^2 - 97^2 + 95^2 - 93^2 + 91^2 - \dots + 3^2 - 1^2$$

$$= \left[99^2 + 95^2 + 91^2 + \dots + 3^2\right] - \left[97^2 + 93^2 + 89^2 \dots + 1^2\right]$$

$$= \sum_{r=1}^{25} (4r-1)^2 - \sum_{r=1}^{25} (4r-3)^2$$ (WRITE IN SIGMA NOTATION)

$$= \sum_{r=1}^{25} \left[(4r-1)^2 - (4r-3)^2\right]$$ (COMBINE SUMMATIONS)

$$= \sum_{r=1}^{25} (4r-1+4r-3)(\cancel{4r}-1-\cancel{4r}+3)$$ (DIFFERENCE OF SQUARES)

$$= \sum_{r=1}^{25} (8r-4)\times 2$$

$$= \sum_{r=1}^{25} (16r-8)$$

$$= 16\sum_{r=1}^{25} r - 8\sum_{r=1}^{25} 1$$ (LINEARITY OF THE OPERATOR)

$$= 16 \times \frac{1}{2} \times 25 \times 26 - 8 \times 25$$ $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

$$= 5200 - 200$$

$$= 5000$$

IYGB – FP1 PAPER W – QUESTION 8

METHOD B

REGROUP THE TERMS AS FOLLOWS

$$= 99^2 - 97^2 + 95^2 - 93^2 + \dots + 3^2 - 1^2$$

$$= (99^2 - 97^2) + (95^2 - 93^2) + (91^2 - 89^2) + \dots + (3^2 - 1^2)$$

$$= (99-97)(99+97) + (95-93)(95+93) + (91-89)(91+89) + \dots (3+1)(3-1)$$

$$= 2(196) + 2(188) + 2(180) + 2(172) + \dots + 2(4)$$

$$= 2\left[4 + 12 + 20 + \dots + 180 + 188 + 196\right]$$

$$= 2 \times 4\left[1 + 3 + 5 + \dots + 45 + 47 + 49\right]$$

$= 8 \times$ ARITHMETIC PROGRESSION WITH $a = 1$, $d = 2$, $n = 25$

$u_n = a + (n-1)d$
$49 = 1 + (n-1) \times 2$
$49 = 1 + 2n - 2$
$50 = 2n$
$n = 25$

$$S_n = \frac{n}{2}\left[a + L\right]$$

$$= 8 \times \frac{25}{2}\left[1 + 49\right]$$

$$= 8 \times \frac{25 \times 50}{2}$$

$$= 5000$$

AS BEFORE

IYGB - FP1 PAPER W - QUESTION 9

PROCEED AS FOLLOWS

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1+\lambda+\mu \\ \lambda-\mu \\ 1-2\lambda+\mu \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 1+\cancel{\lambda}+\cancel{\mu}+\cancel{\lambda}-\cancel{\mu}+1-\cancel{2\lambda}+\mu \\ \lambda-\mu \\ 2+2\lambda+2\mu+\lambda-\mu \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 2+\mu \\ \lambda-\mu \\ 2+3\lambda+\mu \end{pmatrix}$$

ELIMINATE THE PARAMETRICS INTO CARTESIAN

$$\left.\begin{aligned} X &= 2+\mu \\ Y &= \lambda-\mu \\ Z &= 2+3\lambda+\mu \end{aligned}\right\} \implies \mu = X-2$$

SUBSTITUTE INTO THE OTHER TWO

$$Y = \lambda-(X-2) = 2+\lambda-X$$
$$Z = 2+3\lambda+X-2 = 3\lambda+X$$

SOLVE THE "TOP" EQUATION FOR λ

$$\lambda = X+Y-2$$

SUBSTITUTE INTO THE LAST

$$Z = 2+3(X+Y-2)+X-2$$
$$Z = \cancel{2}+3X+3Y-6+X-\cancel{2}$$
$$Z = 4X+3Y-6$$
$$4X+3Y-Z=6$$

IYGB - FP1 PAPER W - QUESTION 10

FOR THE CUBIC $x^3-4x^2-3x-2=0$

$$\alpha+\beta+\gamma = -\frac{b}{a} = -\frac{-4}{1} = 4$$
$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a} = \frac{-3}{1} = -3$$
$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{-2}{1} = 2$$

LET THE THREE ROOTS OF THE REQUIRED CUBIC BE

- $A = \alpha+\beta$
- $B = \beta+\gamma$
- $C = \gamma+\alpha$

- $A+B+C = (\alpha+\beta)+(\beta+\gamma)+(\gamma+\alpha)$
$$= 2(\alpha+\beta+\gamma)$$
$$= 2\times4$$
$$= 8$$

- $AB+BC+CA = (\alpha+\beta)(\beta+\gamma)+(\beta+\gamma)(\gamma+\alpha)+(\gamma+\alpha)(\alpha+\beta)$
$$= \alpha\beta+\alpha\gamma+\beta^2+\beta\gamma$$
$$\alpha\beta+\alpha\gamma+\gamma^2+\beta\gamma$$
$$\alpha\beta+\alpha\gamma+\alpha^2+\beta\gamma$$
$$= (\alpha^2+\beta^2+\gamma^2)+3(\alpha\beta+\beta\gamma+\gamma\alpha)$$
$$= (\alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\beta\gamma+2\gamma\alpha)+(\alpha\beta+\beta\gamma+\gamma\alpha)$$
$$= (\alpha+\beta+\gamma)^2+(\alpha\beta+\beta\gamma+\gamma\alpha)$$
$$= 4^2+(-3)$$
$$= 16-3$$
$$= 13$$

IYGB - FP1 PAPER W - QUESTION 10

- $ABC = (\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$

$= (\alpha+\beta)(\beta\gamma+\alpha\beta+\alpha\gamma+\gamma^2)$

$= \alpha\beta\gamma+\alpha^2\beta+\alpha^2\gamma+\alpha\gamma^2+\beta^2\gamma+\alpha\beta^2+\alpha\beta\gamma+\beta\gamma^2$

$= 2\alpha\beta\gamma+\alpha^2\beta+\alpha\beta^2+\alpha^2\gamma+\alpha\gamma^2+\beta\gamma^2+\beta^2\gamma$

$= 2\alpha\beta\gamma+\alpha\beta(\alpha+\beta)+\alpha\gamma(\alpha+\gamma)+\beta\gamma(\beta+\gamma)$

$= 2\alpha\beta\gamma+\alpha\beta(\alpha+\beta+\gamma)-\alpha\beta\gamma$
$\quad +\alpha\gamma(\alpha+\gamma+\beta)-\alpha\beta\gamma$
$\quad +\beta\gamma(\beta+\gamma+\alpha)-\alpha\beta\gamma$

$= (\alpha\beta+\alpha\gamma+\beta\gamma)(\alpha+\beta+\gamma)-\alpha\beta\gamma$

$= -3\times 4 - 2$

$= -14$

HENCE THE REQUIRED CUBIC WILL BE

$$x^3-(A+B+C)x^2+(AB+BC+CD)x-(ABC)=0$$

$$x^3-8x^2+13x+14=0$$

AS REQUIRED