C3 Trigonometry

(1)

a) $\sec^2 x - \operatorname{cosec}^2 x$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$(1 + \tan^2 x) - (1 + \cot^2 x)$

$1 + \tan^2 x - 1 - \cot^2 x$

$\tan^2 x - \cot^2 x$

(1)

b) $2\cot^2\theta = 7\operatorname{cosec}\theta - 8 \qquad 0 < \theta < 180$

$2(\operatorname{cosec}^2\theta - 1) = 7\operatorname{cosec}\theta - 8$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$2\operatorname{cosec}^2\theta - 2 = 7\operatorname{cosec}\theta - 8$

$2\operatorname{cosec}^2\theta - 7\operatorname{cosec}\theta + 6 = 0$

let $y = \operatorname{cosec}\theta$

$2y^2 - 7y + 6 = 0$

$(2y - 3)(y - 2) = 0$

$y = \frac{3}{2} \qquad y = 2$

$\operatorname{cosec}\theta = \frac{3}{2} \qquad \operatorname{cosec}\theta = 2$

$\frac{1}{\sin\theta} = \frac{3}{2} \qquad \frac{1}{\sin\theta} = 2$

$\sin\theta = \frac{2}{3} \qquad \sin\theta = \frac{1}{2}$

$\theta = \underline{41.8} \qquad \theta = \underline{30}$

$\theta = 180 - 41.8 \qquad \theta = 180 - 30$

$= \underline{138.2} \qquad = \underline{150}$

(2)

a) $\sin^2\theta + \cos^2\theta \equiv 1$

$\div \sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} \equiv \frac{1}{\sin^2\theta}$$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$1 + \cot^2\theta \equiv \frac{1}{\sin^2\theta}$$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$$1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$$

(2)

b) $2\cot^2\theta - 9\operatorname{cosec}\theta = 3 \qquad 0 \leq \theta < 180$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$2(\operatorname{cosec}^2\theta - 1) - 9\operatorname{cosec}\theta = 3$

$2\operatorname{cosec}^2\theta - 2 - 9\operatorname{cosec}\theta = 3$

$2\operatorname{cosec}^2\theta - 9\operatorname{cosec}\theta - 5 = 0$

$y = \operatorname{cosec}\theta$

$2y^2 - 9y - 5 = 0$

$(2y + 1)(y - 5) = 0$

$y = -\frac{1}{2}$	$y = 5$
$\operatorname{cosec}\theta = -\frac{1}{2}$	$\operatorname{cosec}\theta = 5$
	$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$
$\frac{1}{\sin\theta} = -\frac{1}{2}$	$\frac{1}{\sin\theta} = 5$
$\sin\theta = -2$	$\sin\theta = \frac{1}{5}$
No solutions	$\theta = 11.5^\circ$
	$\theta = 180 - 11.5 = 168.5^\circ$

(3)

a) $\sin^2\theta + \cos^2\theta \equiv 1$

$(\div \cos^2\theta)$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} \equiv \frac{1}{\cos^2\theta}$$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

$\frac{1}{\cos\theta} = \sec\theta$

$$\tan^2\theta + 1 \equiv \sec^2\theta$$

$$1 + \tan^2\theta \equiv \sec^2\theta$$

(3)
b)

$2\tan^2\theta + \sec\theta = 1 \qquad 0 \leq \theta < 360$

$1 + \tan^2\theta = \sec^2\theta$

$2(\sec^2\theta - 1) + \sec\theta = 1$

$2\sec^2\theta - 2 + \sec\theta = 1$

$2\sec^2\theta + \sec\theta - 3 = 0$

$y = \sec^2\theta$

$2y^2 + y - 3 = 0$

$(2y + 3)(y - 1) = 0$

$y = \frac{-3}{2} \qquad y = +1$

$\therefore \quad \sec\theta = \frac{-3}{2} \qquad \sec\theta = 1$

$\sec\theta = \frac{1}{\cos\theta}$

$\frac{1}{\cos\theta} = -\frac{3}{2} \qquad \frac{1}{\cos\theta} = 1$

$\cos\theta = -\frac{2}{3} \qquad \cos\theta = 1$

$\theta = 131.8^\circ \qquad \theta = 0^\circ$

$= 360 - 131.8$

$= 228.2^\circ$

(4)

a) $\sin^2\theta + \cos^2\theta \equiv 1$

$(\div \sin^2\theta)$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} \equiv \frac{1}{\sin^2\theta}$$

$\frac{\cos\theta}{\sin\theta} = \cot\theta$

$\frac{1}{\sin\theta} = \operatorname{cosec}\theta$

$$1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$$

$$\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$$

(4)

b) $\operatorname{cosec}^4\theta - \cot^4\theta$

$= \operatorname{cosec}^2\theta \operatorname{cosec}^2\theta - \cot^2\theta \cot^2\theta$

$= (1 + \cot^2\theta)\operatorname{cosec}^2\theta - \cot^2\theta(\operatorname{cosec}^2\theta - 1)$

$= \operatorname{cosec}^2\theta + \cot^2\theta\operatorname{cosec}^2\theta - \cot^2\theta\operatorname{cosec}^2\theta + \cot^2\theta$

$= \operatorname{cosec}^2\theta + \cot^2\theta$

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$90 < \theta < 180$

c) $\operatorname{cosec}^4\theta - \cot^4\theta = 2 - \cot\theta$

from part b

$\operatorname{cosec}^4\theta - \cot^4\theta = \operatorname{cosec}^2\theta + \cot^2\theta$

$\operatorname{cosec}^2\theta + \cot^2\theta = 2 - \cot\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$1 + \cot^2\theta + \cot^2\theta = 2 - \cot\theta$

$2\cot^2\theta + \cot\theta - 1 = 0$

$y = \cot\theta$

$2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$y = \frac{1}{2} \qquad y = -1$

$\therefore \quad \cot\theta = \frac{1}{2} \qquad \cot\theta = -1$

$\frac{1}{\tan\theta} = \frac{1}{2} \qquad \frac{1}{\tan\theta} = -1$

$\tan\theta = 2 \qquad \tan\theta = -1$

$\theta = 63^\circ \qquad \theta = -45$

$\theta = -45 + 180 = 135^\circ$

within the given range