

Year 2 Pure Chapter 7: asinx + bcosx- Exam Questions (Answers)

1. (a) $R^2 = (\sqrt{3})^2 + 1^2 \Rightarrow R = 2$ M1A1
 $\tan \alpha = \sqrt{3} \Rightarrow \alpha =$ accept awrt 1.05 M1 A1 4

(b) $\sin(x + \text{their } \alpha) =$ M1
 $x + \text{their } \alpha =$ A1
 $xi =$ accept awrt 1.57, 5.76 M1 A1 4

2. (a) $5\cos x - 3\sin x = R \cos(x + \alpha), R > 0, 0 < x <$
 $5\cos x - 3\sin x = R \cos x \cos \alpha - R \sin x \sin \alpha$
 Equate cos x: $5 = R \cos \alpha$
 Equate sin x: $3 = R \sin \alpha$
 $R^2 = 5^2 + 3^2$ M1;
 or awrt 5.8 A1

$\tan \alpha =$ $\tan \alpha =$
 $\sin \alpha =$ $\cos \alpha =$ M1
 $\alpha = \text{awrt } 0.54$ or $\alpha = \text{awrt } 0.17\pi$ or
 or A1 4

Hence, $5\cos x - 3\sin x =$

(b) $5\cos x - 3\sin x = 4$

$\cos(x + 0.5404) =$ M1
 $\cos(x + \text{their } \alpha) =$
 $(x + 0.5404) = 0.814826916\dots^\circ$ For applying \cos^{-1} M1
 $x = 0.2744\dots^\circ$ awrt 0.27^c A1
 $(x + 0.5404) = 2\pi - 0.814826916\dots^\circ$
 $\{ = 5.468358\dots^\circ\}$ $2\pi - \text{their } 0.8148$ ddM1
 $x = 4.9279\dots^\circ$ awrt 4.93^c A1 5

Hence, $x = \{0.27, 4.93\}$

[9]

3. (a) $R^2 = 5^2 + 12^2$ M1
 $R = 13$ A1
 $\tan \alpha =$ M1
 $\alpha \approx 1.176$ cao A1 4

(b) $\cos(x - \alpha) =$ M1
 $x - \alpha = \arccos = 1.091 \dots$ A1
 $x = 1.091 \dots + 1.176 \dots \approx 2.267 \dots$ awrt 2.3 A1
 $x - \alpha = -1.091 \dots$ accept ... = 5.19 ... for M M1
 $x = -1.091 \dots + 1.176 \dots \approx 0.0849 \dots$ awrt 0.084 or 0.085 A1 5

(c)	(i)	$R_{\max} = 13$	ft their R	B1ft
	(ii)	At the maximum, $\cos(x - \alpha) = 1$ or $x - \alpha = 0$ $x = \alpha = 1.176 \dots$	awrt 1.2, ft their α	M1 A1ft 3

[12]

4.	(a)	$R \cos \alpha = 12, R \sin \alpha = 4$		
		$R = \sqrt{12^2 + 4^2} = \sqrt{160}$ Accept if just written down, awrt 12.6		M1 A1
		$\tan \alpha = , \Rightarrow \alpha \approx 18.43^\circ$	awrt 18.4°	M1, A1 4

(b)	$\cos(x + \text{their } \alpha) = (\approx 0.5534)$		M1
	$x + \text{their } \alpha = 56.4^\circ$	awrt 56°	A1
	$= \dots, 303.6^\circ \quad 360^\circ - \text{their principal value}$ $x = 38.0^\circ, 285.2^\circ \quad \text{Ignore solutions out of range}$		M1 A1, A1 5

If answers given to more than 1 dp, penalise first time then accept awrt above.

(c)	(i)	minimum value is $-\sqrt{160}$	ft their R	B1ft
	(ii)	$\cos(x + \text{their } \alpha) = -1$		M1

$$x \approx 161.57^\circ \quad \text{cao} \quad \text{A1} \quad 3$$

[12]

5.	(a)	$4 \sin \theta - 3 \cos \theta = R \sin \theta \cos \alpha - R \cos \theta \sin \alpha$		
		sin θ terms give $4 = R \cos \alpha$		
		cos θ terms give $3 = R \sin \alpha$		
		$\tan \alpha = 0.75$		M1
		$\alpha = 36.9^\circ$		A1
		$R^2 = 4^2 + 3^2 = 25 \Rightarrow R = 5$		M1 A1 4

(b)	$5 \sin(\theta - 36.9^\circ) = 3$		
	$\sin(\theta - 36.9^\circ) = 0.6$		M1
	$\theta - 36.9^\circ = 36.9^\circ, 143.1$		A1 M1
	$\theta = 73.7^\circ, 180^\circ$		
	$\text{awrt } 74^\circ$		A1 A15

(c)	Max value 5		B1 1
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(d)	$\sin(\theta - 36.9^\circ) = 1$		M1
	$\theta - 36.9^\circ = 90^\circ$		

$$\theta = 90^\circ + 36.9^\circ = 126.9^\circ$$

A1 2

[12]