1. (a) $\mathbf{T} = \begin{pmatrix} -1 & 0 & -2 \\ 1 & -1 & -1 \\ -2 & -3 & 1 \end{pmatrix}$, $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 4 \end{pmatrix}$

$\Delta\mathbf{T} = 1 + 3 + 6 + 4 = 14$ B1

Minor matrix of $\mathbf{T} = \begin{pmatrix} -4 & -1 & -5 \\ -6 & -5 & 3 \\ -2 & 3 & 1 \end{pmatrix}$ M1 A1

Cofactors matrix of $\mathbf{T} = \begin{pmatrix} -4 & 1 & -5 \\ 6 & -5 & -3 \\ -2 & -3 & 1 \end{pmatrix}$ A1 ft

Adjoint of $\mathbf{T} = \begin{pmatrix} -4 & 6 & -2 \\ 1 & -5 & -3 \\ -5 & -3 & 1 \end{pmatrix}$ A1 ft

$\mathbf{T}^{-1} = \dfrac{1}{14}\begin{pmatrix} -4 & 6 & -2 \\ 1 & -5 & -3 \\ -5 & -3 & 1 \end{pmatrix}$ A1 ft

$\mathbf{BT} = \mathbf{A} \Rightarrow \mathbf{BTT}^{-1} = \mathbf{AT}^{-1} \Rightarrow \mathbf{B} = \mathbf{AT}^{-1}$ M1

$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & 1 & 4 \end{pmatrix}\mathbf{T}^{-1} = \dfrac{1}{14}\begin{pmatrix} -4 & 6 & -2 \\ -3 & -13 & -5 \\ -23 & -11 & -1 \end{pmatrix}$ A1 8

(b) $\mathbf{C} = \mathbf{TAT}^{-1} = \mathbf{TB}$ M1

$= \dfrac{1}{14}\begin{pmatrix} 50 & 16 & 4 \\ 22 & 30 & 8 \\ -6 & 16 & 18 \end{pmatrix}1$ M1 A1 3

(c) $\mathbf{T}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ -1 \end{pmatrix}$

$\mathbf{T}^{-1}\mathbf{T}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \mathbf{T}^{-1}\begin{pmatrix} 5 \\ -4 \\ -1 \end{pmatrix}$ M1

$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \frac{1}{14}\begin{pmatrix} -4 & 6 & -2 \\ 1 & -5 & -3 \\ -5 & -3 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ -4 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ M1

$p = -3$, $q = 2$, $r = -1$ A1 3

[14]

2. (a) $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ B1 1

(b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ B1 1

(c) $\text{T} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & -8 \end{pmatrix}$ M1 A1 2

<u>Note</u>

M1: Accept multiplication of their matrices either way round
(this can be implied by correct answer)
A1: cao

(d) $\text{AB} = \begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix} = \begin{pmatrix} 6k + c & 0 \\ 4k + 2c & -8 \end{pmatrix}$ M1 A1 A1 3

<u>Note</u>

M1: Correct matrix multiplication method implied by one or two correct terms in correct positions.
A1: for three correct terms in correct positions
2nd A1: for all four terms correct and simplified

(e) “ $6k + c = 8$ “ and “ $4k + 2c = 0$ “ Form equations and solve simultaneously — M1

$k = 2$ and $c = –4$ — A1 — 2

Alternative method

M1: AB = **T** $\Rightarrow$ **B** = **A**$^{-1}$**T** = and compare elements to find *k* and *c*.
Then A1 as before.

<u>Note</u>

M1: follows their previous work but must give two equations from which *k* and *c* can be found and there must be attempt at solution getting to *k* = or *c* =.
A1: is cao (but not cso – may follow error in position of $4k + 2c$ earlier).

[9]

3. (a) Use $4a – (–2 \times –1) = 0 \quad \Rightarrow \quad a, = \frac{1}{2}$ — M1, A1 — 2

<u>Note</u>

Allow sign slips for first M1

(b) Determinant: $(3 \times 4) – (–2 \times –1) = 10$ (Δ) — M1

$\mathbf{B}^{-1} = \frac{1}{10}\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ — M1 A1cso — 3

<u>Note</u>

Allow sign slip for determinant for first M1 (This mark may be awarded for 1/10 appearing in inverse matrix.)

Second M1 is for correctly treating the 2 by 2 matrix, ie for $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$

Watch out for determinant $(3 + 4) – (–1 + –2) = 10$ – M0 then final answer is A0

(c)	$\frac{1}{10}\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix}$, = $\frac{1}{10}\begin{pmatrix} 4(k-6)+2(3k+12) \\ (k-6)+3(3k+12) \end{pmatrix}$	M1, A1ft	
	$\begin{pmatrix} k \\ k+3 \end{pmatrix}$ Lies on $y = x + 3$	A1	3
	<u>Alternative:</u>		
	$\begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} x \\ x+3 \end{pmatrix}$, = $\begin{pmatrix} 3x-2(x+3) \\ -x+4(x+3) \end{pmatrix}$,	M1, A1,	
	$\begin{pmatrix} x-6 \\ 3x+12 \end{pmatrix}$, which was of the form (k – 6, 3k + 12)	A1	
	Or $\begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, = $\begin{pmatrix} 3x-2y \\ -x+4y \end{pmatrix} = \begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix}$, and solves simultaneous equations	M1	
	Both equations correct and eliminate one letter to get $x = k$ or $y = k + 3$ or $10x - 10y = -30$ or equivalent.	A1	
	Completely correct work (to $x = k$ and $y = k + 3$), and conclusion lies on $y = x + 3$	A1	

<u>Note</u>

M1 for multiplying matrix by appropriate column vector
A1 correct work (ft wrong determinant)
A1 for conclusion

[8]

4.	(a)	$\mathbf{R}^2 = \begin{Bmatrix} a^2+2a & 2a+2b \\ a^2+ab & 2a+b^2 \end{Bmatrix}$	M1 A1 A1	3

<u>Note</u>

1 term correct: M1 A0 A0
2 or 3 terms correct: M1 A1 A0

(b) Puts their $a^2 + 2a = 15$ or their $2a + b^2 = 15$ M1,

or their $(a^2 + 2a)(2a + b^2) – (a^2 + ab)(2a + 2b) = 225$ (or to 15),

Puts their $a^2 + ab = 0$ or their $2a + 2b = 0$ M1

Solve to find either a or b M1

$a = 3$, $b = –3$ A1, A1 5

Alternative for (b)

Uses $\mathbf{R}^2 \times$ column vector = 15 × column vector, and equates rows to give two equations in a and b only M1, M1
Solves to find either a or b as above method M1 A1 A1

<u>Note</u>

M1 M1 as described in scheme (In the alternative scheme column vector can be general or specific for first M1 but must be specific for 2nd M1)
M1 requires solving equations to find a and/or b (though checking that correct answer satisfies the equations will earn this mark) This mark can be given independently of the first two method marks.
So solving $\mathbf{M}^2 = 15\mathbf{M}$for example gives M0M0M1A0A0 in part (b)
Also putting leading diagonal = 0 and other diagonal = 15 is M0M0M1A0A0 (No possible solutions as a >0)
A1 A1 for correct answers only
Any Extra answers given, e.g. a = –5 and b = 5 or wrong answers – **deduct last A1 awarded**
So the two sets of answers would be A1 A0
Just the answer . $a = –5$ and $b = 5$ is A0 A0
Stopping at two values for a or for b – no attempt at other is A0A0
Answer with no working at all is 0 marks

[8]

5. (a) 45° or $\frac{\pi}{4}$ rotation (anticlockwise), about the origin B1, B1 2

<u>Note</u>

More than one transformation 0/2

(b) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$ M1

$p - q = 6$ and $p + q = 8$ or equivalent M1A1

$p = 7$ and $q = 1$ both correct A1 4

Note

Second M1 for correct matrix multiplication to give two equations

Alternative

$\mathbf{M}^{-1} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ First M1A1

$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$ Second M1 A1

(c) Length of OA (= length of OB) = $\sqrt{7^2 + 1^2}, = \sqrt{50} = 5\sqrt{2}$ M1, A1 2

Note

Correct use of their p and their q award M1

(d) $M^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & = \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & = \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ M1A1 2

(e) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$ so coordinates are $(-4\sqrt{2}, 3\sqrt{2})$ M1A1 2

Note

Accept column vector for final A1.

Order of matrix multiplication needs to be correct to award Ms

[12]