IYGB - FP1 PAPER O - QUESTION 1

a) CARRY OUT THE REQUIRED "MULTIPLICATIONS"

$$\underline{A}^2 = \underline{A}\,\underline{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1\times1+2\times0 & 1\times2+2\times1 \\ 0\times1+1\times0 & 0\times2+1\times1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$$

$$\underline{A}^3 = \underline{A}^2\underline{A} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1\times1+4\times0 & 1\times2+4\times1 \\ 0\times1+1\times0 & 0\times2+1\times1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$$

b) A POSSIBLE FORM OF $\underline{A}^n$ MIGHT BE

$$\underline{A}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$$

- IF $n=1$, $\underline{A}^1 = \underline{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, IE THE RESULT STANDS

- SUPPOSE THAT THE RESULT STANDS FOR $n=k \in \mathbb{N}$

$$\Longrightarrow \underline{A}^k = \begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix}$$

$$\Longrightarrow \underline{A}^k\underline{A} = \begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$\Longrightarrow \underline{A}^{k+1} = \begin{pmatrix} 1\times1+2k\times0 & 1\times2+2k\times1 \\ 0\times1+1\times0 & 0\times2k+1\times1 \end{pmatrix}$$

$$\Longrightarrow \underline{A}^{k+1} = \begin{pmatrix} 1 & 2k+2 \\ 0 & 1 \end{pmatrix}$$

IYGB - FP1 PAPER O - QUESTION 1

$$\Longrightarrow \underline{A}^{k+1} = \begin{pmatrix} 1 & 2(k+1) \\ 0 & 1 \end{pmatrix}$$

- IF THE RESULT HOLDS FOR $n=k \in \mathbb{N}$, THEN IT ALSO HOLDS FOR $n=k+1$

SINCE THE RESULT HOLDS FOR $n=1$, THEN IT MUST HOLD FOR ALL $n \in \mathbb{N}$

IYGB - FP1 PAPER O - QUESTION 2

$$z^4 - 8z^3 + 33z^2 - 68z + 52 = 0 \ , \quad z \in \mathbb{C}$$

AS THE EQUATION HAS REAL COEFFICIENTS, ANY ROOTS IF COMPLEX MUST EXIST AS CONJUGATE PAIRS

$$\therefore \ z_1 = 2 + 3i \implies z_2 = 2 - 3i$$

PROCEED AS FOLLOWS

$$\begin{aligned}
(z - z_1)(z - z_2) &= \left[z - (2+3i)\right]\left[z - (2-3i)\right] \\
&= \left[(z-2) - 3i\right]\left[(z-2) + 3i\right] \\
&= (z-2)^2 - (3i)^2 \\
&= z^2 - 4z + 4 + 9 \\
&= z^2 - 4z + 13
\end{aligned}$$

BY "LONG DIVISION" OR INSPECTION

$$\begin{array}{r|l}
 & z^2 - 4z + 4 \\
\hline
z^2 - 4z + 13 & z^4 - 8z^3 + 33z^2 - 68z + 52 \\
 & -z^4 + 4z^3 - 13z^2 \\
\hline
 & \quad -4z^3 + 20z^2 - 68z + 52 \\
 & \quad +4z^3 - 16z^2 + 52z \\
\hline
 & \qquad 4z^2 - 16z + 52 \\
 & \qquad -4z^2 + 16z - 52 \\
\hline
 & \qquad\qquad 0
\end{array}$$

IYGB - FP1 PAPER O - QUESTION 2

HENCE WE HAVE

$$z^4 - 8z^3 + 33z^2 - 68z + 52 = (z^2 - 4z + 13)(z^2 - 4z + 4)$$
$$= (z - 4z + 13)(z - 2)^2$$

HENCE THE FULL SET OF SOLUTIONS IS

$$z = \begin{cases} 2+3i & \text{(GIVEN)} \\ 2-3i & \\ 2 & \text{(REPEATED)} \end{cases}$$

IYGB - FP1 PAPER O - QUESTION 3

LET THE SMALLER ROOT OF THE QUADRATIC BE α

- THE SUM OF THE ROOTS: $\alpha + (\alpha+3) = -\frac{b}{a} = -\frac{5}{2}$

I.E $\quad 2\alpha + 3 = -\frac{5}{2}$

$2\alpha = -\frac{11}{2}$

$\alpha = -\frac{11}{4}$

- THE PRODUCT OF THE ROOTS: $\alpha(\alpha+3) = \frac{c}{a} = \frac{c}{2}$

I.E. $\quad c = 2\alpha(\alpha+3)$

$c = 2\left(-\frac{11}{4}\right)\left(-\frac{11}{4}+3\right)$

$c = -\frac{11}{2} \times \frac{1}{4}$

$c = -\frac{11}{8}$

ALTERNATIVE - WITHOUT USING DIRECTLY RESULTS ON THE SUM AND PRODUCT OF ROOTS OF A QUADRATIC

- LET THE SMALLER OF THE TWO ROOTS BE α

THEN $\quad 2x^2 + 5x + c = 0$

$\Rightarrow x^2 + \frac{5}{2}x + \frac{c}{2} = 0$

$\Rightarrow (x-\alpha)(x-(\alpha+3)) = 0$

$\Rightarrow x^2 - (\alpha+3)x - \alpha x + \alpha(\alpha+3) = 0$

$\Rightarrow x^2 - (2\alpha+3)x + \alpha(\alpha+3) = 0$

IYGB - FP1 PAPER O - QUESTION 3

- BY COMPARISON WE HAVE

- $\frac{5}{2} = -(2\alpha+3)$

$\Longrightarrow 2\alpha+3 = -\frac{5}{2}$

$\Longrightarrow 4\alpha+6 = -5$

$\Longrightarrow 4\alpha = -11$

$\Longrightarrow \alpha = -\frac{11}{4}$

&

- $\frac{c}{2} = \alpha(\alpha+3)$

$\Rightarrow c = 2\alpha(\alpha+3)$

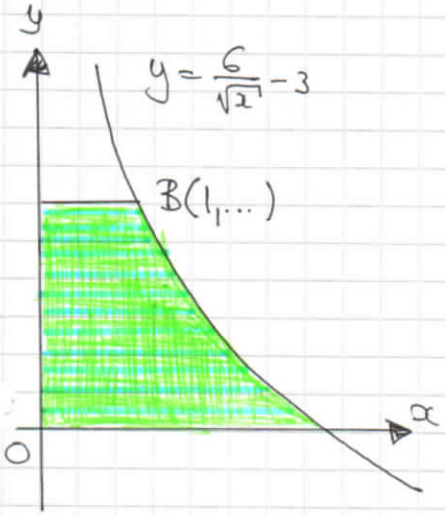
$\Longrightarrow c = 2\left(-\frac{11}{4}\right)\left(-\frac{11}{4}+3\right)$

$\Longrightarrow c = -\frac{11}{2} \times \frac{1}{4}$

$\Longrightarrow c = -\frac{11}{8}$

IYGB - FP1 PAPER O - QUESTION 4

AS THE REVOLUTION IS ABOUT THE y AXIS, REARRANGE THE EQUATION FOR x



$$\Rightarrow y = \frac{6}{\sqrt{x}} - 3$$

$$\Rightarrow y + 3 = \frac{6}{\sqrt{x}}$$

$$\Rightarrow (y+3)^2 = \frac{36}{x}$$

$$\Rightarrow x = \frac{36}{(y+3)^2}$$

$$\Rightarrow x^2 = \frac{1296}{(y+3)^4}$$

BY INSPECTION THE y CO-ORDINATE OF B IS 3

THUS WE HAVE

$$V = \pi \int_{y_1}^{y_2} [x(y)]^2 \, dy = \pi \int_0^3 \frac{1296}{(y+3)^4} \, dy$$

$$= \pi \int_0^3 1296(y+3)^{-4} \, dy = \pi \left[\frac{1296}{-3}(y+3)^{-3} \right]_0^3$$

$$= \pi \left[-\frac{432}{(y+3)^3} \right]_0^3 = 432\pi \left[\frac{1}{(y+3)^3} \right]_3^0$$

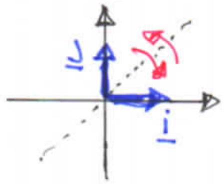
$$= 432\pi \left[\frac{1}{27} - \frac{1}{216} \right] = 432\pi \times \frac{7}{216} = 14\pi$$

AS REQUIRED

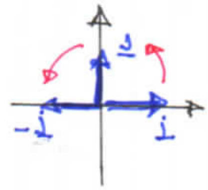
IYGB – FP1 PAPER O – QUESTION 5

- START OBTAINING THE THREE MATRICES

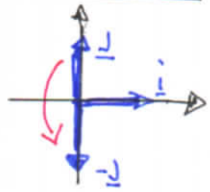
REFLECTION ABOUT $y=x$



NEXT ROTATION BY 90° ANTICLOCKWISE



FINALLY REFLECTION ABOUT x AXIS



$$\underline{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \underline{B} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \underline{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- "MULTIPLY IN THE CORRECT ORDER"

$$\underline{C}\,\underline{B}\,\underline{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

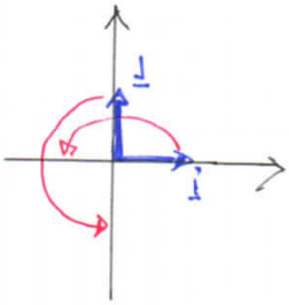
$$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

eg $\quad \underline{i} \longmapsto -\underline{i}$

$\quad\;\; \underline{j} \longmapsto -\underline{j}$

WITH POSITIVE DETERMINANT, SO NO REFLECTION



$\therefore$ ROTATION ABOUT O, BY 180°

IYGB - FP1 PAPER O - QUESTION 6

TIDY UP AS FOLLOWS

$$\Rightarrow \frac{1}{x+iy} + \frac{1}{1+2i} = 1$$

$$\Rightarrow \frac{1}{x+iy} = 1 - \frac{1}{1+2i}$$

$$\Rightarrow \frac{1}{x+iy} = 1 - \frac{1-2i}{(1+2i)(1-2i)}$$

$$\Rightarrow \frac{1}{x+iy} = 1 - \frac{1-2i}{5}$$

$$\Rightarrow \frac{5}{x+iy} = 5 - (1-2i)$$

$$\Rightarrow \frac{5}{x+iy} = 4+2i$$

$$\Rightarrow \frac{x+iy}{5} = \frac{1}{4+2i}$$

$$\Rightarrow \frac{1}{5}(x+iy) = \frac{4-2i}{(4+2i)(4-2i)}$$

$$\Rightarrow \frac{1}{5}(x+iy) = \frac{4-2i}{16+4}$$

$$\Rightarrow \frac{1}{5}(x+iy) = \frac{4-2i}{20}$$

$$\Rightarrow \frac{1}{5}(x+iy) = \frac{1}{5} - \frac{1}{10}i$$

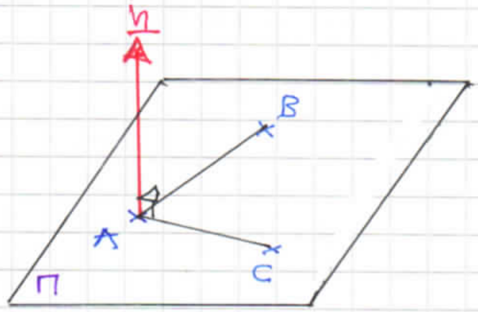
$$\Rightarrow x+iy = 1 - \frac{1}{2}i$$

$\therefore x = 1$

$y = -\frac{1}{2}$ //

IYGB - FP1 PAPER O - QUESTION 7

a) SHOWING THAT $\vec{AB}$ & $\vec{AC}$ ARE BOTH PERPENDICULAR TO THE GIVEN VECTOR



- $\vec{AB} = \underline{b} - \underline{a} = (-1,-2,0) - (1,-3,1)$

$\vec{AB} = (-2,1,-1)$

- $\vec{AC} = \underline{c} - \underline{a} = (0,-1,-4) - (1,-3,1)$

$\vec{AC} = (-1,2,-5)$

$$\left(\begin{array}{l} \vec{AB}\cdot(1,3,1) = (-2,1,-1)\cdot(1,3,1) = -2+3-1 = 0 \\ \vec{AC}\cdot(1,3,1) = (-1,2,-5)\cdot(1,3,1) = -1+6-5 = 0 \end{array}\right)$$

INDEED A NORMAL TO Π

b) THE CARTESIAN EQUATION OF Π MUST BE

$$1x + 3y + 1z = \text{CONSTANT}$$
$$x + 3y + z = \text{CONSTANT}$$

USING $A(1,-3,1)$ GIVES

$$1 + 3(-3) + 1 = \text{CONSTANT}$$
$$-7 = \text{CONSTANT}$$

$$\therefore x + 3y + z = -7$$
$$x + 3y + z + 7 = 0$$

IYGB - FP1 PAPER O - QUESTION 7

c) SOLVING SIMULTANEOUSLY

$\Pi: x+3y+z+7=0$

$\underline{r} = (2,0,1)+\lambda(5,1,2)$

$\underline{r} = (5\lambda+2, \lambda, 2\lambda+1)$

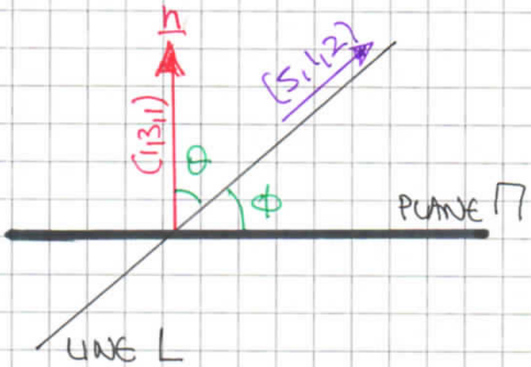
$(5\lambda+2)+3\lambda+(2\lambda+1)+7=0$

$10\lambda+10=0$

$\lambda=-1$

$\therefore \left[5(-1)+2, -1, 2(-1)+1\right]$ YIELDS $(-3,-1,-1)$

d)



LOOKING AT THE DIAGRAM

$\Rightarrow (1,3,1)\cdot(5,1,2) = |1,3,1|\,|5,1,2|\cos\theta$

$\Rightarrow 5+3+2 = \sqrt{1+9+1}\sqrt{25+1+4}\cos\theta$

$\Rightarrow 10 = \sqrt{11}\sqrt{30}\cos\theta$

$\Rightarrow \cos\theta = \frac{10}{\sqrt{11\times30}}$

$\Rightarrow \theta \approx 56.599^\circ\ldots$

$\therefore$ REQUIRED ANGLE IS ϕ

$\Rightarrow \phi = 90-\theta$

$\Rightarrow \phi \approx 33.4^\circ$

IYGB - FP1 PAPER O - QUESTION 8

a) TRIVIALLY WE HAVE

$$u_1 = S_1 = 1^2(1+1)(1+2) = 1 \times 2 \times 3 = 6$$

b) I) USING $S_n - S_{n-1} = u_n$

$$\Rightarrow u_n = n^2(n+1)(n+2) - (n-1)^2[(n-1)+1][(n-1)+2]$$
$$\Rightarrow u_n = n^2(n+1)(n+2) - (n-1)^2 n(n+1)$$
$$\Rightarrow u_n = n(n+1)\left[n(n+2) - (n-1)^2\right]$$
$$\Rightarrow u_n = n(n+1)\left[\cancel{n^2} + 2n - (\cancel{n^2} - 2n + 1)\right]$$
$$\Rightarrow u_n = n(n+1)(4n-1)$$

AS REQUIRED

II) $$\sum_{r=n+1}^{2n} u_r = S_{2n} - S_n$$
$$= (2n)^2(2n+1)(2n+2) - n^2(n+1)(n+2)$$
$$= 4n^2(2n+1) \times 2(n+1) - n^2(n+1)(n+2)$$
$$= n^2(n+1)\left[8(2n+1) - (n+2)\right]$$
$$= n^2(n+1)(8n+8-n-2)$$
$$= n^2(n+1)(15n+6)$$
$$= 3n^2(n+1)(5n+2)$$

AS REQUIRED

IYGB - FP1 PAPER O - QUESTION 9

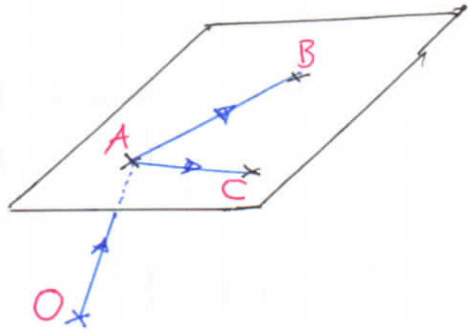
START BY PARAMETRIZING THE PLANE - TAKE ANY 3 POINTS ON THE PLANE SAY $A(6,0,0)$, $B(0,12,0)$ & $C(0,0,-12)$

$$\vec{AB} = \underline{b} - \underline{a} = (0,12,0) - (6,0,0) = (-6,12,0) \quad \text{SCALE IT TO } (-1,2,0)$$
$$\vec{AC} = \underline{c} - \underline{a} = (0,0,-12) - (6,0,0) = (-6,0,-12) \quad \text{SCALE IT TO } (1,0,2)$$

HENCE WE HAVE

$$\underline{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6-\lambda+\mu \\ 2\lambda \\ 2\mu \end{bmatrix}$$



NOW TRANSFORM THE PARAMETRIZED PLANE

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6-\lambda+\mu \\ 2\lambda \\ 2\mu \end{bmatrix} = \begin{bmatrix} 6-\lambda+\mu+4\lambda \\ 6-\lambda+\mu+2\mu \\ 6-\lambda+\mu+2\lambda+2\mu \end{bmatrix} = \begin{bmatrix} 6+3\lambda+\mu \\ 6-\lambda+3\mu \\ 6+\lambda+3\mu \end{bmatrix}$$

$$X = 6+3\lambda+\mu \implies \underline{\mu = X-6-3\lambda}$$
$$Y = 6-\lambda+3\mu$$
$$Z = 6+\lambda+3\mu$$

SUBSTITUTING INTO THE OTHER TWO EQUATIONS

THUS $\quad Y = 6-\lambda+3(X-6-3\lambda)$

$\quad\quad Z = 6+\lambda+3(X-6-3\lambda)$

IYGB - FP1 PAPER O - QUESTION 9

$$\left.\begin{aligned} y &= 6-\lambda+3x-18-9\lambda \\ z &= 6+\lambda+3x-18-9\lambda \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} y &= 3x-12-10\lambda \\ z &= 3x-12-8\lambda \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} 10\lambda &= 3x-y-12 \\ 8\lambda &= 3x-z-12 \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} 40\lambda &= 12x-4y-48 \\ 40\lambda &= 15x-5z-60 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow 12x-4y-48 = 15x-5z-60$$

$$\Rightarrow -3x-4y+5z = -12$$

$$\Rightarrow \underline{3x+4y-5z = 12}$$