1) $$\frac{x+5}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$x+5 = A(x+3) + B(x+2)$$

Let $x = -3$

$$2 = B(-1)$$

$$B = -2$$

Let $x = -2$

$$3 = A$$

$$\int \frac{3}{x+2} - \frac{2}{x+3} = 3\ln|x+2| - 2\ln|x+3| + C$$

$$= \ln\left|\frac{|x+2|^3}{|x+3|^2}\right| + C$$

2) $$\frac{9x-5}{2(2x-1)(x-1)} = \frac{A}{2x-1} + \frac{B}{x-1}$$

$$9x-5 = 2A(x-1) + 2B(2x-1)$$

Let $x = 1$

$$4 = 2B$$

$$B = 2$$

Let $x = \frac{1}{2}$

$$-0.5 = -A$$

$$A = 0.5$$

$$\int \frac{0.5}{2x-1} + \frac{2}{x-1} \qquad \left[\frac{1}{2(2x-1)} + \frac{2}{x-1}\right]$$

$$= \frac{1}{2}(0.5)\ln|2x-1| + 2\ln|x-1| + C$$

$$= \ln\left|\sqrt[4]{|2x-1|}\,|x-1|^2\right| + C$$

3) $$\frac{5x-13}{(x-1)(x-3)^2} = \frac{A}{x-1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$5x-13 = A(x-3)^2 + B(x-1)(x-3) + C(x-1)$$

Let $x=3$

$$2 = 2C$$

$$C = 1$$

Let $x = 1$

$$-8 = 4A$$

$$A = -2$$

Let $x = 0$

$$-13 = 9A + 3B - C$$

$$-13 = -18 + 3B - 1$$

$$6 = 3B$$

$$B = 2$$

$$\int \frac{2}{x-3} - \frac{2}{x-1} + \frac{1}{(x-3)^2}$$

$$= 2\ln|x-3| - 2\ln|x-1| + \frac{(x-3)^{-1}}{-1} + C$$

$$= \ln\left|\frac{|x-3|^2}{|x-1|^2}\right| - \frac{1}{(x-3)} + C$$

4/

$$\frac{5x^2 - 12x - 1}{(x+3)(x-1)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$5x^2 - 12x - 1 = A(x-1)^2 + B(x+3)(x-1) + C(x+3)$$

Let $x = 1$

$$-8 = 4C$$

$$C = -2$$

Let $x = -3$

$$80 = 16A$$

$$A = 5$$

Let $x = 0$

$$-1 = A - 3B + 3C$$

$$-1 = 5 - 3B - 6$$

$$0 = -3B$$

$$B = 0$$

$$\int \frac{5}{x+3} - \frac{2}{(x-1)^2}$$

$$= 5\ln|x+3| - \frac{2(x-1)^{-1}}{-1} + C$$

$$= \ln|x+3|^5 + \frac{2}{(x-1)} + C$$

5) $$\frac{6x^2 - x + 1}{(3x-1)(x+1)} = A + \frac{B}{3x-1} + \frac{C}{x+1}$$

$$6x^2 - x + 1 = A(3x-1)(x+1) + B(x+1) + C(3x-1)$$

Let $x = -1$

$$8 = -4C$$

$$C = -2$$

Let $x = \frac{1}{3}$

$$\frac{4}{3} = \frac{4}{3}B$$

$$B = 1$$

Let $x = 0$

$$1 = -A + B - C$$

$$1 = -A + 1 + 2$$

$$1 = -A + 3$$

$$-2 = -A$$

$$A = 2$$

$$\int 2 + \frac{1}{3x-1} - \frac{2}{x+1}$$

$$= 2x + \frac{1}{3}\ln|3x-1| - 2\ln|x+1| + C$$

$$= 2x + \ln\left|\frac{|3x-1|^{1/3}}{|x+1|^2}\right| + C$$

6 $$\frac{2x^3+7x^2+8x+2}{(x+1)(x+2)} = Ax + B + \frac{C}{x+1} + \frac{D}{x+2}$$

$$2x^3+7x^2+8x+2 = Ax(x+1)(x+2) + B(x+1)(x+2) + C(x+2) + D(x+1)$$

Let $x = -1$

$$-1 = C$$

Let $x = -2$

$$-2 = -D$$

$$D = 2$$

Let $x = 0$

$$2 = 2B + 2C + D$$

$$2 = 2B - 2 + 2$$

$$B = 1$$

Let $x = 1$

$$19 = 6A + 6B + 3C + 2D$$

$$19 = 6A + 6 * -3 + 4$$

$$19 = 6A + 7$$

$$12 = 6A$$

$$A = 2$$

$$\int 2x + 1 - \frac{1}{x+1} + \frac{2}{x+2}$$

$$= \frac{2}{2}x^2 + x - \ln|x+1| + 2\ln|x+2| + C$$

$$= x^2 + x + \ln\left|\frac{|x+2|^2}{|x+1|}\right| + C$$

7) $$\frac{6(x^2+4x+1)}{(2x+3)(3x-1)} = A + \frac{B}{2x+3} + \frac{C}{3x-1}$$

$$6(x^2+4x+1) = A(2x+3)(3x-1) + B(3x-1) + C(2x+3)$$

Let $x = -\frac{3}{2}$

$$-\frac{33}{2} = -\frac{11}{2}B$$

$$B = 3$$

Let $x = \frac{1}{3}$

$$\frac{44}{3} = \frac{11}{3}C$$

$$C = 4$$

Let $x = 0$

$$6 = -3A - B + 3C$$

$$6 = -3A - 3 + 12$$

$$-3 = -3A$$

$$A = 1$$

$$\int 1 + \frac{3}{2x+3} + \frac{4}{3x-1}$$

$$= x + \frac{1}{2}(3)\ln|2x+3| + \frac{1}{3}(4)|3x-1| + C$$

$$= x + \ln\left|(2x+3)^{3/2}(3x-1)^{4/3}\right| + C$$