Paper 3: Statistics and Mechanics Mark Scheme

Question	Scheme	Marks	AOs
1(a)	**Area** = $8\times1.5=12$ cm^2 **Frequency** = 8 so 1 cm^2 = $\frac{2}{3}$ **hour (o.e.)**	M1	3.1a
	Frequency of 12 corresponds to area of 18 so height = $18\div2.5$ = **7.2 (cm)**	A1	1.1b
	Width = 5×0.5 = **2.5 (cm)**	B1cao	1.1b
		(3)	
(b)	$[\bar{y}=]\ \frac{205.5}{31}=$ awrt 6.63	B1cao	1.1b
	$\left[\sigma_y=\right]\sqrt{\frac{1785.25}{31}-\bar{y}^2}$ = $\sqrt{13.644641}$ = **awrt 3.69** allow $[s=]\sqrt{\frac{1785.25-31\bar{y}^2}{30}}$ = **awrt 3.75**	M1 A1	1.1a 1.1b
		(3)	
(c)	Mean of Heathrow is higher than Hurn and standard deviation smaller suggesting Heathrow is more reliable	M1	2.4
	Hurn is South of Heathrow so does <u>not</u> support his belief	A1	2.2b
		(2)	
(d)	$\bar{x}+\sigma\approx10.3$ so number of days is e.g. $\frac{(11-"10.3")}{3}\times8\ (+5)$	M1	1.1b
	= 6.86 so **7 days**	A1	1.1b
		(2)	
(e)	[H = no. of hours] P(H > 10.3) or P(Z > 1) = [0.15865…]	M1	3.4
	Predict $31\times0.15865...$ = **<u>4.9 or 5 days</u>**	A1	1.1b
		(2)	
(f)	(5 or) 4.9 days < (7 or) 6.9 days so model may **not** be suitable	B1	3.5a
		(1)	
			(13 marks)

Question 1 continued	
Notes:	
(a)	
M1:	for clear attempt to relate the area to frequency. Can also award if their height $\times$ their width = 18
A1:	for height = 7.2 (cm)
(b)	
M1:	for a correct expression for σ or s, can ft their value for mean
A1:	awrt 3.69 (allow s = 3.75)
(c)	
M1:	for a suitable comparison of standard deviations to comment on reliability.
A1:	for stating Hurn is south of Heathrow and a correct conclusion
(d)	
M1:	for a correct expression – ft their $\bar{x}+\sigma \approx 10.3$
A1:	for 7 days but accept 6 (rounding down) following a correct expression
(e)	
M1:	for a correct probability attempted
A1:	for a correct prediction
(f)	
B1:	for a suitable comparison and a compatible conclusion

Question	Scheme	Marks	AOs
2(a)	e.g. It requires extrapolation so will be unreliable (o.e.)	B1	1.2
		(1)	
(b)	e.g. Linear association between w and t	B1	1.2
		(1)	
(c)	H_0: $\rho = 0$ H_1: $\rho > 0$	B1	2.5
	Critical value 0.5822	M1	1.1a
	Reject H_0		
	There is evidence that the product moment correlation coefficient is greater than 0	A1	2.2b
		(3)	
(d)	Higher $\bar{t}$ suggests overseas and not Perth…lower wind speed so perhaps not close to the sea so suggest **Beijing**	B1	2.4
		(1)	
			(6 marks)

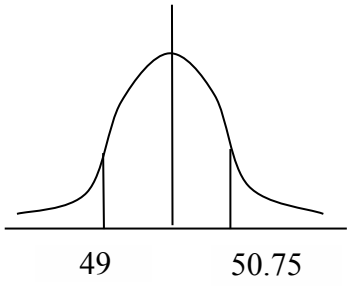
Notes:

(a)
B1: for a correct statement (unreliable) with a suitable reason

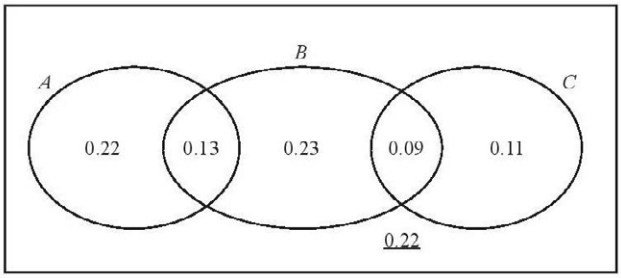
(b)
B1: for a correct statement

(c)
B1: for both hypotheses in terms of ρ
M1: for selecting a suitable 5% critical value compatible with their H_1
A1: for a correct conclusion stated

(d)
B1: for suggesting Beijing with some supporting reason based on t or w
Allow Jacksonville with a reason based just on higher $\bar{t}$

Question	Scheme	Marks	AOs
Q3(a)			
	P ($L > 50.98$) = 0.025	B1cao	3.4
	$\therefore \frac{50.98-\mu}{0.5} = 1.96$	M1	1.1b
	$\therefore \quad \mu = 50$	A1cao	1.1b
	P($49 < L < 50.75$)	M1	3.4
	= 0.9104… awrt **0.910**	A1ft	1.1b
		(5)	
(b)	S = number of strips that cannot be used so S~B(10, 0.090)	M1	3.3
	= P($S \leqslant 3$) = 0.991166… awrt 0.991	A1	1.1b
		(2)	
(c)	$H_0 : \mu = 50.1$ $H_1 : \mu > 50.1$	B1	2.5
	$\bar{X} \sim N\left(50.1, \frac{0.6^2}{15}\right)$ and $\bar{X} > 50.4$	M1	3.3
	$P(\bar{X} > 50.4) = 0.0264$	A1	3.4
	$p = 0.0264 > 0.01$ or $z = 1.936\ldots < 2.3263$ and not significant	A1	1.1b
	There is insufficient evidence that the **mean length** of strips is **greater than 50.1**	A1	2.2b
		(5)	
			(12 marks)

Question 3 continued
Notes:
(a) **1st M1:** for standardizing with μ and 0.5 and setting equal to a z value ($\|z\| > 1$) **2nd M1:** for attempting the correct probability for strips that can be used **2nd A1ft:** awrt 0.910 (allow ft of their μ)
(b) **M1:** for identifying a suitable binomial distribution **A1:** awrt 0.991 (from calculator)
(c) **B1:** hypotheses stated correctly **M1:** for selecting a correct model (stated or implied) **1st A1:** for use of the correct model to find p = awrt 0.0264 (allow z = awrt 1.94) **2nd A1:** for a correct calculation, comparison and correct statement **3rd A1:** for a correct conclusion in context mentioning "mean length" and 50.1

Question	Scheme	Marks	AOs
4(a)	$\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')}$ or $\dfrac{0.33}{0.55}$	M1	3.1a
	$= \dfrac{3}{5}$ or 0.6	A1	1.1b
		(2)	
(b)	e.g. $\mathrm{P}(A) \times \mathrm{P}(B) = \frac{7}{20} \times \frac{9}{20} = \frac{63}{400} \neq \mathrm{P}(A \cap B) = 0.13 = \frac{52}{400}$ or $\mathrm{P}(A' \mid B') = 0.6 \neq \mathrm{P}(A') = 0.65$	B1	2.4
		(1)	
(c)		B1	2.5
		M1	3.1a
		A1	1.1b
		M1	1.1b
		A1	1.1b
		(5)	
(d)	$\mathrm{P}(B \cup C)' = 0.22 + 0.22$ or $1-[0.56]$ or $1-[0.13+0.23+0.09+0.11]$ o.e.	M1	1.1b
	= 0.44	A1	1.1b
		(2)	
			(10 marks)

Notes:

(a)

M1: for a correct ratio of probabilities formula and at least one correct value.

A1: a correct answer

(b)

for a fully correct explanation: correct probabilities and correct comparisons.

(c)

B1: for box with B intersecting A and C but C not intersecting A.(Or accept three intersecting circles, but with zeros entered for $A \cap C$ and $A \cap B \cap C$)No box is B0

M1: for method for finding P($B \cap C$)

A1: for 0.09

M1: for 0.13 and their 0.09 in correct places and method for their 0.23

A1: fully correct

(d)

M1: for a correct expression – ft their probabilities from their Venn diagram.

A1: cao

Question	Scheme	Marks	AOs
5 (a)	The seeds would be destroyed in the process so they would have none to sell	B1	2.4
		(1)	
(b)	[S = no. of seeds out of 24 that germinate, $S \sim B(24, 0.55)$]		
	T = no. of trays with at least 15 germinating. $T \sim B(10, p)$	M1	3.3
	$p = P(S \geqslant 15) = 0.299126...$	A1	1.1b
	So $P(T \geqslant 5) = 0.1487\ldots$ awrt **<u>0.149</u>**	A1	1.1b
		(3)	
(c)	n is large and p close to 0.5	B1	1.2
		(1)	
(d)	$X \sim N(132, 59.4)$	B1	3.4
	$P(X \geqslant 149.5) = P\left(Z \geqslant \frac{149.5-132}{\sqrt{59.4}}\right)$	M1	1.1b
	$= 0.01158\ldots$ awrt **<u>0.0116</u>**	A1cso	1.1b
		(3)	
(e)	e.g The probability is very small therefore there is evidence that the company's claim is incorrect.	B1	2.2b
		(1)	
			(9 marks)

Notes:

(a)
B1: cao

(b)
M1: for selection of an appropriate model for T
1st A1: for a correct value of the parameter p (accept 0.3 or better)
2nd A1: for awrt 0.149

(c)
B1: both correct conditions

(d)
B1: for correct normal distribution
M1: for correct use of continuity correction
A1: cso

(e)
B1: correct statement

Question	Scheme	Marks	AOs
6	Integrate **a** w.r.t. time	M1	1.1a
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of **C**)	A1	1.1b
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1**ft**	1.1b
			(6 marks)

Notes:

1st M1: for integrating **a** w.r.t. time (powers of t increasing by 1)
1st A1: for a correct **v** expression without **C**
2nd A1: for a correct **v** expression including **C**
2nd M1: for putting $t = 4$ into their **v** expression
3rd M1: for finding magnitude of their **v**
3rd A1: ft for 100 m s^{-1}, follow through on an incorrect **v**

Question	Scheme	Marks	AOs
7(a)	$R = mg\cos\alpha$	B1	3.1b
	Resolve parallel to the plane	M1	3.1b
	$-F - mg\sin\alpha = -0.8mg$	A1	1.1b
	$F = \mu R$	M1	1.2
	Produce an equation in μ only and solve for μ	M1	2.2a
	$\mu = \frac{1}{4}$	A1	1.1b
		(6)	
(b)	Compare $\mu mg\cos\alpha$ with $mg\sin\alpha$	M1	3.1b
	Deduce an appropriate conclusion	A1 **ft**	2.2a
		(2)	
			(8 marks)

Notes:

(a)

B1: for $R = mg\cos\alpha$

1st M1: for resolving parallel to the plane

1st A1: for a correct equation

2nd M1: for use of $F = \mu R$

3rd M1: for eliminating F and R to give a value for μ

2nd A1: for $\mu = \frac{1}{4}$

(b)

M1: comparing size of limiting friction with weight component down the plane

A1ft: for an appropriate conclusion from their values

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2} Given answer	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j}\,t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})\,t^2$	A1	1.1b
		(2)	
(c)	Equating the **i** and **j** components of **r**	M1	3.1b
	$\frac{1}{2} \leftarrow 0.7\,t^2 = 0.6\,t - \frac{1}{2} \leftarrow 0.1\,t^2$	A1ft	1.1b
	$t = 1.5$	A1	1.1b
		(3)	
(d)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j})\,t$	M1	3.1b
	Equating the **i** and **j** components of **v**	M1	3.1b
	$t = 0.75$	A1 **ft**	1.1b
		(3)	
			(10 marks)

Notes:

(a)
M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
A1: for given answer correctly obtained

(b)
M1: for use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
A1: for a correct expression for **r** in terms of t

(c)
M1: for equating the **i** and **j** components of their **r**
A1ft: for a correct equation following their **r**
A1: for $t = 1.5$

(d)
M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for a general t
M1: for equating the **i** and **j** components of their **v**
A1ft: for $t = 0.75$, or a correct follow through answer from an incorrect equation

Question	Scheme	Marks	AOs
9(a)	Take moments about A (or any other complete method to produce an equation in S, W and α only)	M1	3.3
	$Wa\cos\alpha + 7W2a\cos\alpha = S2a\sin\alpha$	A1 A1	1.1b 1.1b
	Use of $\tan\alpha = \frac{5}{2}$ to obtain S	M1	2.1
	$S = 3W$ *	A1*	2.2a
		(5)	
(b)	$R = 8W$	B1	3.4
	$F = \frac{1}{4} R\ (= 2W)$	M1	3.4
	$P_{MAX} = 3W + F$ or $P_{MIN} = 3W - F$	M1	3.4
	$P_{MAX} = 5W$ or $P_{MIN} = W$	A1	1.1b
	$W \leq P \leq 5W$	A1	2.5
		(5)	
(c)	M(A) shows that the reaction on the ladder at B is unchanged	M1	2.4
	also R increases (resolving vertically)	M1	2.4
	which increases max F available	M1	2.4
		(3)	
			(13 marks)

Question 9 continued

Notes:

(a)

1st M1: for producing an equation in S, W and α only

1st A1: for an equation that is correct, or which has one error or omission

2nd A1: for a fully correct equation

2nd M1: for use of $\tan\alpha = \frac{5}{2}$ to obtain S in terms of W only

3rd A1*:for given answer $S = 3W$ correctly obtained

(b)

B1: for $R = 8W$

1st M1: for use of $F = \frac{1}{4} R$

2nd M1: for either $P = (3W + \text{their } F)$ or $P = (3W - \text{their } F)$

1st A1: for a correct max or min value for a correct range for P

2nd A1: for a correct range for P

(c)

1st M1: for showing, by taking moments about A, that the reaction at B is unchanged by the builder's assistant standing on the bottom of the ladder

2nd M1: for showing, by resolving vertically, that R increases as a result of the builder's assistant standing on the bottom of the ladder

3rd M1: for concluding that this increases the limiting friction at A

Question	Scheme	Marks	AOs
10(a)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$36 = U t\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-18 = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	Correct strategy for solving the problem by setting up two equations in t and U and solving for U	M1	3.1b
	$U = 15$	A1	1.1b
		(6)	
(b)	Using the model and horizontal motion: $U\cos\alpha$ (12)	B1	3.4
	Using the model and vertical motion: $v^2 = (U\sin\alpha)^2 + 2(-10)(-7.2)$	M1	3.4
	$v = 15$	A1	1.1b
	Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = $\sqrt{(12^2 + 15^2)}$	M1	3.1b
	$\sqrt{369} = 19$ m s^{-1} (2sf)	A1 **ft**	1.1b
		(5)	
(c)	Possible improvement (see below in notes)	B1	3.5c
	Possible improvement (see below in notes)	B1	3.5c
		(2)	
			(13 marks)

Question 10 continued
Notes:
(a) **1st M1:** for use of $s = ut$ horizontally **1st A1:** for a correct equation **2nd M1:** for use of $s = ut + \frac{1}{2}at^2$ vertically **2nd A1:** for a correct equation **3rd M1:** for correct strategy (need both equations) **2nd A1:** for $U = 15$
(b) **B1:** for $U\cos\alpha$ used as horizontal velocity component **1st M1:** for attempt to find vertical component **1st A1:** for 15 **2nd M1:** for correct strategy (need both components) **2nd A1ft:** for 19 m s $^{-1}$ (2sf) following through on incorrect component(s)
(c) **B1, B1:** for any two of e.g. Include air resistance in the model of the motion e.g. Use a more accurate value for g in the model of the motion e.g. Include wind effects in the model of the motion e.g. Include the dimensions of the stone in the model of the motion