

KS5 "Full Coverage": Projectiles (Year 2)

This worksheet is designed to cover one question of each type seen in past papers, for each A Level topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrstmths.com, logging on, *Practise* → *Past Papers* (or *Library* → *Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Determine the vertical height at a given final speed, given an unknown angle of projection.

[Edexcel M2 June 2014 Q6a]

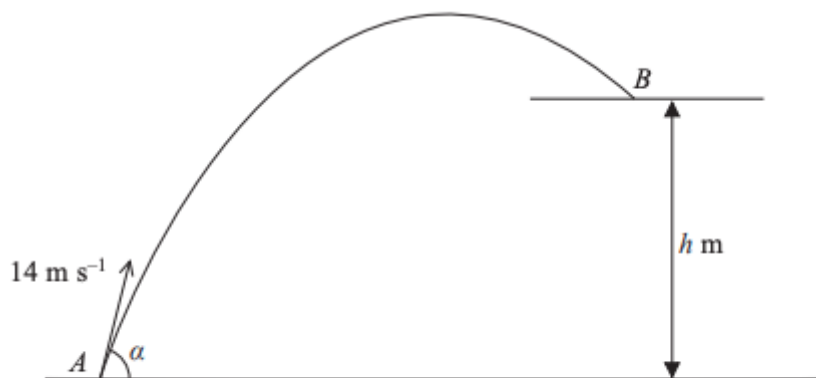


Figure 2

A small ball is projected with speed 14 m s^{-1} from a point A on horizontal ground. The angle of projection is α above the horizontal. A horizontal platform is at height h metres above the ground. The ball moves freely under gravity until it hits the platform at the point B , as shown in Figure 2. The speed of the ball immediately before it hits the platform at B is 10 m s^{-1} .

Find the value of h .

$$h = \dots\dots\dots \text{ m}$$

Question 2

Categorisation: Consider particles projected downwards.

[Edexcel A2 Specimen Papers P3 Q10a]

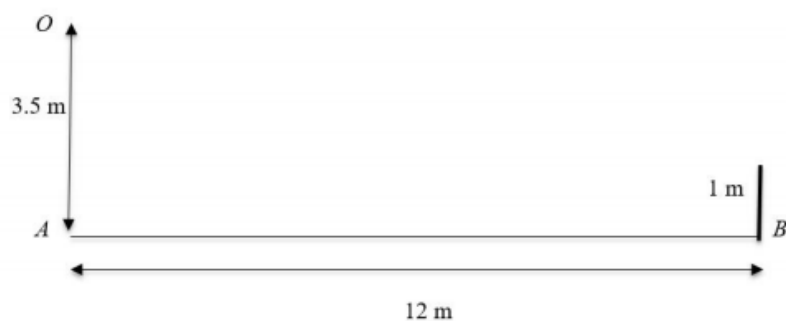


Figure 3

A tennis player serves a ball so as to pass over the net.

The ball is given an initial velocity of 45 m s^{-1} in a direction 10° below the horizontal.

The ball is struck at a point O which is 3.5 m vertically above the point A which is on horizontal ground.

The bottom of the net is the point B which is on the ground and $AB = 12 \text{ m}$.

The height of the net is 1 m, as shown in Figure 3.

The ball is modelled as a particle moving freely under gravity.

The ball passes over the net at a point which is vertically above B .

Using the model, find, in centimetres to 2 significant figures, the distance between the ball and the top of the net, as the ball passes over the net.

$$d = \dots\dots\dots \text{ cm}$$

Question 3

Categorisation: Determine the final speed.

[Edexcel A2 Specimen Papers P3 Q10b] (Continued from above)

Using the model, find, to 2 significant figures, the speed of the ball as it passes over the net.

$$v = \dots\dots\dots \text{ m s}^{-1}$$

Question 4

Categorisation: Determine the initial speed.

[Edexcel A2 SAM P3 Q10a]

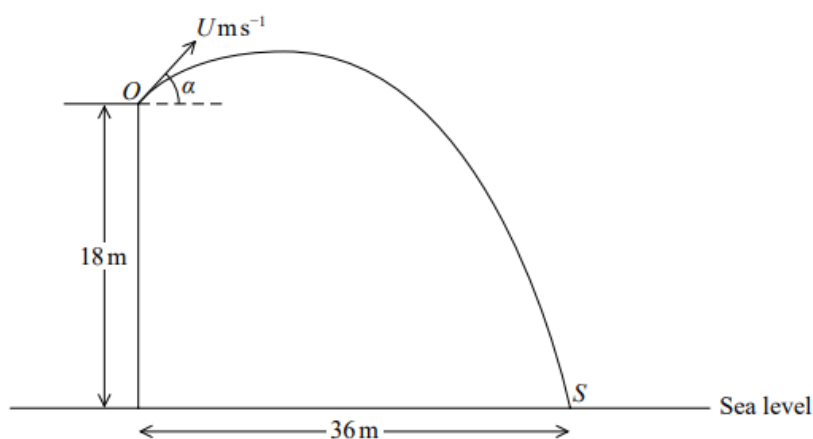


Figure 2

A boy throws a stone with speed $U \text{ m s}^{-1}$ from a point O at the top of a vertical cliff. The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2. The stone is modelled as a particle moving freely under gravity with $g = 10 \text{ m s}^{-2}$

Find the value of U .

$$U = \dots\dots\dots \text{ m s}^{-1}$$

Question 5

Categorisation: Determine the speed at a given height.

[Edexcel A2 SAM P3 Q10b Edited] (Continued from above)

Find the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

$$\dots\dots\dots \text{ m s}^{-1}$$

Question 6

Categorisation: Determine the horizontal distance travelled.

[Edexcel M2 June 2010 Q7b Edited]

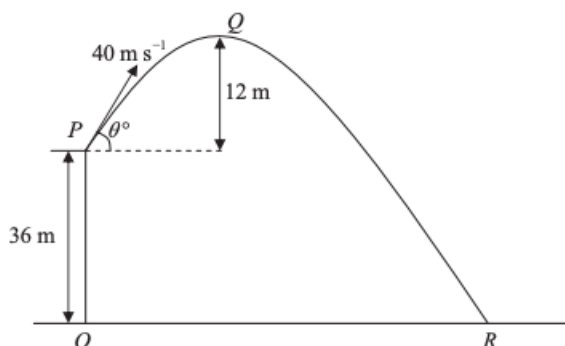


Figure 3

A ball is projected with speed 40 m s^{-1} from a point P on a cliff above horizontal ground.

The point O on the ground is vertically below P and OP is 36 m. The ball is projected at an angle θ° to the horizontal. The point Q is the highest point of the path of the ball and is 12 m above the level of P . The ball moves freely under gravity and hits the ground at the point R , as shown in Figure 3.

Find the distance OR .

..... m

Question 7

Categorisation: Derive a formula involving projectile motion.

[Edexcel M2 June 2011 Q8a Edited] A particle is projected from a point O with speed u at an angle of elevation α above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x , its height above O is y .

Show that

$$y = x \tan \alpha - \frac{gx^2}{A \cos^2 \alpha}$$

where A is an expression to be found.

$A =$

Question 8

Categorisation: As above.

[Edexcel M2 June 2018 Q7a Edited] A particle, of mass 0.3 kg, is projected from a point O on horizontal ground with speed u .

The particle is projected at an angle α above the horizontal, where $\tan \alpha = 2$, and moves freely under gravity. When the particle has moved a horizontal distance x from O , its height above the ground is y .

Show that $y = 2x - \frac{5g}{A}x^2$ where A is an expression to be found.

$A = \dots\dots\dots$

Question 9

Categorisation: Determine the time/speed when the projectile is moving at a given angle.

[Edexcel M2 June 2012 Q7c Edited]

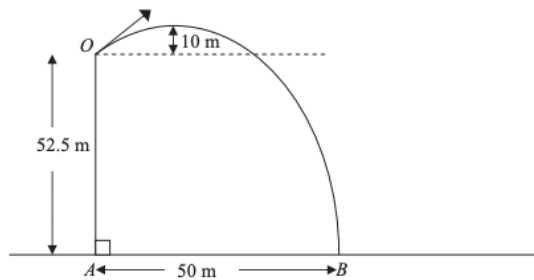


Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA . The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B , where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

It can be shown that the vertical component of the velocity of projection of the stone is 14 m s^{-1} . Find the time after projection when the stone is moving parallel to OB .

$t = \dots\dots\dots$ seconds

Question 10

Categorisation: Consider the collision of multiple projectiles.

[Edexcel M2 June 2017 Q6a]

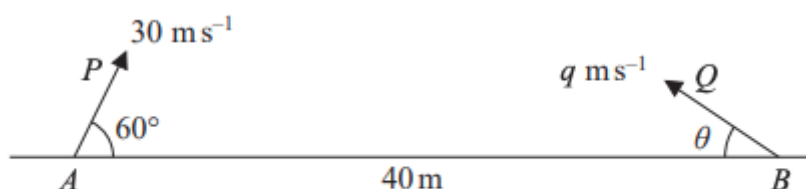


Figure 4

The points A and B lie 40 m apart on horizontal ground. At time $t = 0$ the particles P and Q are projected in the vertical plane containing AB and move freely under gravity. Particle P is projected from A with speed 30 m s^{-1} at 60° to AB and particle Q is projected from B with speed $q \text{ m s}^{-1}$ at angle θ to BA , as shown in Figure 4.

At $t = 2$ seconds, P and Q collide.

Find the size of angle θ and the value of q .

$$\theta = \dots\dots\dots^\circ$$

$$q = \dots\dots\dots$$

Question 11

Categorisation: Use vectors within projectile motion.

[Edexcel M2 June 2016 Q6a]

[In this question, \mathbf{i} is a horizontal unit vector and \mathbf{j} is an upward vertical unit vector.]

A particle P is projected from a fixed origin O with velocity $(3\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$. The particle moves freely under gravity and passes through the point A with position vector $\lambda(\mathbf{i} - \mathbf{j}) \text{ m}$, where λ is a positive constant.

Find the value of λ .

$$\lambda = \dots\dots\dots$$

Question 12

Categorisation: As above.

[Edexcel M2 June 2016 Q6b Edited] (Continued from above)

Find the speed of P at the instant when it passes through A and the direction of motion of P at the instant when it passes through A .

.....

Question 13

Categorisation: Determine the initial speed/angle of projection given a later speed and angle.

[Edexcel M2 June 2015 Q7a]

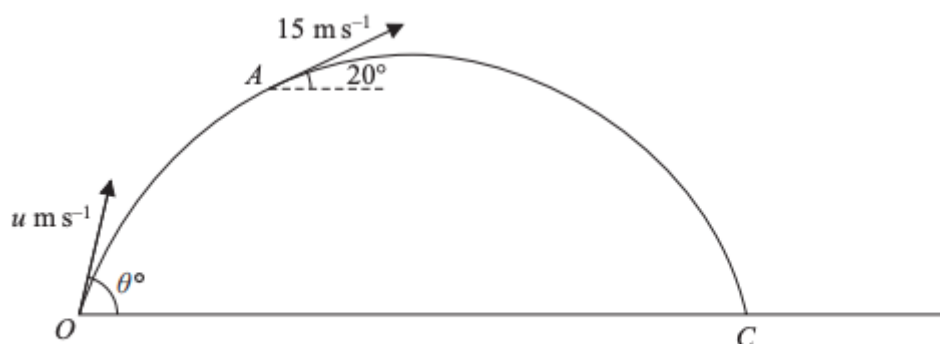


Figure 3

At time $t = 0$, a particle is projected from a fixed point O on horizontal ground with speed $u \text{ m s}^{-1}$ at an angle θ° to the horizontal. The particle moves freely under gravity and passes through the point A when $t = 4 \text{ s}$. As it passes through A , as shown in Figure 3.

Find the value of u and the value of θ .

$$u = \dots\dots\dots \text{ m s}^{-1}$$

$$\theta = \dots\dots\dots^\circ$$

Answers

Question 1

$$h = 4.9 \text{ m}$$

$$\text{Initial } v_y = 14 \sin \alpha \quad \text{Final } v_y = \sqrt{100 - 14^2 \cos^2 \alpha}$$

$$100 - 196 \cos^2 \alpha = 196 \sin^2 \alpha - 2gh$$

$$h = \frac{48}{g} = 4.90$$

M1A2

A1

(4)

Using $v^2 = u^2 + 2as$ on the vertical components of speed.
-1 each error in the unsimplified equation
Accept in exact form. Maximum 3 s.f. if they go in to decimals.

Question 2

$$d = 2.5 \text{ cm}$$

Using the model and horizontal motion: $s = ut$	M1	3.3
$12 = T \times 45 \cos 10^\circ$	A1	1.1b
$T = 0.2707..$	A1	1.1b
Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
$s = 45T \sin 10^\circ + 4.9T^2$	A1	1.1b
Correct strategy: sub for T and find s	M1	3.1b
$d = 3.5 - 2.4752 - 1$	M1	3.1b
$= 2.5 \text{ (cm)} \quad (2 \text{ SF})$	A1	2.2a

Question 3

$$v = 46 \text{ m s}^{-1}$$

Using the model and vertical motion: $v = u + at$	M1	3.3
$v = 45 \sin 10^\circ + 9.8T$	A1	1.1b
Speed $= ((45 \cos 10^\circ)^2 + v^2)^{0.5}$	M1	3.1b
$46 \text{ (m s}^{-1}) \quad (2 \text{ SF})$	A1	1.1b

Question 4

$$U = 15 \text{ m s}^{-1}$$

Using the model and horizontal motion: $s = ut$	M1
$36 = U \cos \alpha$	A1
Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1
$-18 = U \sin \alpha - \frac{1}{2}gt^2$	A1
Correct strategy for solving the problem by setting up two equations in t and U and solving for U	M1
$U = 15$	A1

Question 5

$$19 \text{ m s}^{-1}$$

Using the model and horizontal motion: $U \cos \alpha$ (12)	B1
Using the model and vertical motion: $v^2 = (U \sin \alpha)^2 + 2(-10)(-7.2)$	M1
$v = 15$	A1
Correct strategy for solving the problem by finding the horizontal and vertical components of velocity and combining using Pythagoras: Speed = $\sqrt{(12^2 + 15^2)}$	M1
$\sqrt{369} = 19 \text{ m s}^{-1}$ (2sf)	A1 ft

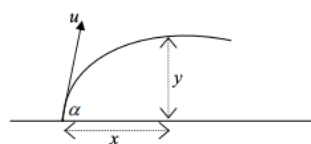
Question 6

$$170 \text{ m}$$

Vert motion $P \rightarrow R$: $s = ut + \frac{1}{2}at^2$ $-36 = 40 \sin \theta t - \frac{9.8}{2}t^2$ $\frac{9.8}{2}t^2 - 40 \sin \theta t - 36 = 0$ $t = \frac{40 \sin 22.54 \pm \sqrt{(40 \sin 22.54)^2 + 4 \times 4.9 \times 36}}{9.8}$ $t = 4.694...$	M1 A1 A1 A1
Horizontal P to R: $s = 40 \cos \theta t$ $= 173 \text{ m}$ (or 170 m)	M1 A1

Question 7

$$A = 2u^2$$



Horiz: $x = u \cos \alpha t$	B1
Vert: $y = u \sin \alpha t - \frac{1}{2}gt^2$ $y = u \sin \alpha \times \frac{x}{u \cos \alpha} - \frac{1}{2}g \times \frac{x^2}{u^2 \cos^2 \alpha}$ $y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$ **	M1 DM1 A1

Question 8

$$A = 2u^2$$

Horizontal distance: $x = u \cos \alpha t$	B1	$\frac{1}{\sqrt{5}}ut$
Vertical distance: $y = u \sin \alpha t - \frac{1}{2}gt^2$	M1A1	$\frac{2}{\sqrt{5}}ut - \frac{1}{2}gt^2$ Condone sign errors and sin/cos confusion
$y = u \sin \alpha \times \frac{x}{u \cos \alpha} - \frac{g}{2} \times \left(\frac{x}{u \cos \alpha} \right)^2$ $= x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha} = 2x - \frac{gx^2}{2u^2} \times \frac{1}{1/5}$ $= 2x - \frac{5g}{2u^2}x^2$	DM1 A1	Substitute for t and α Dependent on previous M1 Obtain given answer from exact working

Question 9

$$t = 2.5 \text{ seconds}$$

$\tan OBA = \frac{52.5}{50} = 1.05$	B1	Correct direction o.e. (accept reciprocal)
$v_y = 1.05 \times 10 = 10.5$	M1	Use trig. with their u_H and correct interpretation of direction to find the vertical component of speed. Working with distances is M0. (condone $10 \div 1.05$)
$(\uparrow), -10.5 = 14 - gt$	DM1	Use suvat to form an equation in t. Dependent on the preceding M.
$t = 2.5$	A1	Correct equation for their u_H .
	A1	For incorrect direction give A0 here. only

Question 10

$$\theta = 79^\circ \text{ and } q = 26.5$$

$30 \cos 60 \times 2 + q \cos \theta \times 2 = 40$	M1	Equation for horizontal distance Need to be using the 40 m
	A1	Correct unsimplified
$30 \sin 60 \times 2 - 4.9 \times 4 = q \sin \theta \times 2 - 4.9 \times 4$ $30 \sin 60 = q \sin \theta$	M1	Equal vertical distance or initial vertical components of velocity
	A1	Correct unsimplified (no error seen)
$q \cos \theta = \pm 5$ $q \sin \theta = 15\sqrt{3}$		
$\tan \theta = 3\sqrt{3}$ ($\tan \theta = 6 \sin 60$)	DM1	Solve for q or θ Dependent on both preceding M marks
$\theta = 79.1 \text{ (79)}$		(1.38 radians) or better
$q = 26.45... = 26.5$	A1	(26 or better) ($10\sqrt{7}$) Both correct and no error seen

Question 11

$$\lambda = 4.3$$

Horizontal motion: $x = 3t$	B1	
Vertical motion: $y = 4t - \frac{g}{2}t^2$	M1	Correct use of <i>suvat</i> . Condone sign error(s)
	A1	
$\left(y = 4 \times \frac{x}{3} - \frac{g}{2} \times \frac{x^2}{9} \right), \lambda = - \left(\frac{4\lambda}{3} - \frac{g\lambda^2}{18} \right)$	M1	Use $y = -x$ and form an equation in one variable
$\frac{7\lambda}{3} = \frac{g\lambda^2}{18}$	M1	solve for λ
$\lambda = \frac{42}{g} \text{ or } 4.3 \text{ (4.29)}$	A1 (6)	Not $\frac{30}{7}$

Question 12

"73.3" and "below"

At A: $v \rightarrow 3 \text{ (m s}^{-1}\text{)}$	B1	
$v \uparrow \quad 4 - g \times \frac{14}{g}$	M1	Complete method using <i>suvat</i> to find $v \uparrow$ with their t or λ
$= -10 \text{ (m s}^{-1}\text{)}$	A1	Accept +10 with direction confirmed by diagram
Speed $= \sqrt{(\text{their } 10)^2 + (3)^2}$	DM1	Dependent on the first M1 in (b)
$= \sqrt{109} \text{ (m s}^{-1}\text{)}$	A1	(10.4) Allow for $v \uparrow = 10$
$\tan^{-1}\left(\frac{\text{their } 10}{3}\right)$ or $\tan^{-1}\left(\frac{3}{\text{their } 10}\right)$	DM1	Use trig to find a relevant angle. Dependent on the first M1 in (b)
Direction $= 73.3^\circ$ below the horizontal	A1	(1.28 radians) Accept direction $3\mathbf{i} - 10\mathbf{j}$ Do not accept a bearing

Question 13

$u = 72 \text{ m s}^{-1}$ and $\theta = 47^\circ$

After 4 seconds from O, horizontal speed $= u \cos \theta$	B1	
Vertical component of speed at A $= u + at$	M1	Complete method using <i>suvat</i> to find v .
$= u \sin \theta - 4g$	A1	
At A, components are $15 \cos 20$ (horizontal) and $15 \sin 20$ (vertical)	B1	
$u \cos \theta = 15 \cos 20$ $u \sin \theta = 15 \sin 20 + 4g$	DM1	Form simultaneous equations in u and θ and attempt to solve for u or θ . Depends on the previous M1
$\theta = 72.4$ (72)	A1	Remember - A0 for the first overspecified answer
$u = 46.5$ (47)	A1	