Integration

Remember, integration is the reverse process of differentiation — so powers should increase when you integrate (I've been doing integration for years and I've only just thought of that way of remembering it).

1 Find $\int (4x^3 + 6x + 3)\,dx$.

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(3 marks)

2 Find $\int \left(2\sqrt{x} + \frac{1}{x^3}\right) dx$.

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(3 marks)

3 Find $\int \left(\frac{x^2 + 3}{\sqrt{x}}\right) dx$.

.....

(3 marks)

4 A curve that passes through the point (0, 0) has derivative $\frac{dy}{dx} = 3x^2 + 6x - 4$. Find the equation of the curve.

You're given the derivative of the curve — so integrate to find its equation.

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(4 marks)

5 The curve C has the equation $y = f(x)$, $x > 0$. $f'(x)$ is given as $2x + 5\sqrt{x} + \frac{6}{x^2}$.

A point P on curve C has the coordinates (3, 7). Find $f(x)$, giving your answer in its simplest form.

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(6 marks)

Integration

6 Evaluate $\int_{p}^{4p} \left(\frac{1}{\sqrt{x}} - 4x^3\right) dx$, where $p > 0$, leaving your answer in terms of p.

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(4 marks)

7 Region A is bounded by the curve $y = \frac{2}{\sqrt{x^3}}$ $(x > 0)$, the x-axis and the lines $x = 2$ and $x = 4$.

Show that the area of A is $2\sqrt{2} - 2$.

(5 marks)

8 Find the possible values of k that satisfy $\int_{\sqrt{2}}^{2} (8x^3 - 2kx)\, dx = 2k^2$, where k is a constant.

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(5 marks)

Integration

9 A curve has equation $y = f(x)$, where $\frac{dy}{dx} = 4(1 - x)$.
The curve passes through the point A, with coordinates (2, 6).

a) Find the equation of the curve.

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(4 marks)

b) Hence find the area of the region under the curve that lies above the x-axis.

Sketch the graph first to help you see what area you need to find.

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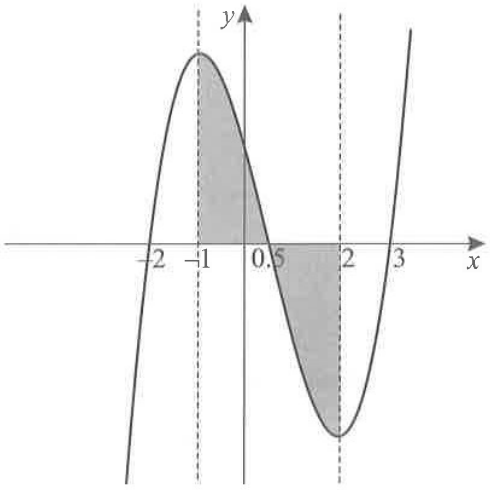
(6 marks)

10 The curve $y = 2x^3 - 3x^2 - 11x + 6$ is shown below. It crosses the x-axis at (–2, 0), (0.5, 0) and (3, 0).
Find the area of the shaded region bounded by the curve, the x-axis and the lines $x = -1$ and $x = 2$.



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(6 marks)

EXAM TIP

Knowing your index laws inside out is a big help when it comes to integration — always write out fractions and roots as powers of x before integrating (it'll save a lot of heartache further down the line). The examiners might try to confuse you by sneaking some extra algebra in there or wording the question weirdly, but the basic integration process is the same each time.

Score