- ARITHMETIC SEQUENCES

Difference between consecutive terms is THE SAME

U_1	U_2	U_3	U_4	U_n
2	8	14	20	$a+(n-1)d$
+6	+6	+6		

U_n = n^{th} term U_1 = 1st term $(n=1)$

a = 1st term

d = common difference

- SUM OF ARITHMETIC SERIES

5, 7, 9, 11 sequence

5 + 7 + 9 + 11 series

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

or

$$S_n = \frac{n}{2}(a + l)$$

l = last term

– PROVE OF SUM (ARITHMETIC SERIES)

$$S_n = a + a+d + a+2d + \ldots\ldots + a+(n-1)d$$

switch order $\quad S_n = a+(n-1)d + \ldots\ldots + a+2d + a+d + a$

Add together $\quad 2S_n = 2a+(n-1)d + 2a+(n-1)d + \ldots\ldots + 2a(n-1)d$

simplify $\quad 2S_n = n[2a+(n-1)d]$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

– RECURRANCE RELATIONS

Rule to get to the next term ONLY.

$$U_{n+1} = U_n + 4$$

$$U_1 = 7$$

$$U_2 = U_1 + 4 = 7 + 4 = 11$$

$$U_3 = U_2 + 4 = 11 + 4 = 15$$

etc...

NOTE:

$U_n + 1 > U_n$ sequence increasing

$U_n + 1 < U_n$ sequence decreasing

$U_n + k = U_n$ terms repeat in cycle (order = number of terms in a cycle)

– GEOMETRIC SEQUENCES

Difference between consecutive terms is COMMON MULTIPLIER

U_1	U_2	U_3	U_4	U_n
2	4	8	16	ar^{n-1}

$$2 \underbrace{\ \ }_{\times 2} 4 \underbrace{\ \ }_{\times 2} 8 \underbrace{\ \ }_{\times 2} 16$$

a = 1st term

r = common ratio/multipler

– GEOMETRIC SERIES

$$S_n = \frac{a(1-r^n)}{1-r}$$

or

$$S_n = \frac{a(r^n-1)}{r-1}$$

– SUM TO INFINITY OF GEOMETRIC SERIES

$$S_\infty = \frac{a}{1-r}$$

$|r| < 1$ series is convergent
(terms in series get smaller)

– SIGMA NOTATION

$\sum$ sum of

$\sum_{r=1}^{20}$ sum of first 20 terms

$\sum_{r=1}^{4} 3r+1$

1ST term	$3(1)+1 = 4$
2nd term	$3(2)+1 = 7$
3rd term	$3(3)+1 = 10$
4th term	$3(4)+1 = 13$

– PROVE OF SUM (GEOMETRIC SERIES)

A $\quad S_n = a + ar + ar^2 + \ldots\ldots + ar^{n-1}$

B $\quad rS_n = ar + ar^2 + ar^3 + \ldots\ldots + ar^n$

A - B $\quad S_n - rS_n = a - ar^n$

factorise $\quad S_n(1-r) = a(1-r^n)$

$$S_n = \frac{a(1-r^n)}{1-r}$$