Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

A level Further Mathematics
Core Pure Mathematics
Practice Paper 3

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **7** questions in this question paper. The total mark for this paper is **70**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (a) Express $\dfrac{2}{4r^2 - 1}$ in partial fractions.

(2)

(b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{4r^2 - 1} = \frac{n}{2n+1}$$

(3)

(Total 5 marks)

2.

$$z_1 = 3\mathrm{i} \text{ and } z_2 = \frac{6}{1 + \mathrm{i}\sqrt{3}}$$

(a) Express z_2 in the form $a + \mathrm{i}b$, where a and b are real numbers.

(2)

(b) Find the modulus and the argument of z_2, giving the argument in radians in terms of π.

(4)

(c) Show the three points representing z_1, z_2 and $(z_1 + z_2)$ respectively, on a single Argand diagram.

(2)

(Total 8 marks)

---

3. The curve C_1 has equation $y = 3\sinh 2x$, and the curve C_2 has equation $y = 13 - 3e^{2x}$.

(a) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes.

(4)

(b) Solve the equation $3\sinh 2x = 13 - 3e^{2x}$, giving your answer in the form $\frac{1}{2} \ln k$, where k is an integer.

(5)

(Total 9 marks)

---

4. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (3r^2 + 8r + 3) = \frac{1}{2} n(2n+5)(n+3)$$

for all positive integers n.

(5)

Given that

$$\sum_{r=1}^{12} \left(3r^2 + 8r + 3 + k(2^{r-1})\right) = 3520$$

(b) find the exact value of the constant k.

(4)

(Total 9 marks)

5.

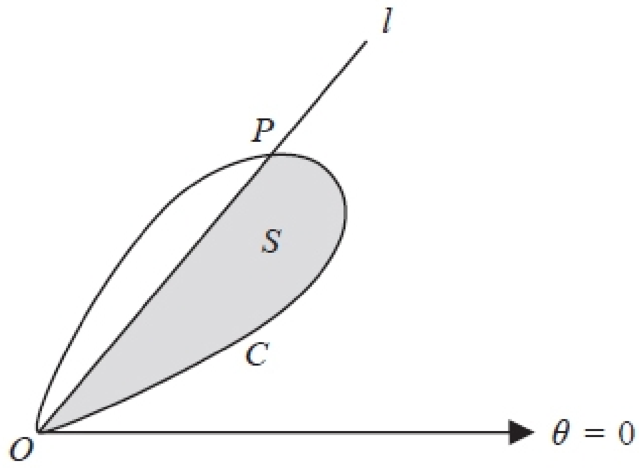

Figure 1

Figure 1 shows a curve C with polar equation $r = a\sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, and a half-line l.

The half-line l meets C at the pole O and at the point P. The tangent to C at P is parallel to the initial line. The polar coordinates of P are (R, ϕ).

(a) Show that $\cos\phi = \dfrac{1}{\sqrt{3}}$

(6)

(b) Find the exact value of R.

(2)

The region S, shown shaded in Figure 1, is bounded by C and l.

(c) Use calculus to show that the exact area of S is

$$\frac{1}{36}a^2\left(9\arccos\left(\frac{1}{\sqrt{3}}\right)+\sqrt{2}\right)$$

(7)

(Total 15 marks)

6. $$f(n) = 2^n + 6^n$$

(a) Show that $f(k + 1) = 6f(k) - 4(2^k)$.

(3)

(b) Hence, or otherwise, prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is divisible by 8.

(4)

(Total 7 marks)

---

7. At the start of the year 2000, a survey began of the number of foxes and rabbits on an island.

At time t years after the survey began, the number of foxes, f, and the number of rabbits, r, on the island are modelled by the differential equations

$$\frac{df}{dt} = 0.2f + 0.1r$$

$$\frac{dr}{dt} = -0.2f + 0.4r$$

(a) Show that $\frac{d^2 f}{dt^2} - 0.6\frac{df}{dt} + 0.1f = 0$

(3)

(b) Find a general solution for the number of foxes on the island at time t years.

(4)

(c) Hence find a general solution for the number of rabbits on the island at time t years.

(3)

At the start of the year 2000 there were 6 foxes and 20 rabbits on the island.

(d) (i) According to this model, in which year are the rabbits predicted to die out?

(ii) According to this model, how many foxes will be on the island when the rabbits die out?

(iii) Use your answers to parts (i) and (ii) to comment on the model.

(7)

(Total 17 marks)

TOTAL FOR PAPER: 70 MARKS