KS5 "Full Coverage": Exponentials & Logarithms

This worksheet is designed to cover one question of each type seen in past papers, for each GCSE Higher Tier topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com, logging on, *Practise → Past Papers* (or *Library → Past Papers* for teachers), and using the 'Revision' tab.

Question 1
Categorisation: Solve equations of the form $a^x = b$.

[Edexcel C2 May 2011 Q3a]

Find, giving your answer to 3 significant figures where appropriate, the value of x for which

$$5^x = 10$$

$x = $ ....

Question 2
Categorisation: Solve equations of the form $log_x a = b$.

[Edexcel C2 Jan 2010 Q5a]

Find the positive value of x such that

$$log_x 64 = 2$$

....

Question 3

Categorisation: Solve equations of the form $e^{f(x)} = a$

[Edexcel C3 June 2017 Q2a]

Find the exact solution, in their simplest form, to the equation

$$e^{3x-9} = 8$$

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Question 4

Categorisation: Solve modulus equations involving exponential functions.

[Edexcel C3 June 2015 Q2c]

Let $(x) = 2e^x - 5$, $x \in \mathbb{R}$.

Find the exact solutions of the equation $|f(x)| = 2$

....

Question 5

Categorisation: Solve general log equations.

[Edexcel AS Specimen Papers P1 Q9]

Find any real values of x such that

$$2 \log_4(2 - x) - \log_4(x + 5) = 1$$

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Question 6

Categorisation: As above.

[Edexcel C3 June 2017 Q2b] Find the exact solution, in their simplest form, to the equation

$$ln\,(2y+5) = 2 + \,ln\,(4-y)$$

....

Question 7

Categorisation: Solve equations involving ln.

[Edexcel C3 June 2013 Q6a]

Find algebraically the exact solutions to the equation

$$ln\,(4-2x) + \,ln\,(9-3x) = 2\,ln\,(x+1)\,, \quad -1 < x < 2$$

....

Question 8

Categorisation: Solve equations where the resulting non-log equation contains a square root.

[Edexcel C2 May 2014(R) Q8ii]

Use algebra to find the values of x for which

$$log_{\,2}(x+15) - 4 = \frac{1}{2}\,log_{\,2}x$$

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Question 9

Categorisation: Change the subject of an equation involving logs.

[Edexcel C2 May 2016 Q8i] Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, a > 2,$$

express b in terms of a .

$$b = \dots\dots\dots\dots\dots\dots$$

Question 10

Categorisation: Solve log equations in the context of geometric series.

[Edexcel C2 May 2014 Q6c Edited]

The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$. The sum to infinity of the series is S_∞ .

The sum to N terms of the series S_N .

Given that $S_\infty = 160$, find the smallest value of N for which $S_\infty - S_N < 0.5$.

$$N = \dots\dots\dots\dots\dots\dots$$

Question 11

Categorisation: As above.

[Edexcel C2 May 2011 Q6d Edited] The second and third terms of a geometric series are 192 and 144 respectively. Given that the common ratio is $\frac{3}{4}$ and that the first term is 256 , find the smallest value of n for which the sum of the first n terms of the series exceeds 1000.

$n = $ ....

Question 12

Categorisation: Solve an equation which is quadratic in an exponential function.

[Edexcel C2 Jan 2011 Q8b]

Solve the equation

$$7^{2x} - 4(7^x) + 3 = 0$$

giving your answers to 2 decimal places where appropriate.

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Question 13

Categorisation: As above, but with a more complex power.

[Edexcel AS SAM P1 Q12b]

A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

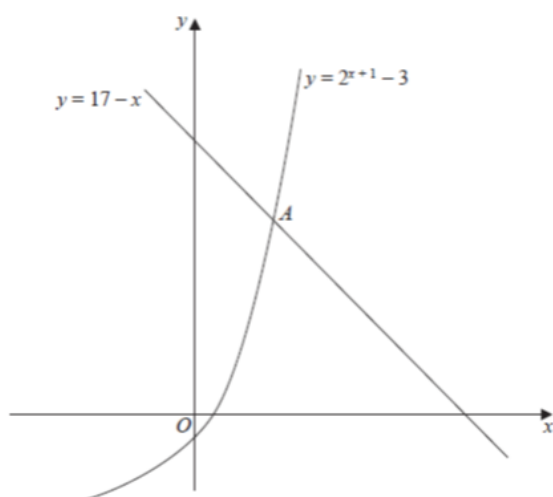Find the exact solution to the equation.

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Question 14

Categorisation: Consider the intersection of an exponential graph with another curve.

[Edexcel C3 June 2015 Q6a Edited] Figure 1 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point A.

Show that the x-coordinate of A satisfies the equation

$$x = \frac{\ln (a - x)}{\ln b} - 1$$

where a and b are constants to be found.

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Question 15

Categorisation: Solve an equation involving exponentials with different bases.

[Edexcel C3 June 2014 Q2b]

Find the exact solution, in its simplest form, to the equation

$$3^x e^{4x} = e^7$$

$x = $ ....

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Question 16

Categorisation: As above.

[Edexcel C3 June 2013 Q6b]

Find algebraically the exact solutions to the equation

$$2^x e^{3x+1} = 10$$

....

Question 17

Categorisation: Determine a composite function involving ln or e^x

[Edexcel A2 SAM P2 Q4a] Given

$f(x) = e^x, \qquad x \in \mathbb{R}$

$g(x) = 3 \ln x, \qquad x > 0, \ x \in \mathbb{R}$

find an expression for $gf(x)$, simplifying your answer.

$gf(x) =$ ....

Question 18

Categorisation: Determine the inverse of a logarithmic or exponential equation, and determine the domain or range of an exponential function.

[Edexcel C3 June 2014(R) Q6b] The function f is defined by

$f : x \to e^{2x} + k^2, \quad x \in \mathbb{R}, \quad k$ is a positive constant.

Find f^{-1} and state its domain.

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Question 27

Categorisation: Determine the inverse of a logarithm function.

[Edexcel C3 June 2007 Q5b Edited] The functions f and g are defined by

$$f : x \rightarrow ln\,(2x - 1)\,, \quad x \in \mathbb{R}\,, \quad x > \frac{1}{2}$$

$$g : x \rightarrow \frac{2}{x-3}\,, \quad x \in \mathbb{R},\quad x \neq 3$$

Find the inverse function $f^{-1}(x)$.

$f^{-1}(x) =$ ....

Question 19

Categorisation: Solve an equation involving use of a function.

[Edexcel C3 June 2014(R) Q6c] The function g is defined by

$$g : x \rightarrow ln\,(2x)\,, \quad\quad x > 0$$

Solve the equation

$$g(x) + g(x^2) + g(x^3) = 6$$

giving your answer in its simplest form.

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Question 20

Categorisation: Determine the range of a composite function involving exponentials or logarithms.

[Edexcel C3 June 2011 Q4d] The function f is defined by

$$f:x \rightarrow 4 - \ln(x+2), \quad x \in \mathbb{R}, \quad x \geq -1$$

The function g is defined by

$$g:x \rightarrow e^{x^2} - 2, \quad x \in \mathbb{R}$$

Find the range of g .

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Question 21

Categorisation: Simplify a logarithmic expression.

[Edexcel C2 May 2013(R) Q6b Edited] Given that $\log_3 x = a$, find $\log_3\left(\frac{x^5}{81}\right)$ in terms of a , giving each answer in its simplest form.

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Question 22

Categorisation: Use a cubic/quadratic factorisation to solve a related log/exponential equation.

[Edexcel C2 May 2013 Q3c Edited]

$$f(x) = 2x^3 - 5x^2 - 9x + 18$$

$$f(x) = (x-3)(2x-3)(x+2)$$

Given that

$$g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18$$

find the values of y that satisfy $g(y) = 0$, giving your answers to 2 decimal places where appropriate.

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Question 23

Categorisation: As above.

[Edexcel A2 Specimen Papers P1 Q5c Edited]

$$f(x) = x^3 + 4x^2 - 4x + 48$$

It can be shown that $f(x) = (x + 6)(x^2 - 2x + 8)$

It can also be shown that $2 \log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3$ is equivalent to $x^3 + 4x^2 - 4x + 48 = 0$.

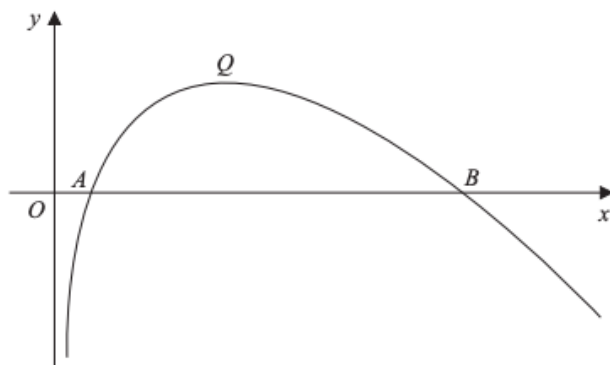How many roots as the equation $2 \log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3$?

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Question 24

Categorisation: Determine the x or y intercepts of a logarithmic/exponential function.

[Edexcel A2 Specimen Papers P2 Q6a]

Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x) \ln x, \ x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 2.

Find the x coordinate of A and the x coordinate of B.

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Question 25

Categorisation: Determine the equation of the normal/tangent to a curve.

[Edexcel AS SAM P1 Q15]
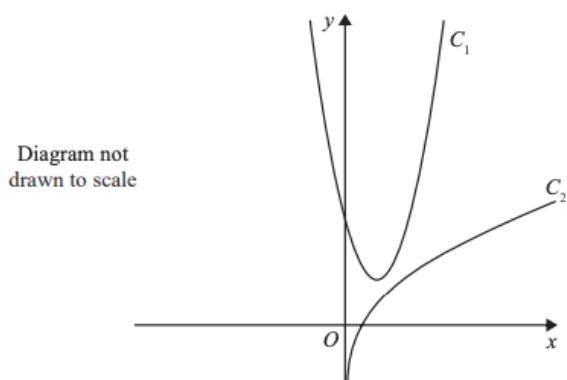

Diagram not drawn to scale

Figure 3

The curve C_1 , shown in Figure 3, has equation $y = 4x^2 - 6x + 4$

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1 . The curve C_2 , also shown in Figure 3, has equation $y = \frac{1}{2}x + ln\,(2x)$. The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q. *(Solutions based entirely on graphical or numerical methods are not acceptable.)*

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Question 28

Categorisation: Determine the tangent to an exponential curve.

[Edexcel C3 June 2008 Q1b Edited]

The point P lies on the curve with equation

$$y = 4e^{2x+1}$$

The y -coordinate of P is 8 .   The x -coordinate of P is $\frac{1}{2}(\,ln\,2 - 1)$.
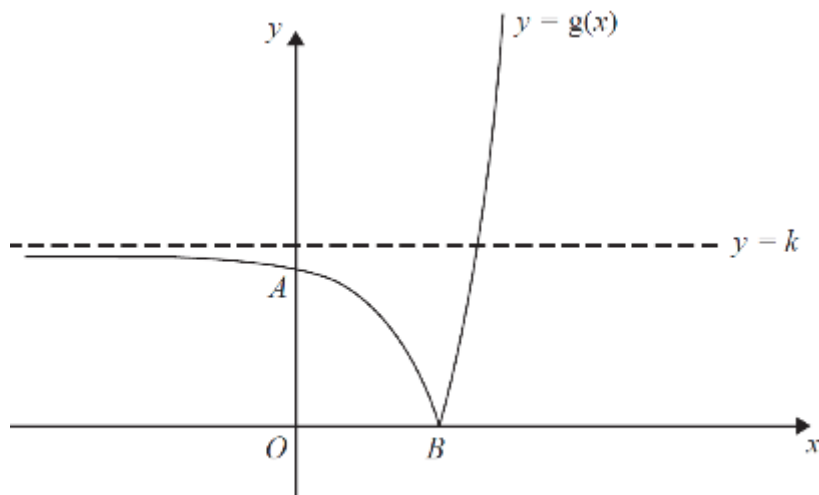
Find the equation of the tangent to the curve at the point P in the form $= ax + b$, where a and b are exact constants to be found.

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Question 26

Categorisation: Consider roots/y-intercepts of exponential graphs involving a modulus.

[Edexcel C3 June 2016 Q4b Edited]

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = g(x)$, where

$$g(x) = |4e^{2x} - 25|, \quad x \in \mathbb{R}.$$

The curve cuts the y -axis at the point A and meets the x -axis at the point B . The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1.

The equation $g(x) = 2x + 43$ has a positive root at $x = \alpha.$

α is a solution of $x = a \, ln \, (bx + c)$ where a , b and c are to be found.

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Answers

Question 1

$x = 1.43$

Question 2

$x = 8$

Question 3

$x = \ln 2 + 3$

Question 4

$x = \ln\left(\frac{7}{2}\right)$ or $x = \ln\left(\frac{3}{2}\right)$

Question 5

$x = 4 - 4\sqrt{2}$

Question 6

$y = \dfrac{4e^2 - 5}{2 + e^2}$

Question 7

$x = \dfrac{7}{5}$

Question 8

$x = 1$ or $x = 225$

Question 9

$b = \dfrac{a-5}{9}$

Question 10

$N = 44$

Question 11

$n = 14$

Question 12

$x = 0.56$ or $x = 0$

Question 13

$x = \dfrac{\log 9}{\log 2} - 4$

Question 14

$a = 20$, $b = 2$

Question 15

$x = \frac{7}{4 + \ln 3}$

Question 16

$x = \dfrac{-1 + \ln 10}{3 + \ln 2}$

Question 17

$gf(x) = 3x$

Question 18

$f^{-1}(x) = \frac{1}{2} \ln(x - k^2)$,
$Domain := x > k^2$

Question 27

$f^{-1}(x) = \frac{1}{2}(e^x + 1)$

Question 19

$\frac{e}{\sqrt{2}}$

Question 20

$fg(x) \le 4$

Question 21

$\log_3\left(\frac{x^5}{81}\right) = 5a - 4$

Question 22

$y = 1$ or $y = 0.37$

Question 23

"0 OR none OR zero"

Question 24

$x = 1$ and $x = 8$

Question 25

$\left(\frac{1}{2}e^{\frac{7}{4}}, \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}\right)$

Question 28

$y = 16x + 16 - 8\ln 2$

Question 26

$a = \frac{1}{2}$, $b = \frac{1}{2}$, $c = 17$