1a) $$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = e^x$ $\qquad$ $v = 2x + 1$

$\frac{du}{dx} = e^x$ $\qquad$ $\frac{dv}{dx} = 2$

$$\frac{dy}{dx} = \frac{e^x(2x+1) - 2e^x}{(2x+1)^2}$$

b/ $u = \ln(x^2+1)$ $\qquad$ $v = 3x + 2$

$\frac{du}{dx} = \frac{2x}{x^2+1}$ $\qquad$ $\frac{dv}{dx} = 3$

$$\frac{\frac{2x(3x+2)}{x^2+1} - 3\ln(x^2+1)}{(3x+2)^2}$$

$$\frac{2x(3x+2) - 3(x^2+1)\ln(x^2+1)}{(3x+2)^2(x^2+1)}$$

2

$$y = \cancel{2} \frac{3x}{2x-1}$$

$u = 3x \qquad v = 2x - 1$

$\frac{du}{dx} = 3 \qquad \frac{dv}{dx} = 2$

$$\frac{dy}{dx} = \frac{3(2x-1) - 2(3x)}{(2x-1)^2}$$

$$= \frac{6x - 3 - 6x}{(2x-1)^2}$$

$$= \frac{-3}{(2x-1)^2}$$

when $x = 1 \qquad \frac{dy}{dx} = \frac{-3}{1}$

$$= -3$$

$$y = \frac{3}{1} = 3$$

$y = -3x + c \qquad (1, 3)$

$3 = -3 + c$

$6 = c$

$$\underline{y = -3x + 6}$$

3/

$$y = \frac{e^x + 1}{e^x + 3}$$

$u = e^x + 1$ $\quad$ $v = e^x + 3$

$\frac{du}{dx} = e^x$ $\quad$ $\frac{dv}{dx} = e^x$

$$\frac{dy}{dx} = \frac{e^x(e^x + 3) - e^x(e^x + 1)}{(e^x + 3)^2}$$

$$= \frac{e^{2x} + 3e^x - e^{2x} - e^x}{(e^x + 3)^2}$$

$$= \frac{2e^x}{(e^x + 3)^2}$$

when $x = 0$ $\quad$ $\frac{dy}{dx} = \frac{2}{16} = \frac{1}{8}$

$$y = \frac{1 + 1}{1 + 3} = \frac{2}{4} = \frac{1}{2}$$

gradient of normal : -8

$y = -8x + c$ $\quad$ $(0, \frac{1}{2})$

$\underline{\underline{y = -8x + \frac{1}{2}}}$

4)

$$y = \frac{x}{9+x^2}$$

$u = x \qquad v = 9 + x^2$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = 2x$

$$\frac{dy}{dx} = \frac{9 + x^2 - 2x^2}{(9+x^2)^2}$$

$$= \frac{9 - x^2}{(9+x^2)^2}$$

stationary points where $\frac{dy}{dx} = 0$

$$\frac{9-x^2}{(9+x^2)^2} = 0$$

$$9 - x^2 = 0$$

$$(3+x)(3-x) = 0$$

$x = -3 \qquad x = 3$

$$\frac{dy}{dx} = \frac{9 - x^2}{(9+x^2)^2}$$

$u = 9 - x^2 \qquad v = (9+x^2)^2$

$\frac{du}{dx} = -2x \qquad \frac{dv}{dx} = 2(9+x^2)\,2x$

$= 4x(9+x^2)$

$$\frac{d^2y}{dx^2} = \frac{-2x(9+x^2)^2 - (9-x^2)(4x)(9+x^2)}{(9+x^2)^4}$$

$$= \frac{-2x(9+x^2) - 4x(9-x^2)}{(9+x^2)^3}$$

$$= \frac{-18x - 2x^3 - 36x + 4x^3}{(9+x^2)^3}$$

$$= \frac{2x^3 - 54x}{(9+x^2)^3}$$

when $x = -3$ $\quad \dfrac{d^2y}{dx^2} = \dfrac{2(-3)^3 - 54(-3)}{(9 + (-3)^2)^3}$

$= \dfrac{1}{54}$ Min

when $x = 3$ $\quad \dfrac{d^2y}{dx^2} = -\dfrac{1}{54}$ Max.

when $x = -3$ $\quad y = \dfrac{-3}{9 + (-3)^2} = -\dfrac{1}{6}$

$x = 3$ $\quad y = \dfrac{1}{6}$

Minimum point at $\left(-3, -\dfrac{1}{6}\right)$

Maximum point at $\left(3, \dfrac{1}{6}\right)$

5a/

$$f(x) = \frac{x-5}{2x+3} + \frac{2(\cancel{x+2})}{(2x+3)(\cancel{x+2})}$$

$$= \frac{x-5}{2x+3} + \frac{2}{2x+3}$$

$$= \frac{x-3}{2x+3}$$

b/

$u = x - 3$ $\qquad$ $v = 2x + 3$

$\frac{du}{dx} = 1$ $\qquad$ $\frac{dv}{dx} = 2$

$$f'(x) = \frac{2x+3 - 2(x-3)}{(2x+3)^2}$$

$$= \frac{2x+3-2x+6}{(2x+3)^2}$$

$$= \frac{9}{(2x+3)^2}$$

6a/

$$f(x) = \frac{4x+1}{2x+3} - \frac{2(2x-3)}{(2x+3)(2x-3)}$$

$$= \frac{4x+1}{2x+3} - \frac{2}{2x+3}$$

$$= \frac{4x-1}{2x+3}$$

b/

$u = 4x - 1$ $\qquad$ $v = 2x + 3$

$\frac{du}{dx} = 4$ $\qquad$ $\frac{dv}{dx} = 2$

$$f'(x) = \frac{4(2x+3) - 2(4x-1)}{(2x+3)^2}$$

$$= \frac{8x + 12 - 8x + 2}{(2x+3)^2}$$

$$= \frac{14}{(2x+3)^2}$$