Trigonometric identities

Gold

Show that:

$$(\cos x + \tan x \sin x)^2 \equiv \sec^2 x .$$

Hence explain why $\cos x + \tan x \sin x = \frac{1}{2}$ has no solutions.

Silver

Solve the equation:

$$\sec\theta \cot\theta = 2$$

for $0 \leq \theta \leq 2\pi$.

Bronze

Simplify fully:

$$\frac{1}{\cot\theta}\cos^2\theta \operatorname{cosec}\theta .$$