- 1 Given vectors $\mathbf{a} = 3\mathbf{i} \mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 6\mathbf{i} 3\mathbf{j} 2\mathbf{k}$ and $\mathbf{c} = \mathbf{i} + \mathbf{j} 3\mathbf{k}$, work out
 - a i $\mathbf{a} \mathbf{b}$ ii $2\mathbf{a} + 5\mathbf{c}$ iii $|\mathbf{b}|$ iv $|\mathbf{c} \mathbf{a}|$ v unit vector $\hat{\mathbf{b}}$
 - vi the angle between b and the positive x-, y- and z-directions. Show your working.
 - b a vector parallel to b with magnitude 28
 - **c** The values of p, q and r if $p\mathbf{a} + q\mathbf{c} = 3\mathbf{i} 5\mathbf{j} + r\mathbf{k}$
- 2 The vector r has magnitude 8 and makes angles of 27°, 85° and 63.5° with the positive x-, y- and z-directions respectively. Express r in component form.
- 3 A vector p has magnitude 12 and makes angles of 68° and 75° with the positive y- and z-directions respectively.
 - a Work out the two possible angles between p and the positive x-direction.
 - **b** Use your answer to part **a** to express the two possible vectors **p** in component form.
- 4 Points A, B and C have position vectors

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} \text{ and } \mathbf{c} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix} \text{ respectively.}$$

a Work out the lengths of the sides of triangle ABC

- b Deduce that the triangle is right-angled.
- c State one other fact about the triangle.
- 5 Points A and B have position vectors a = 7i + 2j + 5k and b = 5i - 6j + k respectively. C is the midpoint of AB. Work out
 - a The position vector, c, of C,
 - **b** The distance of C from the origin,
 - **c** The unit vector, $\hat{\mathbf{c}}$, in the direction of \mathbf{c} .
- 6 Points A and B have position vectors a = 9i + 2j - 4k and b = 3i + 5j + k respectively. The point C lies on AB, and AC: CB = 7:3. Work out the position vector of C
- 7 Given vectors

$$\mathbf{p} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}, \mathbf{q} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}, \text{ work out }$$

- a p+q-2r,
- $|\mathbf{p} \mathbf{q}|$
- c A vector of magnitude 15 in the direction of r,
- **d** The angle between **r** and the positive *x*-direction,
- e The values of λ and μ if $\lambda \mathbf{p} + \mu \mathbf{q} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$

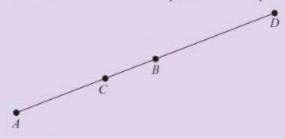
- Points A, B and C have positions vectors 2i+3j-3k, i-4j+2k and 3i-5j+k respectively. Prove that ABC is a right-angled triangle.
- 2 The points A, B and C with position vectors a = i + 2j, b = 2i + j - 2k and c = 3i - j + k are three vertices of a parallelogram. Work out all possible positions of the fourth vertex, D
- 3 The vector V has magnitude 6 and makes the same angle with each of the positive x-, y- and z-directions. Evaluate the possible values of V
- In this question east, north and upwards are the positive x-, y- and z-directions respectively. A child, standing at the origin O, flies a toy drone. She first sends it to A, 25 m north and at a height of 15 m, then for 35 m in the direction of the vector 6i + 3j + 2k to B
 - a Work out the angle of elevation of B from O
 - b At B the drone's battery runs out and it falls to the ground. How far does she have to walk to retrieve it?
- 5 ABCD is a tetrahedron. The position vectors of its vertices are a, b, c and d respectively.

- P, Q and R are the respective midpoints of AB, AD and BC. S divides PC in the ratio 1:2. T is the midpoint of QR
- a Show that D, T and S are collinear.
- **b** Work out the ratio DT: TS

Challenge

6 The points *A*, *B*, *C* and *D* lie on a straight line, as shown.

$$AC: CB = AD: BD = \lambda: \mu$$
, where $\lambda > \mu$



- a If $\lambda = 3$, $\mu = 2$ and A and B have position vectors $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $7\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$ respectively, work out the length of CD.
- **b** Show that in general $CD:AB=2\lambda\mu:(\lambda^2-\mu^2)$