1 Given vectors $\mathbf{a} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 6\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{c} = \mathbf{i} + \mathbf{j} - 3\mathbf{k}$, work out

a **i** $\mathbf{a} - \mathbf{b}$ **ii** $2\mathbf{a} + 5\mathbf{c}$ **iii** $|\mathbf{b}|$
iv $|\mathbf{c} - \mathbf{a}|$ **v** unit vector $\hat{\mathbf{b}}$
vi the angle between $\mathbf{b}$ and the positive x-, y- and z-directions. Show your working.

b a vector parallel to $\mathbf{b}$ with magnitude 28

c The values of p, q and r if $p\mathbf{a} + q\mathbf{c} = 3\mathbf{i} - 5\mathbf{j} + r\mathbf{k}$

2 The vector $\mathbf{r}$ has magnitude 8 and makes angles of 27°, 85° and 63.5° with the positive x-, y- and z-directions respectively. Express $\mathbf{r}$ in component form.

3 A vector $\mathbf{p}$ has magnitude 12 and makes angles of 68° and 75° with the positive y- and z-directions respectively.

a Work out the two possible angles between $\mathbf{p}$ and the positive x-direction.

b Use your answer to part **a** to express the two possible vectors $\mathbf{p}$ in component form.

4 Points A, B and C have position vectors $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}$ respectively.

a Work out the lengths of the sides of triangle ABC

b Deduce that the triangle is right-angled.

c State one other fact about the triangle.

5 Points A and B have position vectors $\mathbf{a} = 7\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} - 6\mathbf{j} + \mathbf{k}$ respectively. C is the midpoint of AB. Work out

a The position vector, $\mathbf{c}$, of C,

b The distance of C from the origin,

c The unit vector, $\hat{\mathbf{c}}$, in the direction of $\mathbf{c}$.

6 Points A and B have position vectors $\mathbf{a} = 9\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 5\mathbf{j} + \mathbf{k}$ respectively. The point C lies on AB, and $AC : CB = 7 : 3$. Work out the position vector of C

7 Given vectors $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$, work out

a $\mathbf{p} + \mathbf{q} - 2\mathbf{r}$,

b $|\mathbf{p} - \mathbf{q}|$,

c A vector of magnitude 15 in the direction of $\mathbf{r}$,

d The angle between $\mathbf{r}$ and the positive x-direction,

e The values of λ and μ if $\lambda\mathbf{p} + \mu\mathbf{q} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$

1 Points A, B and C have positions vectors $2\mathbf{i}+3\mathbf{j}-3\mathbf{k}$, $\mathbf{i}-4\mathbf{j}+2\mathbf{k}$ and $3\mathbf{i}-5\mathbf{j}+\mathbf{k}$ respectively. Prove that ABC is a right-angled triangle.

2 The points A, B and C with position vectors $\mathbf{a}=\mathbf{i}+2\mathbf{j}$, $\mathbf{b}=2\mathbf{i}+\mathbf{j}-2\mathbf{k}$ and $\mathbf{c}=3\mathbf{i}-\mathbf{j}+\mathbf{k}$ are three vertices of a parallelogram. Work out all possible positions of the fourth vertex, D

3 The vector $\mathbf{V}$ has magnitude 6 and makes the same angle with each of the positive x-, y- and z-directions. Evaluate the possible values of $\mathbf{V}$

4 In this question east, north and upwards are the positive x-, y- and z-directions respectively. A child, standing at the origin O, flies a toy drone. She first sends it to A, 25 m north and at a height of 15 m, then for 35 m in the direction of the vector $6\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ to B

a Work out the angle of elevation of B from O

b At B the drone's battery runs out and it falls to the ground. How far does she have to walk to retrieve it?

5 $ABCD$ is a tetrahedron. The position vectors of its vertices are $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ respectively. P, Q and R are the respective midpoints of AB, AD and BC. S divides PC in the ratio $1:2$. T is the midpoint of QR

a Show that D, T and S are collinear.

b Work out the ratio $DT:TS$

Challenge

6 The points A, B, C and D lie on a straight line, as shown.

$AC:CB=AD:BD=\lambda:\mu$, where $\lambda>\mu$

a If $\lambda=3$, $\mu=2$ and A and B have position vectors $2\mathbf{i}+\mathbf{j}+3\mathbf{k}$ and $7\mathbf{i}+3\mathbf{j}+9\mathbf{k}$ respectively, work out the length of CD.

b Show that in general $CD:AB=2\lambda\mu:(\lambda^2-\mu^2)$