Pearson
Edexcel GCE

A level Further Mathematics
Further Mechanics 1
Practice Paper 1

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **7** questions in this question paper. The total mark for this paper is **75**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. A van of mass 900 kg is moving down a straight road that is inclined at an angle θ to the horizontal, where $\sin \theta = \dfrac{1}{30}$. The resistance to motion of the van has constant magnitude 570 N. The engine of the van is working at a constant rate of 12.5 kW.

At the instant when the van is moving down the road at 5 m s^{-1}, the acceleration of the van is a m s^{-2}.

Find the value of a.

(Total 5 marks)

2. A ball of mass 0.4 kg is moving in a horizontal plane when it is struck by a bat. The bat exerts an impulse $(-5\mathbf{i} + 3\mathbf{j})$ N s on the ball. Immediately after receiving the impulse the ball has velocity $(12\mathbf{i} + 15\mathbf{j})$ m s^{-1}.

Find

(a) the speed of the ball immediately before the impact,

(4)

(b) the size of the angle through which the direction of motion of the ball is deflected by the impact.

(3)

(Total 7 marks)

3. A small smooth ball of mass m is falling vertically when it strikes a fixed smooth plane which is inclined to the horizontal at an angle α, where $0° < \alpha < 45°$. Immediately before striking the plane the ball has speed u. Immediately after striking the plane the ball moves in a direction which makes an angle of 45° with the plane. The coefficient of restitution between the ball and the plane is e. Find, in terms of m, u and e, the magnitude of the impulse of the plane on the ball.

(Total 11 marks)

4.
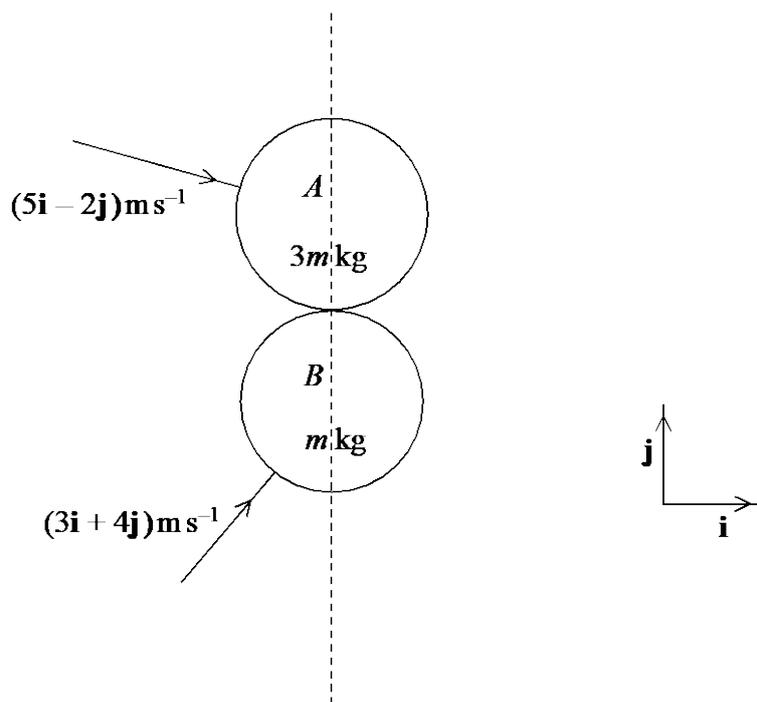
Figure 1

Two smooth uniform spheres A and B have masses $3m$ kg and m kg respectively and equal radii. The spheres are moving on a smooth horizontal surface. Initially, sphere A has velocity $(5\mathbf{i} - 2\mathbf{j})$ m s^{-1} and sphere B has velocity $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1}. When the spheres collide, the line joining their centres is parallel to $\mathbf{j}$, as shown in Figure 1.

The coefficient of restitution between the two spheres is e.

The kinetic energy of sphere B immediately after the collision is 85% of its kinetic energy immediately before the collision.

Find

(a) the velocity of each sphere immediately after the collision,

(9)

(b) the value of e.

(3)

(Total 12 marks)

5. A particle P of mass $2m$ is moving in a straight line with speed $3u$ on a smooth horizontal table. A second particle Q of mass $3m$ is moving in the opposite direction to P along the same straight line with speed u. The particle P collides directly with Q. The direction of motion of P is reversed by the collision. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\dfrac{u}{5}(8e+3)$.

(6)

(b) Find the range of possible values of e.

(4)

The total kinetic energy of the particles before the collision is T. The total kinetic energy of the particles after the collision is kT. Given that $e = \dfrac{1}{2}$,

(c) find the value of k.

(4)

(Total 14 marks)

6. The ends of a light elastic string, of natural length 0.4 m and modulus of elasticity λ newtons, are attached to two fixed points A and B which are 0.6 m apart on a smooth horizontal table. The tension in the string is 8 N.

(a) Show that $\lambda = 16$

(3)

A particle P is attached to the midpoint of the string. The particle P is now pulled **horizontally** in a direction perpendicular to AB to a point 0.4 m from the midpoint of AB. The particle is held at rest by a **horizontal** force of magnitude F newtons acting in a direction perpendicular to AB, as shown in Figure 5 below.
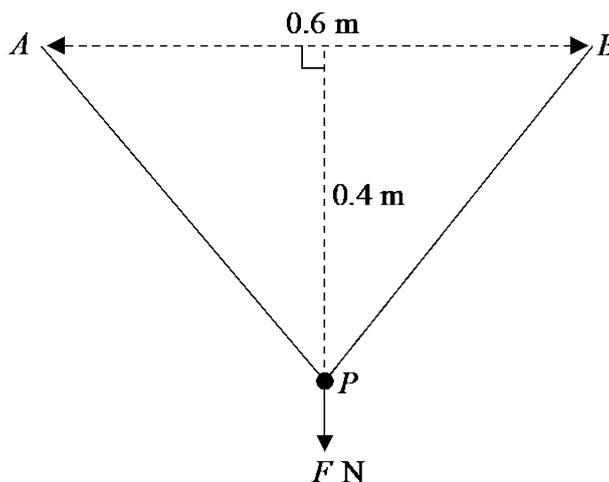
Figure 5

(b) Find the value of F.

(4)

The particle is released from rest. Given that the mass of P is 0.3 kg,

(c) find the speed of P as it crosses the line AB.

(6)

(Total 13 marks)

Mark scheme for Question 6

Examiner comment

7.
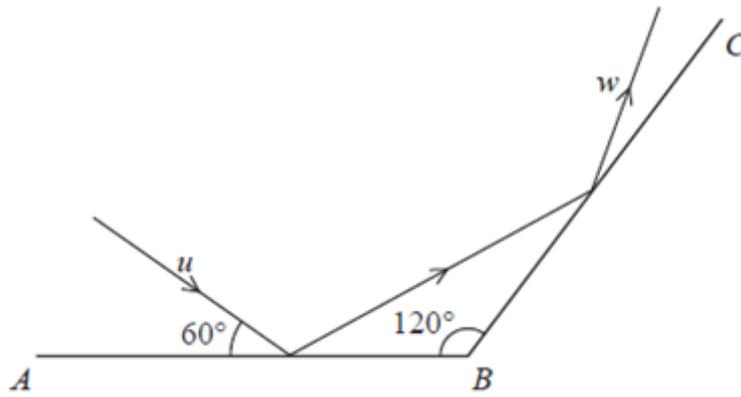
Figure 2

Figure 2 represents the plan view of part of a smooth horizontal floor, where AB and BC are smooth vertical walls. The angle between AB and BC is 120°. A ball is projected along the floor towards AB with speed u m s^{-1} on a path at an angle of 60° to AB. The ball hits AB and then hits BC. The ball is modelled as a particle. The coefficient of restitution between the ball and each wall is $\frac{1}{2}$.

(a) Show that the speed of the ball immediately after it has hit AB is $\dfrac{\sqrt{7}}{4}u$.

(6)

The speed of the ball immediately after it has hit BC is w m s^{-1}.

(b) Find w in terms of u.

(7)

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

A level Further Mathematics – Further Mechanics 1 – Practice Paper 01 – Mark scheme –

Question	Scheme	Marks
1	$12500 = 5F$	**B1**
	$F + 900g \sin \theta - 570 = 900a$	**M1A2**
	$A = 2.47 \ (2.5)$	**A1**
		(5)
		(5 marks)

Question	Scheme	Marks
2(a)	$0.4\mathbf{u} + (-5\mathbf{i} + 3\mathbf{j}) = 0.4(12\mathbf{i} + 15\mathbf{j})$	**M1**
	$\mathbf{u} = \dfrac{9.8\mathbf{i} + 3\mathbf{j}}{0.4} = 24.5\mathbf{i} + 7.5\mathbf{j}$	**A1**
	Speed $= \sqrt{24.5^2 + 7.5^2} = 25.6 \text{ m s}^{-1}$	**M1A1**
		(4)
(b)	$\tan^{-1}\left(\dfrac{15}{12}\right), \quad \tan^{-1}\left(\dfrac{7.5}{24.5}\right)$	**M1A1**
	$= 34.319...°$ or $0.59899....$rad	**A1**
		(3)
		(7 marks)

Question	Scheme		Marks
3	$v\cos 45° = u\sin\alpha$	parallel	**M1A1**
	$v\sin 45° = eu\cos\alpha$	perpendicular	**M1A1**
	$e = \tan\alpha$	Or square and add	**M1A1**
	$I = m\,(v\cos 45° + u\cos\alpha)$	Impulse	**M1A1**
	$\quad = mu(\sin\alpha + \cos\alpha\,)$	In terms of u, α	**M1**
	$\quad = \dfrac{mu(1+e)}{\sqrt{1+e^2}}$	In terms of u, e	**M1A1**
			(11)
			(11 marks)

Question	Scheme	Marks
4(a)		
	For A, component perpendicular to loc = 5	**B1**
	For B, component perpendicular to loc = 3	**B1**
	$\dfrac{1}{2}m \times 25 \times \dfrac{85}{100} = \dfrac{1}{2}m\left(3^3 + v^2\right)$	**M1**
	$\dfrac{85}{4} = 9 + v^2, \quad v^2 = \dfrac{49}{4}$	**A1**
	$-6m + 4m = 3mw - mv \ (= 3mw - 3.5m)$	**M1**
	$w = 0.5$	**A1ft**
	Select correct root and state velocities:	**DM1**
	$\mathbf{v}_B = \left(3\mathbf{i} - 3.5\mathbf{j}\right)(\mathrm{m\,s^{-1}})$	**A1**
	$\mathbf{v}_A = \left(5\mathbf{i} + 0.5\mathbf{j}\right)(\mathrm{m\,s^{-1}})$	**A1**
		(9)
(b)	$v + w = e(2 + 4)$	**M1**
	$0.5 + 3.5 = 6e$	**A1ft**
	$e = \dfrac{2}{3}$	**A1**
		(3)
		(12 marks)
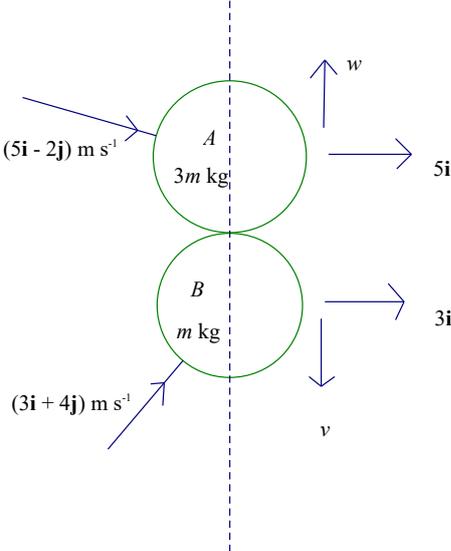
Question	Scheme	Marks
5(a)		
	$6mu - 3mu = 3my - 2mx$	**M1**
	$3y - 2x = 3u$	**A1**
	$4ue = x + y$	**M1A1**
	$y = \dfrac{u}{5}(8e + 3)$ **	**DM1** **A1**
		(6)
(b)	$x = 4ue - y$	**M1**
	$x = \dfrac{1}{5}u(20e - 8e - 3) = \dfrac{3}{5}u(4e - 1)$	**A1**
	$x > 0 \Rightarrow e > \dfrac{1}{4}$	**M1**
	$\therefore \dfrac{1}{4} < e \leqslant 1$	**A1**
		(4)
(c)	$e = \dfrac{1}{2} \quad x = \dfrac{3u}{5} \quad y = \dfrac{u}{5}(4 + 3) = \dfrac{7u}{5}$	**B1**
	$T = \dfrac{1}{2} \times 2m \times 9u^2 + \dfrac{1}{2} \times 3mu^2 \quad \left(= \dfrac{21mu^2}{2} \right)$	**M1**
	$kT = \dfrac{1}{2} \times 2m \times \dfrac{9u^2}{25} + \dfrac{1}{2} \times 3m \times \dfrac{49u^2}{25} \left(= \dfrac{165mu^2}{50} \right)$	**M1**
	$k = \dfrac{\dfrac{1}{2} \times 2 \times \dfrac{9}{25} + \dfrac{1}{2} \times 3 \times \dfrac{49}{25}}{\dfrac{1}{2} \times 2 \times 9 + \dfrac{1}{2} \times 3} \quad \left(\text{or } \dfrac{165}{50} \div \dfrac{21}{2} \right) = \dfrac{11}{35}$	**A1**
		(14 marks)

Question	Scheme	Marks
6(a)	$8 = \dfrac{\lambda \times 0.20}{0.40}$	M1A1
	$\lambda = 16$ *	A1cso
		(3)
(b)	Length of string $= 1\,\text{m}$ or $100\,\text{cm}$	
	$T = \dfrac{\lambda \times 0.6}{0.4}$, $= 24$ (or use half string)	M1A1
	$2T\cos\theta = F$	M1
	$F = 2 \times 24 \times \dfrac{4}{5} = 38.4$, $\dfrac{192}{5}$ or $38\dfrac{2}{5}$	A1
		(4)
(c)	Initial EPE $= \dfrac{16 \times 0.6^2}{2 \times 0.4}\left(= \dfrac{36}{5}\right)$ Final EPE $= \dfrac{16 \times 0.2^2}{2 \times 0.4}\left(= \dfrac{4}{5}\right)$	B1
	$\dfrac{16 \times 0.6^2}{2 \times 0.4} - \dfrac{16 \times 0.2^2}{2 \times 0.4} = \dfrac{1}{2}0.3v^2$	M1A1 A1
	$0.3v^2 = 40\left(0.6^2 - 0.2^2\right)$	
	$v = 6.531...$ Accept $6.5\,(\text{m s}^{-1})$ or better or exact value $8\sqrt{\dfrac{2}{3}}\,(\text{m s}^{-1})$	dM1 A1cso
		(6)
		(13 marks)

Question	Scheme	Marks
7(a)	Resolve parallel to barrier - condone sin/cos confusion	M1
	$\quad u\cos 60 = v\cos\theta$	A1
	Resolve perpendicular to the barrier - condone consistent sin/cos confusion. Use e correctly	M1
	$eu\sin 60 = v\sin\theta$	A1
	$v^2 = u^2\cos^2 60 + e^2 u^2\sin^2 60 = \dfrac{u^2}{4} + \dfrac{3u^2}{16} = \dfrac{7u^2}{16}$	M1
(b)	$v = \dfrac{\sqrt{7}}{4}u$	A1
		(6)
	Angle of approach with $BC = 19.1°$	B1
	$v\cos 19.1 = w\cos\phi$	M1
	$\dfrac{1}{2}v\sin 19.1 = w\sin\phi$	M1
		A1
	Form equation in v and ϕ	M1
	$w^2 = v^2\left(\dfrac{1}{4}\sin^2 19.1 + \cos^2 19.1\right)$	A1
	$0.634u$	A1
		(7)
		(13 marks)

A level Further Mathematics – Further Mechanics 1 – Practice Paper 01 – Examiner report –

1.   This was a straightforward power question that was well answered, with many candidates scoring full marks. The most common error was to have the weight acting up the slope in the equation of motion, which suggests either a lack of basic understanding or a failure to read the question carefully. Some solutions revealed a poor choice of notation, using '*F*' to represent both the driving force and the resultant force.

2.   Part (a) Most candidates started with a correct impulse-momentum equation to find the velocity of the ball before the impact, but many did not go no to find the corresponding speed.
     Part (b) Candidates with a clear diagram usually found the correct angle, and it was pleasing to see a number of candidates with enough knowledge of vectors to use the scalar product (although this method was not expected). Some candidates found the angle between a velocity and the impulse, and some found the angle between a velocity and a fixed direction (usually the unit vector **i**).

3.   All students understood that they needed to start by considering the motion parallel and perpendicular to the plane. In forming the equation for the impulse, some solutions did not take account of the change in direction of the motion perpendicular to the plane due to the impact. Using the initial equations to form an expression for *I* in terms of *m*, *u* and *e* proved to be quite challenging, with most students making some progress but only a few reaching the correct conclusion.

4.   The responses to this question underline the need to read questions carefully and to use clear diagrams. Part (a) was often made more difficult by poor diagrams and ill-defined components of velocity after collision. Some candidates were confused by the line of centres of the spheres being parallel to j. Some set about describing the components of the velocities after the collision without noting that the components in the **i** direction were 5 and 3, so they created more unknown variables than necessary. In forming the equation for the kinetic energy, several candidates used 85% of the total initial kinetic energy, rather than using B only, as described in the question. The majority of candidates who obtained $v^2 = \dfrac{49}{4}$ did select the correct component, $-\dfrac{7}{2}$. However some candidates selected the wrong root and did not seem to notice that in their solution the spheres had passed through each other.
     Part (b), the impact law was usually stated correctly but the signs of the components caused problems in some cases.

5.   Part (a) Many candidates earned full marks for correct work in this part of the question, although there were some slips in forming the equations and some inconsistent use of signs, usually from candidates with poor diagrams or no diagram at all.

Part (b) Although there have been similar questions to this in the past, some candidates struggled to find a way of introducing an inequality in e. Most of the marks in this part of the question depend on using the information about the direction of motion of P after the collision. The initial inequality does need to match the candidate's velocity for P, whether or not they have already taken account of the change in direction. For the final mark they also need to state the maximum value for e.

Part (c) The work on kinetic energy was usually very good, and often resulted in a correct answer for k.

6.   Almost all students gained full marks for part (a). Part (b) caused problems for a few who confused a half string with a full string when substituting into Hooke's Law – they used extension of one and natural length of another. There were occasional instances where the students forgot which was which and their natural length was in the numerator. The vast majority obtained full marks though students should be reminded to always check their working carefully. However, part (c) was not well answered. The formula for EPE was not always known – the square or the 2 in the denominator being missed. Despite the fact that the motion was on a horizontal table and that the word horizontal was in bold twice more in the question, many students treated this as a vertical problem. This meant that they included a GPE term in their energy equation which lost them nearly all the marks. Some did not realise that there was still elastic energy when the particle reached AB so missed out one term, also losing nearly all the marks.

7.   (a) Most candidates showed a sound knowledge of the topic and gave good solutions. Many found the angle first and then the rebound velocity. Most candidates understood that they needed to show exact working to support an answer given in exact form.

(b) Although exact working was not required in this part, many candidates worked through the entire question in exact form. The more usual solution in this part was to calculate the angle of approach and then apply the same method as used in part (a) to find the final velocity as a multiple of u.