Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

A level Further Mathematics
Core Pure Mathematics

Practice Paper 4

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **8** questions in this question paper. The total mark for this paper is **70**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

$$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$$

Given that the matrix **M** is singular, find the possible values of x.

(Total 4 marks)

2.

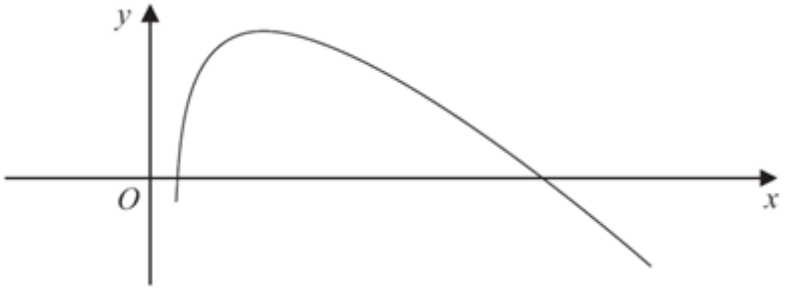

Figure 1

Figure 1 shows part of the curve with equation

$$y = 40 \operatorname{arcosh} x - 9x, \qquad x \geq 1$$

Use calculus to find the exact coordinates of the turning point of the curve, giving your answer in the form $\left(\frac{p}{q}, r \ln 3 + s \right)$ where p, q, r and s are integers.

(Total 7 marks)

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3. The curve C has equation

$$y = \frac{1}{\sqrt{x^2 + 2x - 3}}, \qquad x > 1$$

(a) Find $\int y \, \mathrm{d}x$

(3)

The region R is bounded by the curve C, the x-axis and the lines with equations $x = 2$ and $x = 3$. The region R is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated. Give your answer in the form $p\pi \ln q$, where p and q are rational numbers to be found.

(4)

(Total 7 marks)

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4. $f(x) = 5\cosh x - 4\sinh x, \quad x \in \mathbb{R}.$

(a) Show that $f(x) = \frac{1}{2}(e^x + 9e^{-x})$.

(2)

Hence

(b) solve $f(x) = 5$,

(4)

(c) show that $\int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5\cosh x - 4\sinh x}\, dx = \frac{\pi}{18}$.

(5)

(Total 11 marks)

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5. (a) Find the general solution of the differential equation

$$3\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = x^2.$$

(8)

(b) Find the particular solution for which, at $x = 0$, $y = 2$ and $\frac{dy}{dx} = 3$.

(6)

(Total 14 marks)

6.

$$y = \ln\left(\frac{1}{1-2x}\right), \qquad |x| < \frac{1}{2}$$

(a) Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and $\frac{d^3y}{dx^3}$

(4)

(b) Hence, or otherwise, find the series expansion of $\ln\left(\frac{1}{1-2x}\right)$ about $x = 0$, in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(3)

(c) Use your expansion to find an approximate value for $\ln\left(\frac{3}{2}\right)$, giving your answer to 3 decimal places.

(3)

(Total 10 marks)

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7. The line l_1 has equation $\frac{x-2}{4} = \frac{y-4}{-2} = \frac{z+6}{1}$

The plane Π has equation $x - 2y + z = 6$

The line l_2 is the reflection of the line l_1 in the plane Π.

Find a vector equation of the line l_2

(Total 7 marks)

8.

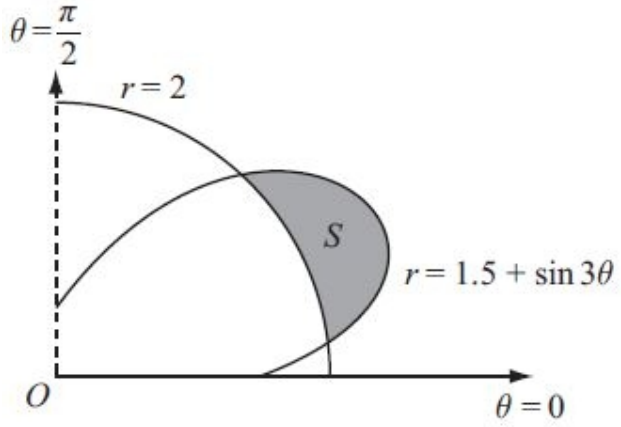

Figure 1

Figure 1 shows the curves given by the polar equations

$$r = 2, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2},$$

$$\text{and} \qquad r = 1.5 + \sin 3\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

(a) Find the coordinates of the points where the curves intersect.

(3)

The region S, between the curves, for which $r > 2$ and for which $r < (1.5 + \sin 3\theta)$, is shown shaded in Figure 1.

(b) Find, by integration, the area of the shaded region S, giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are simplified fractions.

(7)

(Total 10 marks)

TOTAL FOR PAPER: 70 MARKS