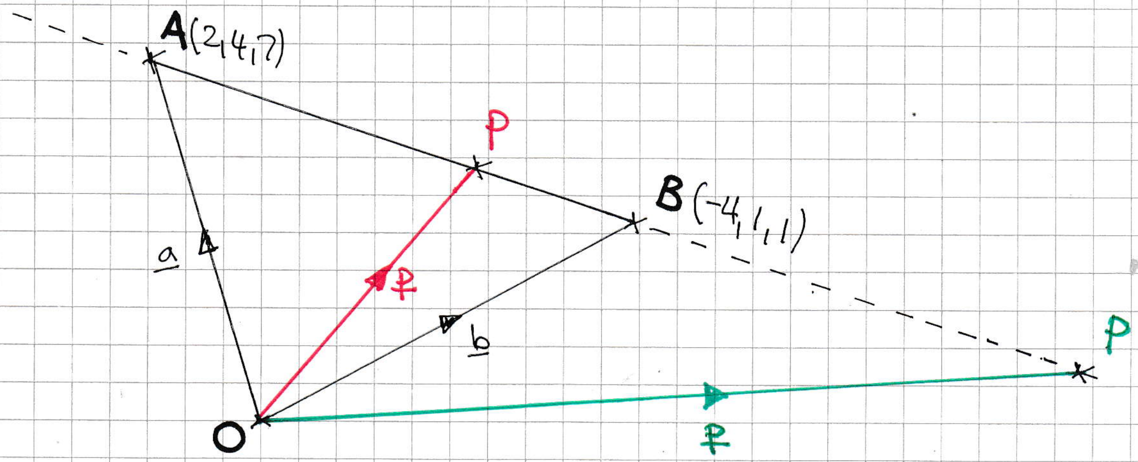
IYGB – MP2 PAPER L – QUESTION 1

LOOKING AT THE DIAGRAM BELOW



$$\vec{OP} = \vec{OA} + \vec{AP}$$

$$\Rightarrow \vec{OP} = \vec{OA} + \frac{2}{3}\vec{AB}$$

$$\Rightarrow \underline{p} = \underline{a} + \frac{2}{3}(\underline{b} - \underline{a})$$

$$\Rightarrow \underline{p} = (2,4,7) + \frac{2}{3}\left[(-4,1,1) - (2,4,7)\right]$$

$$\Rightarrow \underline{p} = (2,4,7) + \frac{2}{3}(-6,-3,-6)$$

$$\Rightarrow \underline{p} = (2,4,7) + (-4,-2,-4)$$

$$\Rightarrow \underline{p} = (-2,2,3)$$

$$\therefore \vec{OP} = -2\underline{i} + 2\underline{j} + 3\underline{k}$$

$$\vec{OP} = \vec{OB} + \vec{BP}$$

$$\Rightarrow \vec{OP} = \vec{OB} + \vec{AB}$$

$$\Rightarrow \underline{p} = \underline{b} + (\underline{b} - \underline{a})$$

$$\Rightarrow \underline{p} = 2\underline{b} - \underline{a}$$

$$\Rightarrow \underline{p} = 2(-4,1,1) - (2,4,7)$$

$$\Rightarrow \underline{p} = (-10,-2,-5)$$

$$\therefore \vec{OP} = -10\underline{i} - 2\underline{j} - 5\underline{k}$$

IYGB - MP2 PAPER L - QUESTION 2

$x = 4at^2$ $\qquad\qquad$ $y = a(2t+1)$

USING THE CO-ORDINATES $(4,8)$ TO FORM TWO EQUATIONS

$4 = 4at^2$ $\qquad\qquad$ $8 = a(2t+1)$

$at^2 = 1$ $\qquad\qquad$ $\frac{8}{a} = 2t+1$

$t^2 = \frac{1}{a}$ $\qquad\qquad$ $2t = \frac{8}{a} - 1$

$\downarrow \times 4$ $\qquad\qquad$ $\downarrow$ SQUARING

$4t^2 = \frac{4}{a}$ $\qquad\qquad$ $4t^2 = \left(\frac{8}{a} - 1\right)^2$

$$\frac{4}{a} = \left(\frac{8}{a} - 1\right)^2$$

$$\frac{4}{a} = \frac{64}{a^2} - \frac{16}{a} + 1$$

MULTIPLY THROUGH BY a^2

$\Longrightarrow 4a = 64 - 16a + a^2$

$\Longrightarrow 0 = a^2 - 20a + 64$

$\Longrightarrow 0 = (a-4)(a-16)$

$\Longrightarrow a = \begin{cases} 4 \\ 16 \end{cases}$

IYGB - MP2 PAPER L - QUESTION 3

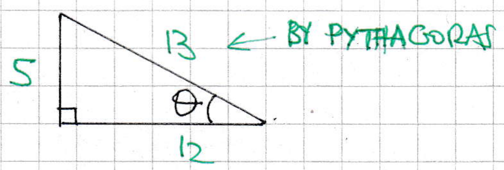
THIS EQUATION IS ABOUT COMPOUND ANGLE & NOT ABOUT DOUBLE ANGLES AS THE ARGUMENTS ARE 2θ THROUGHOUT

$$\Longrightarrow 6 + 13\sin(2\theta+\alpha) = 5\cos 2\theta$$

$$\Longrightarrow 6 + 13\left[\sin 2\theta\cos\alpha + \cos 2\theta\sin\alpha\right] = 5\cos 2\theta$$

$$\Longrightarrow 6 + 13\sin 2\theta\cos\alpha + 13\cos 2\theta\sin\alpha = 5\cos 2\theta$$

NOW $\tan\alpha = \frac{5}{12}$, α ACUTE



$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

RETURNING TO THE MAIN LINE

$$\Longrightarrow 6 + 13\sin 2\theta \times \frac{12}{13} + 13\cos 2\theta \times \frac{5}{13} = 5\cos 2\theta$$

$$\Longrightarrow 6 + 12\sin 2\theta + \cancel{5\cos 2\theta} = \cancel{5\cos 2\theta}$$

$$\Longrightarrow 12\sin 2\theta = -6$$

$$\Longrightarrow \sin 2\theta = -\frac{1}{2}$$

$$\arcsin\left(-\tfrac{1}{2}\right) = -30°$$

$$\left(\begin{array}{l} 2\theta = -30° \pm 360n \\ 2\theta = 210° \pm 360n \end{array}\right. \quad n = 0,1,2,3,\ldots$$

$$\left(\begin{array}{l} \theta = -15° \pm 180n \\ \theta = 105° \pm 180n \end{array}\right.$$

$\theta = 165°, 345°, 105°, 285°$

IYGB – MP2 PAPER L – QUESTION 4

a) DIFFERENTIATE WITH RESPECT TO x BY PRODUCT RULE & SET TO ZERO

$$\Rightarrow y = x\cos x$$

$$\Rightarrow \frac{dy}{dx} = 1 \times \cos x + x(-\sin x)$$

$$\Rightarrow 0 = \cos x - x\sin x$$

$$\Rightarrow \cos x = x\sin x$$

$$\Rightarrow \frac{\cos x}{x\cos x} = \frac{x\sin x}{x\cos x}$$

$$\Rightarrow \frac{1}{x} = \tan x$$

$$\Rightarrow x = \arctan\left(\frac{1}{x}\right)$$ // AS REQUIRED

b) WRITE ABOVE EQUATION AS A FUNCTION $f(x) = x - \arctan\left(\frac{1}{x}\right)$

$f(0.8) = -0.096055\ldots < 0$

$f(1) = +0.214601\ldots > 0$

AS $f(x)$ IS CONTINUOUS IN THE INTERVAL $(0.8, 1)$, AND THERE IS A CHANGE OF SIGN, THERE MUST BE AT LEAST ONE SOLUTION IN THE INTERVAL //

c) USING THE FORMULA GIVEN WITH $x_1 = 0.9$

$$x_{n+1} = \arctan\left(\frac{1}{x_n}\right)$$

$x_1 = 0.9$

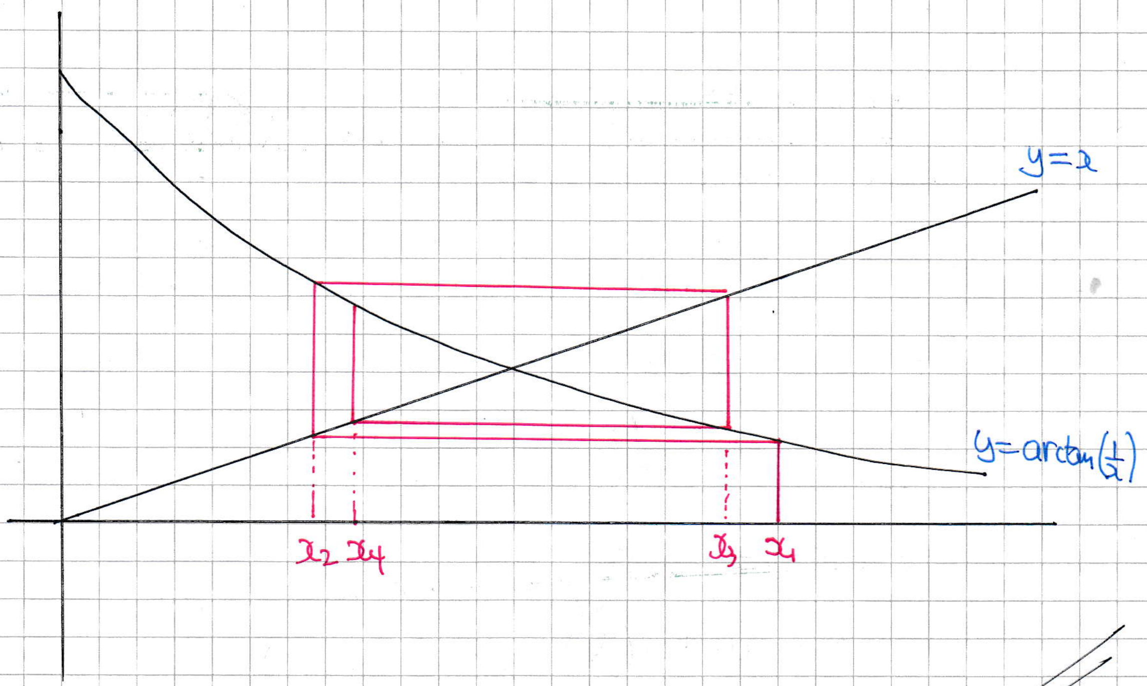
$x_2 \approx 0.838$

$x_3 \approx 0.873$

$x_4 \approx 0.853$ //

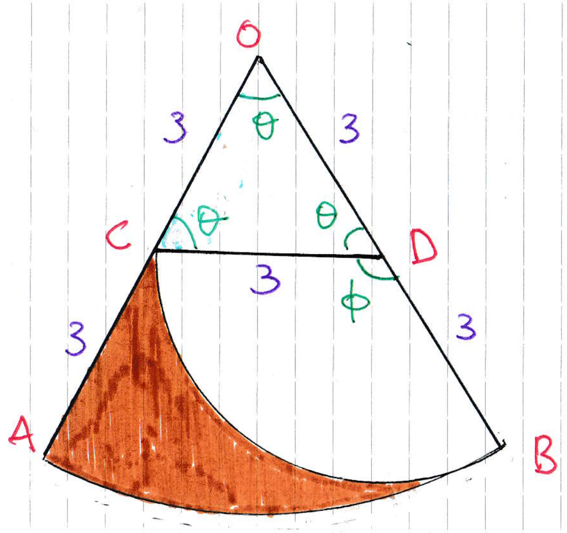
IYGB - MP2 PAPER L - QUESTION 4

d)



IYGB - MP2 PAPER L - QUESTION 5

a)



LOOKING AT THE FIGURE

$\overset{\triangle}{OCD}$ IS EQUILATERAL

$\therefore \hat{COD} = \theta = 60° = \frac{\pi}{3}$

$\hat{CDB} = \phi = 120° = \frac{2\pi}{3}$

USING "$L = r\theta$"

$\overset{\frown}{AB} = 6 \times \frac{\pi}{3} = 2\pi$

$\overset{\frown}{BC} = 3 \times \frac{2\pi}{3} = 2\pi$

REQUIRED PERIMETER

$P = |\overset{\frown}{AB}| + |\overset{\frown}{BC}| + |AC|$

$P = 2\pi + 2\pi + 3$

$P = 4\pi + 3$

AS REQUIRED

b)

AREA OF $\overset{\triangledown}{OAB}$ $= $ "$\frac{1}{2}r^2\theta^c$"

$= \frac{1}{2} \times 6^2 \times \frac{\pi}{3}$

$= 6\pi$

AREA OF $\overset{\triangledown}{CDB}$ $=$ "$\frac{1}{2}r^2\phi^c$"

$= \frac{1}{2} \times 3^2 \times \frac{2\pi}{3}$

$= 3\pi$

AREA OF $\overset{\triangle}{OCD}$ $= \frac{1}{2}|OC||OD|\sin\left(\frac{\pi}{3}\right)$

$= \frac{1}{2} \times 3 \times 3 \times \frac{\sqrt{3}}{2}$

$= \frac{9}{4}\sqrt{3}$

REQUIRED AREA $= \overset{\triangledown}{OAB} - \overset{\triangle}{OCD} - \overset{\triangledown}{DCB}$

$= 6\pi - 3\pi - \frac{9}{4}\sqrt{3}$

$= 3\pi - \frac{9}{4}\sqrt{3}$

$= \frac{3}{4}\left[4\pi - 3\sqrt{3}\right]$

IYGB - MP2 PAPER L - QUESTION 6

START BY RELATING DERIVATIVES

$$\Rightarrow \frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt}$$

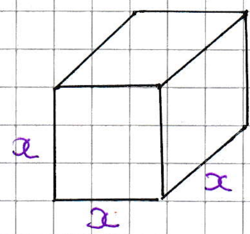
$$\Rightarrow \frac{dV}{dt} = \frac{dV}{dA} \times 0.45$$

$$\Rightarrow \frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dA} \times 0.45$$

$$\Rightarrow \frac{dV}{dt} = 3\cancel{x^2}^{x} \times \frac{1}{12\cancel{x}} \times 0.45$$

$$\Rightarrow \frac{dV}{dt} = \frac{9x}{80}$$

$$\Rightarrow \left.\frac{dV}{dt}\right|_{x=8} = \frac{9 \times 8}{80} = 0.9 \text{ cm}^3\text{s}^{-1}$$



- $V = x^3$

$$\frac{dV}{dx} = 3x^2$$

- $A = 6x^2$

$$\frac{dA}{dx} = 12x$$

$$\frac{dx}{dA} = \frac{1}{12x}$$

ALTERNATIVE METHOD

$$\Rightarrow \frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt}$$

$$\Rightarrow \frac{dV}{dt} = \frac{dV}{dA} \times 0.45$$

$$\Rightarrow \frac{dV}{dt} = \frac{\sqrt{6}}{36} \times \frac{3}{2} A^{\frac{1}{2}} \times 0.45$$

$$\Rightarrow \frac{dV}{dt} = \frac{\sqrt{6}}{24} A^{\frac{1}{2}} \times 0.45$$

$$\Rightarrow \frac{dV}{dt} = \frac{\sqrt{6}}{24} (6x^2)^{\frac{1}{2}} \times 0.45$$

OBTAIN A RELATIONSHIP BETWEEN V & A

$$V = x^3 \qquad A = 6x^2 \Rightarrow V^2 = x^6 \qquad A^3 = 216x^6$$

DIVIDE THE EQUATIONS

$$\frac{A^3}{V^2} = 216$$

$$A^3 = 216V^2$$

$$V = +\left(\frac{A^3}{216}\right)^{\frac{1}{2}} = \frac{\sqrt{6}}{36} A^{\frac{3}{2}}$$

IYGB − MP2 PAPER L − QUESTION 6

$$\Longrightarrow \frac{dw}{dt} = \frac{\sqrt{6}}{24} \times \sqrt{6}\,x \times 0.45$$

$$\Longrightarrow \frac{dw}{dt} = \frac{1}{4}x \times 0.45$$

$$\Longrightarrow \frac{dw}{dt} = \frac{9x}{80}$$

$$\Longrightarrow \left.\frac{dw}{dt}\right|_{x=8} = 0.9\ \text{cm}^3\,\text{s}^{-1}$$

// AS BEFORE

IYGB - MP2 PAPER L - QUESTION 7

SOLVE BY SEPARATING THE VARIABLES

$$\Rightarrow x\frac{dy}{dx} = y(y+1)$$

$$\Rightarrow x\,dy = y(y+1)\,dx$$

$$\Rightarrow \frac{1}{y(y+1)}\,dy = \frac{1}{x}\,dx$$

$$\Rightarrow \int \frac{1}{y(y+1)}\,dy = \int \frac{1}{x}\,dx$$

OBTAIN PARTIAL FRACTIONS, IN THE L.H.S

$$\frac{1}{y(y+1)} \equiv \frac{A}{y} + \frac{B}{y+1}$$

$$1 \equiv A(y+1) + By$$

- If $y=0$: $1 = A$
- If $y=-1$: $1 = -B$

RETURNING TO THE "MAIN LINE"

$$\Rightarrow \int \frac{1}{y} - \frac{1}{y+1}\,dy = \int \frac{1}{x}\,dx$$

$$\Rightarrow \ln|y| - \ln|y+1| = \ln|x| + \ln k$$ ← INTEGRATION CONSTANT

$$\Rightarrow \ln\left|\frac{y}{y+1}\right| = \ln|kx|$$

$$\Rightarrow \frac{y}{y+1} = kx$$

IYGB - MP2 PAPER L - QUESTION 7

PUTTING THE BOUNDARY CONDITION $\left(\frac{1}{3}, \frac{1}{2}\right)$

$$\Longrightarrow \frac{\frac{1}{2}}{\frac{1}{2}+1} = k \times \frac{1}{3}$$

$$\Longrightarrow \frac{1}{3} = \frac{1}{3}k$$

$$\Longrightarrow k = 1$$

FINALLY WE MAY TIDY

$$\Longrightarrow \frac{y}{y+1} = x$$

$$\Longrightarrow y = (y+1)x$$

$$\Longrightarrow y = xy + x$$

$$\Longrightarrow y - xy = x$$

$$\Longrightarrow y(1-x) = x$$

$$\Longrightarrow y = \frac{x}{1-x}$$

// AS REQUIRED

IYGB - MP2 PAPER L - QUESTION 8

a) LOOKING AT THE PATTERN

$$\begin{array}{ccccc} u_1 & & u_2 & & u_3 \\ q(4p+1) & \xrightarrow{\times r} & q(2p+3) & \xrightarrow{\times r} & q(2p-3) \end{array}$$

FORMING TWO EQUATIONS

$$\left.\begin{array}{l} q(4p+1)\times r = q(2p+3) \\ q(2p+3)\times r = q(2p-3) \end{array}\right. \Longrightarrow \begin{array}{l} qr(4p+1) = q(2p+3) \\ qr(2p+3) = q(2p-3) \end{array}$$

DIVIDING SIDE BY SIDE

$$\frac{\cancel{qr}(4p+1)}{\cancel{qr}(2p+3)} = \frac{\cancel{q}(2p+3)}{\cancel{q}(2p-3)} \Longrightarrow (4p+1)(2p-3) = (2p+3)(2p+3)$$

$$\Longrightarrow 8p^2 - 10p - 3 = 4p^2 + 12p + 9$$

$$\Longrightarrow 4p^2 - 22p - 12 = 0$$

$$\Longrightarrow 2p^2 - 11p - 6 = 0$$

$$\Longrightarrow (2p+1)(p-6)$$

$$\Longrightarrow p = \begin{cases} 6 \\ -\frac{1}{2} \end{cases}$$

b) LOOKING AGAIN AT THE FIRST 3 TERMS

- IF $p = -\frac{1}{2}$ $\quad u_1 = -q,\ u_2 = 2q,\ u_3 = -4q \quad \Longrightarrow r = -2$
- IF $p = 6$ $\quad u_1 = 25q,\ u_2 = 15q,\ u_3 = 9q \quad \Longrightarrow r = \frac{3}{5}$

IYGB - MP2 PAPER L - QUESTION 8

AS THERE EXISTS A SUM TO INFINITY $-1 < r < 1$, IE $r = \frac{3}{5}$

$$\Rightarrow S_{\infty} = \frac{a}{1-r}$$

$$\Rightarrow 250 = \frac{25d}{1-\frac{3}{5}}$$

$$\Rightarrow 250 = \frac{25d}{2/5}$$

$$\Rightarrow 100 = 25d$$

$$\Rightarrow d = 4$$

IYGB – MP2 PAPER L – QUESTION 9

$$f(x) = 2x+3, \quad x \in \mathbb{R}, \quad x \leqslant 8$$
$$g(x) = x^2 - 1, \quad x \in \mathbb{R}, \quad x \geqslant 0$$

TO FIND THE DOMAIN, LOOK AT THE DIAGRAM BELOW

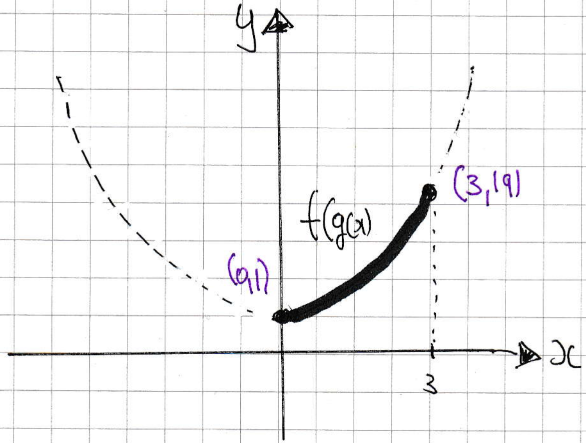
$$\xrightarrow{x \geqslant 0} \boxed{g(x)} \xrightarrow{g(x)} \qquad \xrightarrow{x \leqslant 8} \boxed{f(x)} \xrightarrow{f(g(x))}$$

THE DOMAIN OF $f(g(x))$ MUST SATISFY $x \geqslant 0$ AND $g(x) \leqslant 8$

$$\Rightarrow g(x) \leqslant 8$$
$$\Rightarrow x^2 - 1 \leqslant 8$$
$$\Rightarrow x^2 \leqslant 9$$
$$\Rightarrow -3 \leqslant x \leqslant 3 \qquad (x \geqslant 0)$$
$$\Rightarrow 0 \leqslant x \leqslant 3$$

TO FIND THE RANGE IT MIGHT BE HELPFUL TO OBTAIN THE COMPOSITION FIRST, IN ORDER TO SKETCH IT FOR THE ABOVE DOMAIN

$$f(g(x)) = f(x^2-1)$$
$$= 2(x^2-1)+3$$
$$= 2x^2+1$$



LOOKING AT THE DIAGRAM, THE RANGE OF $f(g(x))$ IS

$$1 \leqslant f(g(x)) \leqslant 19$$

IYGB - MP2 PAPER L - QUESTION 10

a) FILL IN THE TABLE

x	0	8	16	24	32
y	$\frac{3}{4}$	$\frac{35}{29}$	$\frac{67}{52}$	$\frac{99}{76}$	$\frac{131}{100}$

b) USING THE TRAPEZIUM RULE

$$\int_0^{32} \frac{4x+3}{3x+4}\,dx \approx \text{"} \frac{\text{THICKNESS}}{2}\Big[\text{FIRST} + \text{LAST} + 2 \times \text{REST}\Big]\text{"}$$

$$\approx \frac{8}{2}\left[\frac{3}{4} + \frac{131}{100} + 2\left(\frac{35}{29} + \frac{67}{52} + \frac{99}{76}\right)\right]$$

$$\approx 4\left[9.6559\ldots\right]$$

$$\approx 38.6239\ldots$$

$$\approx 38.6$$

c) USING THE SUBSTITUTION GIVEN

- $u = 3x+4$

$\frac{du}{dx} = 3$

$\boxed{dx = \frac{du}{3}}$

- $x=0 \quad u=4$

$x=32 \quad u=100$

TRANSFORMING THE INTEGRAL

$$\int_0^{32} \frac{4x+3}{3x+4}\,dx = \int_4^{100} \frac{4x+3}{u}\left(\frac{du}{3}\right)$$

$$= \int_4^{100} \frac{4x+3}{3u}\,du$$

$$= \int_4^{100} \frac{12x+9}{9u}\,du$$

NOW WE HAVE

$\Rightarrow u = 3x+4$

$\Rightarrow 4u = 12x+16$

$\Rightarrow 4u-7 = 12x+9$

IYGB - MP2 PAPER L - QUESTION 10

RETURNING TO THE INTEGRAL

$$\ldots = \int_4^{100} \frac{4u-7}{9u}\,du = \frac{1}{9}\int_4^{100} \frac{4u}{u} - \frac{7}{u}\,du = \frac{1}{9}\int_4^{100} 4 - \frac{7}{u}\,du$$

$$= \frac{1}{9}\Big[4u - 7\ln|u|\Big]_4^{100} = \frac{1}{9}\Big[(400 - 7\ln 100) - (16 - 7\ln 4)\Big]$$

$$= \frac{1}{9}\Big[400 - 16 + 7\ln 4 - \ln 100\Big] = \frac{1}{9}\Big[384 + 7\ln\frac{1}{25}\Big]$$

$$= \frac{1}{9}\Big[384 + 7\ln 5^{-2}\Big] = \frac{1}{9}\Big[384 - 14\ln 5\Big]$$

ALTERNATIVE METHOD

$$\int_0^{32} \frac{4x+3}{3x+4}\,dx = \frac{1}{3}\int_0^{32} \frac{12x+9}{3x+4}\,dx = \frac{1}{3}\int_0^{32} \frac{4(3x+4)-7}{3x+4}\,dx = \frac{1}{3}\int_0^{32} 4 - \frac{7}{3x+4}\,dx$$

$$= \frac{1}{3}\Big[4x - \frac{7}{3}\ln|3x+4|\Big]_0^{32} = \frac{1}{3}\Big[\big(128 - \tfrac{7}{3}\ln 100\big) - \big(0 - \tfrac{7}{3}\ln 4\big)\Big]$$

$$= \frac{1}{3}\Big[128 - \frac{7}{3}\ln 100 + \frac{7}{3}\ln 4\Big] = \frac{1}{3}\Big[128 + \frac{7}{3}\ln\frac{4}{100}\Big]$$

$$= \frac{1}{3}\Big[128 + \frac{7}{3}\ln\frac{1}{25}\Big] = \frac{1}{3}\Big[128 + \frac{7}{3}\ln(5^{-2})\Big]$$

$$= \frac{1}{3}\Big[128 - \frac{14}{3}\ln 5\Big] = \frac{1}{9}\Big[384 - 14\ln 5\Big]$$

IYGB - MP2 PAPER L - QUESTION 11

DIFFERENTIATING IMPLICITLY WITH RESPECT TO x

$$\Rightarrow \quad y^2 - x^2 = 1$$

$$\Rightarrow \quad 2y\frac{dy}{dx} - 2x = 0$$

$$\Rightarrow \quad y\frac{dy}{dx} = x$$

DIFFERENTIATE AGAIN WITH RESPECT TO x

$$\Rightarrow \quad \frac{d}{dx}\left(y\frac{dy}{dx}\right) = \frac{d}{dx}(x)$$

$$\Rightarrow \quad \frac{dy}{dx}\frac{dy}{dx} + y\frac{d^2y}{dx^2} = 1$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} = 1 - \left(\frac{dy}{dx}\right)^2$$

BUT WE FOUND THAT $\frac{dy}{dx} = \frac{x}{y}$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2 = \frac{x^2}{y^2}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2 = \frac{y^2-1}{y^2}$$

$y^2 - x^2 = 1$
$y^2 - 1 = x^2$

FINALLY WE HAVE

$$y\frac{d^2y}{dx^2} = 1 - \frac{y^2-1}{y^2}$$

$$y\frac{d^2y}{dx^2} = \frac{y^2-(y^2-1)}{y^2}$$

$$y\frac{d^2y}{dx^2} = \frac{1}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{y^3}$$

IYGB – MP2 PAPER L – QUESTION 12

FIRSTLY OBTAIN THE INTERCEPTS WITH THE AXES

- $x=0$

$(y+2)\ln(3y+4)=0$

EITHER

$\cancel{y=-2}$ OR $\ln(3y+4)=0$

BECAUSE $\ln(3y+4)$ IS NOT DEFINED

$3y+4=e^0$

$3y+4=1$

$3y=-3$

$y=-1$

$\therefore Q(0,-1)$

- $y=0$

$x=2\ln 4$

$x=4\ln 2$

$\therefore P(4\ln 2, 0)$

NOT ACTUALLY NEEDED

DIFFERENTIATE x WITH RESPECT TO y, USING PRODUCT RULE

$\Rightarrow x=(y+2)\ln(3y+4)$

$\Rightarrow \frac{dx}{dy} = 1\times\ln(3y+4) + (y+2)\times\frac{1}{3y+4}\times 3$

$\Rightarrow \frac{dx}{dy} = \ln(3y+4) + \frac{3(y+2)}{3y+4}$

EVALUATING & RECIPROCATING AFTER

$\left.\frac{dx}{dy}\right|_{y=0} = \ln 4 + \frac{6}{4}$ $\therefore \left.\frac{dy}{dx}\right|_{P} = \frac{1}{2\ln 2+\frac{3}{2}} = \frac{2}{4\ln 2+3}$

$\left.\frac{dx}{dy}\right|_{y=-1} = \cancel{\ln 1} + 3$ $\therefore \left.\frac{dy}{dx}\right|_{Q} = \frac{1}{3}$