KS5 "Full Coverage": Functions

This worksheet is designed to cover one question of each type seen in past papers, for each A Level topic. This worksheet was automatically generated by the DrFrostMaths Homework Platform: students can practice this set of questions interactively by going to www.drfrostmaths.com, logging on, *Practise* → *Past Papers* (or *Library* → *Past Papers* for teachers), and using the 'Revision' tab.

Question 1

Categorisation: Find the expression of a function for some arbitrary algebraic input, e.g. $f(x^2)$.

[Edexcel C3 June 2014(R) Q6c] The function g is defined by

$g: x \rightarrow \ ln\,(2x)\,, \qquad x > 0$

Solve the equation

$$g(x) + g(x^2) + g(x^3) = 6$$

giving your answer in its simplest form.

$x =$ .....

Question 2

Categorisation: Use $f(a) = b$ for known a and b to find the value of unknown coefficients in a function.

[Edexcel A2 Specimen Papers P1 Q5ai Edited]

$f(x) = x^3 + ax^2 - ax + 48$, where a is a constant

Given that $f(-6) = 0$, find the value of a .

.....

Question 3

Categorisation: Some functional equations involving exponential terms.

[Edexcel C3 June 2012 Q6c] The functions f and g are defined by

$f: x \rightarrow e^x + 2 \quad x \in \mathbb{R}$

$g: x \rightarrow \ln x \quad x > 0$

Find the exact value of x for which $f(2x + 3) = 6$.

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Question 4

Categorisation: Find the output of a composite function.

[Edexcel A2 Specimen Papers P1 Q10c]

The function f is defined by

$$f: x \rightarrow \frac{3x - 5}{x + 1}, x \in \mathbb{R}, x \neq -1$$

The function g is defined by

$$g: x \rightarrow x^2 - 3x, \quad x \in \mathbb{R}, 0 \leq x \leq 5$$

Find the value of $fg(2)$.

$fg(2) =$ .....

Question 5

Categorisation: Determine a composite function.

[Edexcel A2 Specimen Papers P1 Q10b] The function f is defined by

$$f: x \to \frac{3x-5}{x+1}, \qquad x \in \mathbb{R}, x \neq -1$$

Show that

$$ff(x) = \frac{x+a}{x-1}, \qquad x \in \mathbb{R}, x \neq \pm 1$$

where a is an integer to be found.

.....

Question 6

Categorisation: Solve a functional equation involving a composite function.

[Edexcel C3 June 2014(R) Q6e] The function f is defined by

$f: x \to e^{2x} + k^2$, $x \in \mathbb{R}$, k is a positive constant.

The function g is defined by

$g: x \to ln(2x)$, $x > 0$

Find, in terms of the constant k, the solution of the equation $fg(x) = 2k^2$.

$x =$ .....

Question 7

Categorisation: Solve an equation involving a modulus function within a composite function.

[OCR C3 June 2016 Q8iii Edited]

The functions f and g are defined for all real values of x by

$f(x) = |2x + a| + 3a$ and $g(x) = 5x - 4a$

where a is a positive constant.

Solve for x the equation $gf(x) = 31a$.

.....

Question 8

Categorisation: As above.

[OCR C3 June 2015 Q8iii]

The functions f and g are defined as follows:

$f(x) = 2 + \ ln\ (x + 3)$ for $x \geq 0$

$g(x) = ax^2$ for all real values of x , where a is a positive constant.

Given that $ff(e^N - 3) = \ ln\ (53e^2)$, find the value of N .

.....

Question 9

Categorisation: Appreciate that $fg(x)$ is not necessarily the same as $gf(x)$.

[Edexcel A2 SAM P2 Q4b Edited]

Given

$f(x) = e^x$, $\quad x \in \mathbb{R}$

$g(x) = 3\ ln\ x$, $\quad x > 0$, $\ x \in \mathbb{R}$

It can be shown that $gf(x) = 3x$.

Show that there is only one real value of x for which $gf(x) = fg(x)$, stating this solution.

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Question 10

Categorisation: Appreciate that $g^2(x)$ is not the same as $[g(x)]^2$.

[Edexcel C3 June 2013(R) Q4d]

The functions f and g are defined by

$f: x \rightarrow 2|x| + 3$, $\quad x \in \mathbb{R}\ R\ \ g: x \rightarrow 3 - 4x$, $\quad x \in \mathbb{R}$

Solve the equation

$$gg(x) + \big(g(x)\big)^2 = 0$$

.....

Question 11

Categorisation: Determine the range of a composite function.

[Edexcel C3 June 2011 Q4d]

The function f is defined by

$f: x \rightarrow 4 - ln(x + 2), \quad x \in \mathbb{R}, \quad x \geq -1$

The function g is defined by

$g: x \rightarrow e^{x^2} - 2, \quad x \in \mathbb{R}$

Find the range of fg .

.....

Question 12

Categorisation: As above.

[Edexcel C3 Jan 2006 Q8c Edited]

The functions f and g are defined by

$f: x \rightarrow 2x + ln\, 2, \quad x \in \mathbb{R}$

$g: x \rightarrow e^{2x}, \quad x \in \mathbb{R}$

It can be shown that $gf(x) = 4e^{4x}$

Write down the range of gf .

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Question 13

Categorisation: Use a graph to find the output of a composite function.

[Edexcel C3 Jan 2013 Q3a]

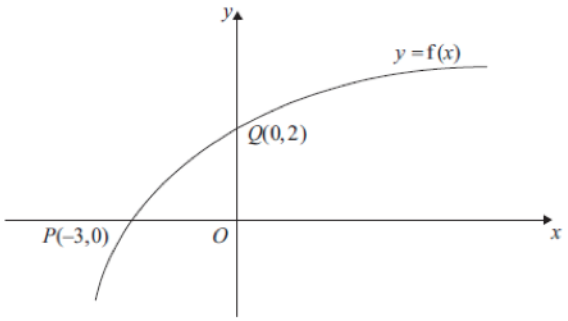


Figure 1

Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.
The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

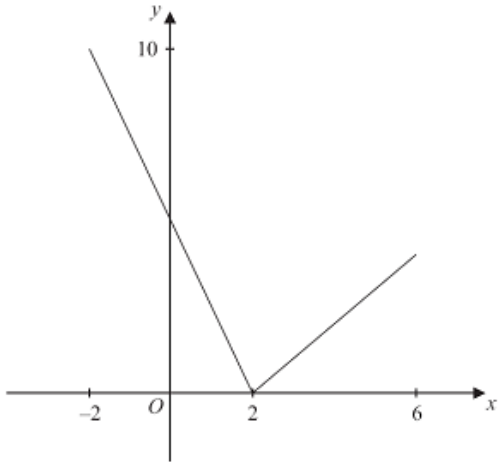
Find the value of $ff(-3)$.

$ff(-3) =$ .....

Question 14

Categorisation: As above.

[Edexcel C3 June 2013 Q7b] The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2,10)$ to $(2,0)$ and from $(2,0)$ to $(6,4)$. A sketch of the graph $y = f(x)$ is shown in the figure.



Find $ff(0)$.

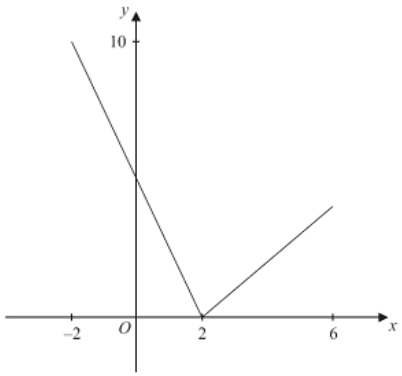
$ff(0) =$ .....

Question 15

Categorisation: Use a function expressed graphically combined with a function expressed algebraically to solve an equation involving a composite function.

[Edexcel C3 June 2013 Q7d]

The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2,10)$ to $(2,0)$ and from $(2,0)$ to $(6,4)$. A sketch of the graph $y = f(x)$ is shown in the figure.



The function g is defined by $g: x \rightarrow \frac{4+3x}{5-x}$, and it can be shown that $g^{-1}(x) = \frac{5x-4}{3+x}$

Solve the equation $gf(x) = 16$.

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Question 16

Categorisation: Determine whether a function has an inverse.

[Edexcel A2 Specimen Papers P1 Q10e Edited]

The function g is defined by

$$g: x \rightarrow x^2 - 3x, \qquad x \in \mathbb{R}, 0 \leq x \leq 5$$

Decide whether the function g has an inverse.

[ ] It has an inverse

[ ] It has not an inverse

Question 17

Categorisation: Determine an inverse function.

[Edexcel A2 Specimen Papers P1 Q10a]

The function f is defined by

$$f: x \to \frac{3x-5}{x+1}, \qquad x \in \mathbb{R}, x \neq -1$$

Find $f^{-1}(x)$

$f^{-1}(x) =$ .....

Question 18

Categorisation: Solve an equation involving an inverse function.

[Edexcel C3 June 2014 Q5c]

$g(x) = \frac{x+1}{x-2}, \quad x > 3$

Find the exact value of a for which $g(a) = g^{-1}(a)$

$a =$ .....

Question 19

Categorisation: Solve an equation involving an inverse trig function.

[Edexcel C3 June 2016 Q7b]

$g(x) = \ arcsin\, x\,, \quad -1 \leq x \leq 1$

Find the exact value of x for which

$$3g(x+1) + \pi = 0$$

.....

Question 20

Categorisation: Find the inverse of a logarithmic function.

[Edexcel C3 Jan 2007 Q6a Edited]

The function f is defined by

$f: x \rightarrow ln\,(4 - 2x)\,, \quad x < 2$ and $x \in \mathbb{R}$

Find the inverse function of f .

$f^{-1}(x) =$ .....

Question 21

Categorisation: Determine the domain or range given parametric functions.

[Edexcel A2 Specimen Papers P2 Q10a]

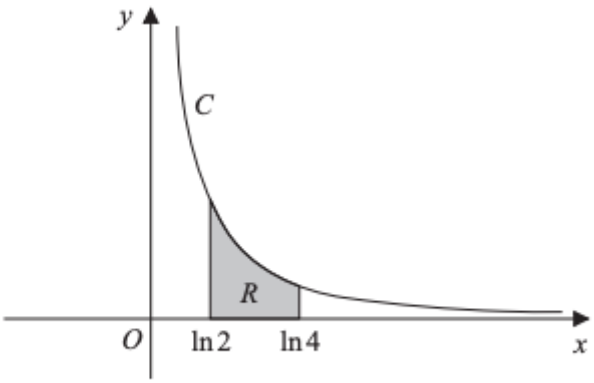

Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = ln\,(t + 2), \qquad y = \frac{1}{t+1}, \qquad t > -\frac{2}{3}$$

State the domain of values of x for the curve C .

.....

Question 22

Categorisation: Determine the range of a quadratic function.

[Edexcel A2 Specimen Papers P1 Q10d]

The function g is defined by

$$g: x \rightarrow x^2 - 3x, \qquad x \in \mathbb{R}, 0 \leq x \leq 5$$

Find the range of g .

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Question 23

Categorisation: As above.

[Edexcel C3 June 2010 Q4d]

The function g is defined by

$g: x \rightarrow x^2 - 4x + 1$, $\qquad x \in \mathbb{R}$, $\qquad 0 \leq x \leq 5$.

Find the range of g .

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Question 24

Categorisation: Determine the range of a function involving a square root.

[Edexcel C3 June 2017 Q3a]

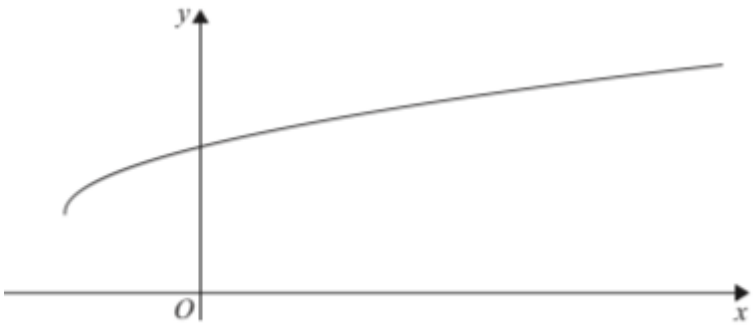

Figure 1

Figure 1 shows a sketch of part of the graph of $y = g(x)$, where

$g(x) = 3 + \sqrt{x+2}$, $x \geq -2$

State the range of g .

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Question 25

Categorisation: Determine the domain of an inverse function.

[Edexcel C3 June 2014(R) Q6b]

The function f is defined by

$f: x \rightarrow e^{2x} + k^2$, $x \in \mathbb{R}$, k is a positive constant.

Find f^{-1} and state its domain.

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Question 26

Categorisation: Determine the range of a function involving an exponential term.

[Edexcel C3 June 2014(R) Q6a] The function f is defined by

$f: x \rightarrow e^{2x} + k^2$, $x \in \mathbb{R}$, k is a positive constant.

State the range of f.

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Question 27

Categorisation: Determine the range of a modulus function.

[Edexcel C3 June 2013(R) Q4a] The functions f and g are defined by

$f: x \rightarrow 2|x| + 3$, $x \in \mathbb{R}$
$g: x \rightarrow 3 - 4x$, $x \in \mathbb{R}$

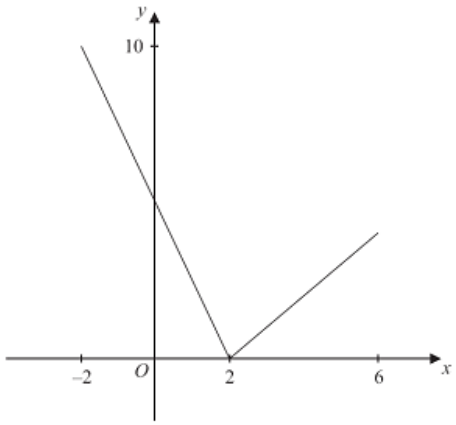
State the range of f.

.....

Question 28

Categorisation: Determine a domain/range given a graph.

[Edexcel C3 June 2013 Q7a] The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2,10)$ to $(2,0)$ and from $(2,0)$ to $(6,4)$. A sketch of the graph $y = f(x)$ is shown in the figure.



Write down the range of f.

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Question 29

Categorisation: Appreciate that the domain of an inverse function is the same as the range of the original function (except with modified notation).

[Edexcel C3 June 2011 Q4b] The function f is defined by

$f: x \rightarrow 4 - \ln(x+2), \quad x \in \mathbb{R}, \ x \geq -1$

Find the domain of f^{-1}.

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Question 30

Categorisation: Determine the range of a composite function.

[Edexcel C3 Jan 2009 Q5c Edited]

The functions f and g are defined by

$f: x \rightarrow 3x + \ln x, \ x > 0, \ x \in \mathbb{R}$

$g: x \rightarrow e^{x^2}, \ x \in \mathbb{R}$

It can be shown that $fg: x \rightarrow x^2 + 3e^{x^2}, \ x \in \mathbb{R}$

Write down the range of fg.

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Question 31

Categorisation: Consider the number of points of intersection of a modulus graph with another graph.

[Edexcel A2 SAM P2 Q11c]

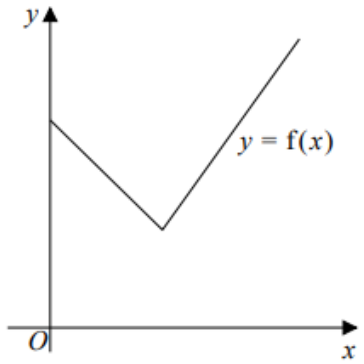

Figure 2

Figure 2 shows a sketch of part of the graph $y = f(x)$, where

$f(x) = 2|3 - x| + 5\,,\ \ x \geq 0$

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots, state the set of possible values for k .

.....

Question 32

Categorisation: Solve an equation involving a modulus function.

[Edexcel A2 SAM P2 Q11b] (Continued from above)

$f(x) = 2|3 - x| + 5\,,\ \ x \geq 0$

Solve the equation $f(x) = \frac{1}{2}x + 30$

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Question 33

Categorisation: Find the range of a modulus function.

[Edexcel A2 SAM P2 Q11a] (Continued from above)

$f(x) = 2|3 - x| + 5$, $x \geq 0$

State the range of f .

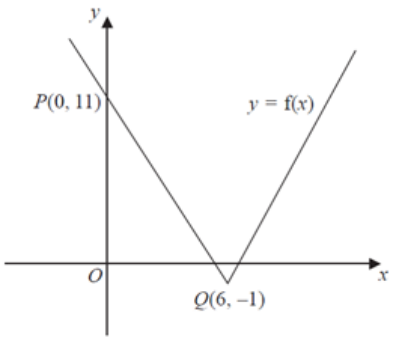
.....

Question 34

Categorisation: Use a given modulus graph to find unknowns within the equation.

[Edexcel C3 June 2014 Q4c]

The figure shows part of the graph with equation $y = f(x)$, $x \in \mathbb{R}$



Given that $f(x) = a|x - b| - 1$, where a and b are constants, state the value of a and the value of b .

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Question 35

Categorisation: The reverse: Use an equation involving a modulus expression to find unknown points within its graph.

[Edexcel C3 June 2005 Q6c]

Figure 1

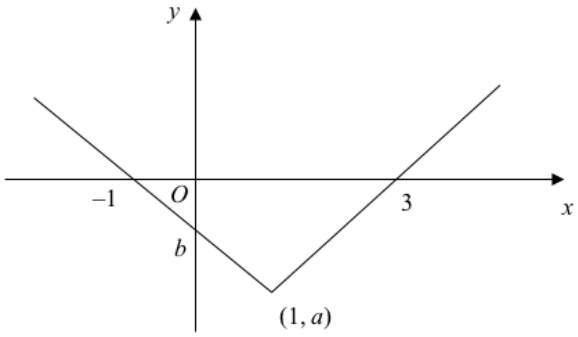

Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the *x*-axis at (3, 0). The other line meets the x-axis at (–1, 0) and the y-axis at $(0, b)$, $b < 0$.

Given that $f(x) = |x - 1| - 2$, find the value of a and the value of b.

.....

Question 36

Categorisation: Solve an equation involving a modulus expression and unknown constants.

[Edexcel C3 June 2017 Q6b]

Given that a and b are positive constants, and that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$, find c in terms of a.

$c =$ .....

Question 37

Categorisation: Reason about an asymptote in a modulus graph.

[Edexcel C3 June 2016 Q4aiii]

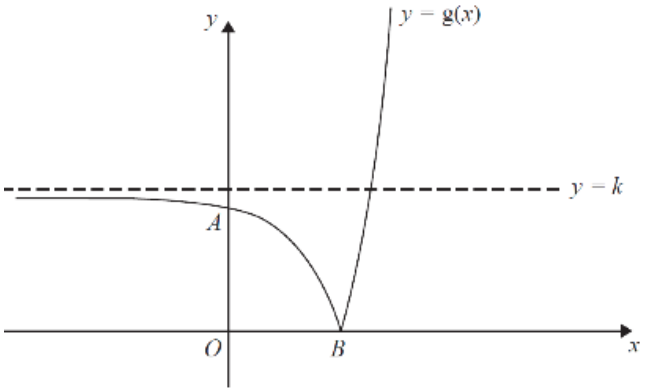

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = g(x)$, where

$g(x) = |4e^{2x} - 25|$, $\quad x \in \mathbb{R}$.

The curve cuts the y -axis at the point A and meets the x -axis at the point B . The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1.

Find the value of the constant k , giving your answer in its simplest form.

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Question 38

Categorisation: Solve modulus equations involving an exponential term.

[Edexcel C3 June 2015 Q2c]

Let $f(x) = 2e^x - 5$, $\quad x \in \mathbb{R}$.

Find the exact solutions of the equation $|f(x)| = 2$

.....

Question 39

Categorisation: Solve modulus equations involving a reciprocal graph.

[Edexcel C3 June 2007 Q5d] The functions f and g are defined by

$f: x \to \ ln\,(2x-1)\,,\ \ x \in \mathbb{R}\,,\ \ x > \frac{1}{2}$

$g: x \to \frac{2}{x-3},\ \ x \in \mathbb{R}\,,\ \ x \neq 3$

Find the exact values of x for which $\left|\frac{2}{x-3}\right| = 3$

.....

Question 40

Categorisation: Solve equations where the modulus term is being subtracted.

[Edexcel C3 June 2008 Q3d]

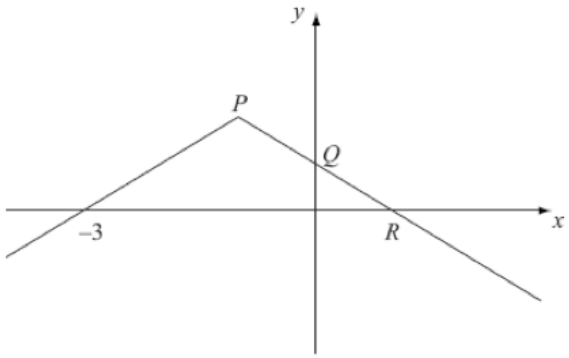

Figure 1

Figure 1 shows the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point P.

The graph cuts the y-axis at the point Q and the x-axis at the points (–3, 0) and R.

Given that $f(x) = 2 - |x+1|$, solve $f(x) = \frac{1}{2}x$.

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Question 41

Categorisation: Solve an inequality involving a modulus term.

[Edexcel C3 June 2014(R) Q5c] By a suitable sketch of $y = 4x - 3$ or otherwise, find the complete set of values of x for which

$$|4x - 3| > \frac{3}{2} - 2x$$

.....

Answers

Question 1

$x = \frac{e}{\sqrt{2}}$

(c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 8x^6 = 6$	M1
	$\Rightarrow 8x^6 = e^6 \Rightarrow x = ..$	M1
	$\Rightarrow x = \left(\frac{e}{\sqrt[6]{8}}\right) = \frac{e}{\sqrt{2}}$ (Ignore any reference to $-\frac{e}{\sqrt{2}}$)	A1

Question 2

$$a = 4$$

$f(x) = x^3 + ax^2 - ax + 48,\ x \in \mathbb{R}$		
$f(-6) = (-6)^3 + a(-6)^2 - a(-6) + 48$	M1	1.1b
$= -216 + 36a + 6a + 48 = 0 \Rightarrow 42a = 168 \Rightarrow a = 4$ *	A1*	1.1b

Question 3

$$x = \frac{ln\ 4 - 3}{2}$$

(c)	$e^{2x+3} + 2 = 6 \Rightarrow e^{2x+3} = 4$	M1A1
	$\Rightarrow 2x + 3 = \ln 4$	
	$\Rightarrow x = \frac{\ln 4 - 3}{2}$ or $\ln 2 - \frac{3}{2}$	M1A1

Question 4

$fg(2) =$ 11

$fg(2) = f(4-6) = f(-2) = \frac{3(-2)-5}{-2+1}; = 11$	M1	1.1b
	A1	1.1b

Question 5

$$a = -5$$

$ff(x) = \frac{3\left(\frac{3x-5}{x+1}\right) - 5}{\left(\frac{3x-5}{x+1}\right) + 1}$	M1	1.1a
$= \frac{\frac{3(3x-5) - 5(x+1)}{x+1}}{\frac{(3x-5) + (x+1)}{x+1}}$	M1	1.1b
	A1	1.1b
$= \frac{9x - 15 - 5x - 5}{3x - 5 + x + 1} = \frac{4x - 20}{4x - 4} = \frac{x-5}{x-1}$ (note that $a = -5$)	A1	2.1

Question 6

$x = \frac{k}{2}$

(e)	$\text{fg}(x) = 2k^2 \Rightarrow 4x^2 + k^2 = 2k^2$	
	$\Rightarrow 4x^2 = k^2 \Rightarrow x = ..$	M1
	$\Rightarrow x = \frac{k}{2}$ **only**	A1

Question 7

$x = -\frac{5}{2}a$ or $x = \frac{3}{2}a$

Either Attempt composition of functions the right way round	M1
Obtain $5\lvert 2x+a\rvert + 11a = 31a$ or equiv	A1
Or Apply their g^{-1} to $31a$	M1
Obtain $\lvert 2x+a\rvert + 3a = 7a$ or equiv	A1
Either Solve $2x+a=4a$ and obtain $\frac{3}{2}a$	B1 FT
Solve linear equation in which signs of (their) $2x$ and (their) $4a$ are different	M1
Obtain $-\frac{5}{2}a$	A1
Or Square both sides and obtain $4x^2 + 4ax - 15a^2 = 0$	B1 FT
Solve 3-term quadratic equation to obtain two values	M1
Obtain $-\frac{5}{2}a$, $\frac{3}{2}a$	A1

Question 8

$N = 48$

Apply f once to obtain $2+N$	B1
Apply f to their expression involving N	M1
Obtain $2+\ln(N+5)$ or $2+\ln(2+N+3)$	A1
Attempt solution of equation of form $2+\ln(pN+q) = \ln(53e^2)$	M1
Obtain 48 from correct work	A1

Question 9

$x = \sqrt{3}$

$\text{gf}(x) = \text{fg}(x) \Rightarrow 3x = x^3$	M1
$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1
$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x=0$ and $-\sqrt{3}$	M1

Question 10

$x = 0$ or $x = 0.5$

(d)	$[g(x)]^2 = (3-4x)^2$	B1
	$gg(x) = 3-4(3-4x)$	M1
	$gg(x)+[g(x)]^2 = 0 \Rightarrow -9+16x+9-24x+16x^2 = 0$	
	$16x^2 - 8x = 0$	A1
	$8x(2x-1) = 0 \Rightarrow x = 0, 0.5$ oe	M1A1

Question 11

$$fg(x) \leq 4$$

(d)	$fg(x) \leq 4$	B1ft

Question 12

$$gf(x) > 0$$

(c)	Range is $\square_+$	Accept $\text{gf}(x) > 0, y > 0$	B1	**(1)**

Question 13

$ff(-3) =$ 2

(a) ff(-3)= f(0),=2	M1,A1

Question 14

$ff(0) = 3$

(b)	ff(0) = f(5), = 3	B1,B1

Question 15

$x = 0.4$ or $x = 6$

(d)	$\text{gf}(x) = 16 \Rightarrow \text{f}(x) = \text{g}^{-1}(16) = 4$ oe	M1A1
	$\text{f}(x) = 4 \Rightarrow x = 6$	B1
	$\text{f}(x) = 4 \Rightarrow 5-2.5x = 4 \Rightarrow x = 0.4$ oe	M1A1

Question 16

It has not an inverse

E.g. • the function g is many-one • the function g is not one-one • the inverse is one-many • $g(0) = g(3) = 0$	B1	2.4

Question 17

$f^{-1}(x) = \frac{x+5}{3-x}$

$y = \frac{3x-5}{x+1} \Rightarrow y(x+1) = 3x-5 \Rightarrow xy + y = 3x - 5 \Rightarrow y + 5 = 3x - xy$	M1	1.1b
$\Rightarrow y + 5 = x(3-y) \Rightarrow \frac{y+5}{3-y} = x$	M1	2.1
Hence $f^{-1}(x) = \frac{x+5}{3-x}$, $x \in \mathbb{R}$, $x \neq 3$	A1	2.5

Question 18

$a = \frac{3+\sqrt{13}}{2}$

(c)	Attempt to set Either $g(x) = x$ or $g(x) = g^{-1}(x)$ or $g^{-1}(x) = x$ or $g^2(x) = x$ $\frac{(x+1)}{(x-2)} = x$ $\frac{x+1}{x-2} = \frac{2x+1}{x-1}$ $\frac{2x+1}{x-1} = x$ $\frac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2} = x$	M1
	$x^2 - 3x - 1 = 0 \Rightarrow x = \ldots$	A1, dM1
	$a = \frac{3+\sqrt{13}}{2}$ oe $\left(1.5 + \sqrt{3.25}\right)$ cso	A1

Question 19

$$x = -1 - \frac{\sqrt{3}}{2}$$

(b)	$3\arcsin(x+1) + \pi = 0 \Rightarrow \arcsin(x+1) = -\frac{\pi}{3}$	M1
	$\Rightarrow (x+1) = \sin\left(-\frac{\pi}{3}\right)$	
	$\Rightarrow x = -1 - \frac{\sqrt{3}}{2}$	dM1A1

Question 20

$f^{-1}(x) = 2 - \frac{1}{2}e^x$

(a) $y = \ln(4-2x)$ $e^y = 4 - 2x$ leading to $x = 2 - \frac{1}{2}e^y$ Changing subject and removing ln		M1 A1
$y = 2 - \frac{1}{2}e^x \Rightarrow f^{-1} \mapsto 2 - \frac{1}{2}e^x$ *	cso	A1
Domain of f^{-1} is		B1

Question 21

$$x > \ ln\left(\frac{4}{3}\right)$$

$x > \ln\left(\frac{4}{3}\right)$	B1	2.2a

Question 22

$g(x) \geq -2.25$ or $g(x) \leq 10$

$g(x) = x^2 - 3x = (x-1.5)^2 - 2.25$. Hence $g_{min} = -2.25$	M1	2.1
Either $g_{min} = -2.25$ or $g(x) \geqslant -2.25$ or $g(5) = 25 - 15 = 10$	B1	1.1b
$-2.25 \leqslant g(x) \leqslant 10$ or $-2.25 \leqslant y \leqslant 10$	A1	1.1b

Question 23

$g(x) \geq -3$ and $g(x) \leq 6$

(d)	$g(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $g_{min} = -3$	M1
	Either $g_{min} = -3$ or $g(x) \geqslant -3$	B1
	or $g(5) = 25 - 20 + 1 = 6$	
	$-3 \leqslant g(x) \leqslant 6$ or $-3 \leqslant y \leqslant 6$	A1

Question 24

$$y \geq 3$$

3.(a)	$y \geq 3$	B1

Question 25

$f^{-1}(x) = \frac{1}{2}\ ln\ (x - k^2)$, $Domain: = x > k^2$

(b)	$y = e^{2x} + k^2 \Rightarrow e^{2x} = y - k^2$	M1
	$\Rightarrow x = \frac{1}{2}\ln(y - k^2)$	dM1
	$\Rightarrow f^{-1}(x) = \frac{1}{2}\ln(x - k^2), \quad x > k^2$	A1

Question 26

$$f(x) > k^2$$

6.(a)	$f(x) > k^2$	B1

Question 27

$$f(x) \geq 3$$

4.(a)	$f(x) \geqslant 3$	M1A1

Question 28

$f(x) \geq 0$ and $f(x) \leq 10$

7(a)	$0 \leqslant f(x) \leqslant 10$	B1

Question 29

$$x \leq 4$$

(b)	$x \leq 4$	B1

Question 30

$$fg(x) \geq 3$$

(c)	$fg(x) \geq 3$	B1

Question 31

$k > 5$ or $k \leq 11$

Makes the connection that there must be two intersections. Implied by either end point $k > 5$ or $k \leqslant 11$	M1
$\{k : k \in \mathbb{R}, 5 < k \leqslant 11\}$	A1

Question 32

$$x = \frac{62}{3}$$

Uses $-2(3-x)+5 = \frac{1}{2}x + 30$	M1
Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x = 31$	M1
$x = \frac{62}{3}$ only	A1

Question 33

$$f(x) \geq 5$$

$f(x) \geqslant 5$	B1

Question 34

$a = 2$, $b = 6$

(c)	One of $a = 2$ or $b = 6$	B1
	$a = 2$ and $b = 6$	B1

Question 35

$a = -2$, $b = -1$

(c)	$a = -2, \quad b = -1$	B1B1

Question 36

$c = 4a$

(b)	States or uses $a+b=8$	B1
	Attempts to solve $\lvert 2x-a\rvert+b=\frac{3}{2}x+8$ in either x or with $x=c$	
	$2c-a+b=\frac{3}{2}c+8 \Rightarrow kc=\mathrm{f}(a,b)$	M1
	Combines $kc=\mathrm{f}(a,b)$ with $a+b=8 \quad \Rightarrow c=4a$	dM1 A1

Question 37

$$k = 25$$

(iii) 25	B1

Question 38

$x = \ ln\left(\frac{7}{2}\right)$ or $x = \ ln\left(\frac{3}{2}\right)$

(c)	$2e^{x}-5=-2 \Rightarrow (x)=\ln\left(\frac{3}{2}\right)$	M1A1
	$(x)=\ln\left(\frac{7}{2}\right)$	B1

Question 39

$x = \frac{11}{3}$ or $x = \frac{7}{3}$

(d)	$\frac{2}{x-3}=3 \quad \Rightarrow x=3\frac{2}{3}$ or exact equiv.	B1
	$\frac{2}{x-3}=-3$, $\Rightarrow x=2\frac{1}{3}$ or exact equiv.	M1, A1

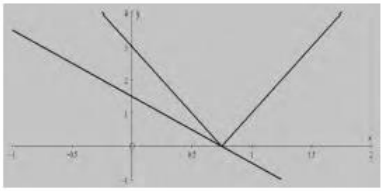
Question 40

$x = \frac{2}{3}$ or $x = -6$

(d)	$x>-1; \quad 2-x-1=\frac{1}{2}x$	M1 A1
	Leading to $x=\frac{2}{3}$	A1
	$x<-1; \quad 2+x+1=\frac{1}{2}x$	M1
	Leading to $x=-6$	A1

Question 41

$$x\ ne\ \frac{3}{4}$$

(c)	 Draws graph Or solves $\lvert 4x-3\rvert = 1\frac{1}{2} - 2x$ to give one soln $x = ¾$	M1
	Accept for all values of x except $x = \frac{3}{4}$ Or $(x \in \mathbb{R},)\ x \neq \frac{3}{4}$, or $x < \frac{3}{4}, x > \frac{3}{4}$	A1