IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper R

Difficulty Rating: 4.015/1.4106

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 11 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Relative to a fixed origin O, the point A has coordinates $(k,3,5)$, where k is a scalar constant.

The points B and C are such so that $\overrightarrow{BA} = 3\mathbf{i} - 2\mathbf{j}$ and $\overrightarrow{BC} = 2\mathbf{i} + c\mathbf{j} - 4\mathbf{k}$, where c is a scalar constant.

If the coordinates of C are $(1,4k,1)$, determine the distance BC. **(6)**

Question 2

The functions f and g are given by

$$f(x) = x^2 + 2kx + 4, \ x \in \mathbb{R}$$

$$g(x) = 3 - kx, \ x \in \mathbb{R}.$$

where k is a non zero constant.

a) Find, in terms of k, the range of f. **(3)**

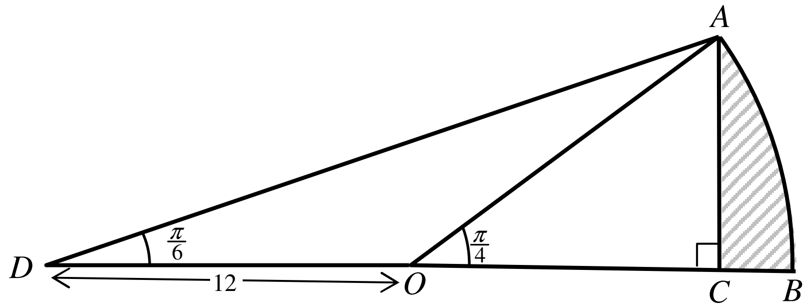
b) Given further that $fg(2) = 4$, determine the value of k. **(3)**

Question 3

The 2^{nd}, 3^{rd} and 9^{th} term of an arithmetic progression are three consecutive terms of a geometric progression.

Find the common ratio of the geometric progression. **(8)**

Question 4



The figure above shows a triangle OAC with $\measuredangle ACO = \frac{1}{2}\pi$ and $\measuredangle AOC = \frac{1}{2}\pi$.

Another triangle AOD is drawn next to the triangle OAC, so that DOC is a straight line, $|DO| = 12$ units and $\measuredangle ADO = \frac{1}{6}\pi$.

Finally a circular sector OAB is drawn, centred at O, with radius OA, so that $DOCB$ is a straight line.

a) Show that the length of OA is

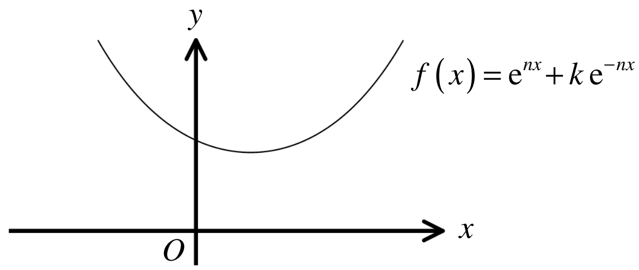
$$6\left(\sqrt{6}+\sqrt{2}\right).$$ **(4)**

b) Find the exact area of the sector OAB. **(3)**

c) Hence show that the area of the shaded region ACB is

$$18\left(2+\sqrt{3}\right)(\pi-2).$$ **(4)**

Question 5



The figure above shows the graph of the curve with equation

$$f(x) = e^{nx} + k\,e^{-nx}, \ x \in \mathbb{R} \ , \ k > 1, \ n > 0 .$$

Find the range of $f(x)$ in exact form. **(6)**

Question 6

The value of a machine, in thousands of pounds, t years after it was purchased is denoted by £ V .

The value of this machine at any given time is depreciating at rate proportional to its value **squared**, at that time.

a) Given that the initial value of the machine was £12000, show that

$$V = \frac{12}{at+1},$$

where a is a positive constant. **(5)**

b) Given further that the machine depreciated by £4000 two years after it was bought, find its value after a **further** period of ten years has elapsed. **(4)**

Question 7

$$y = \frac{(1+\sin x)^2}{\cos^2 x}.$$

a) Calculate the two missing values of the following table.

x	$\frac{\pi}{6}$		$\frac{\pi}{4}$	$\frac{7\pi}{24}$	$\frac{\pi}{3}$
y	3		5.8284	8.6784	13.9282

(2)

b) Use the trapezium rule with all the values from the completed table of part **(a)** to find an estimate for

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{(1+\sin x)^2}{\cos^2 x}\, dx.$$ **(3)**

c) Use trigonometric identities to find the exact value of

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{(1+\sin x)^2}{\cos^2 x}\, dx.$$ **(6)**

Question 8

At the point P which lies on the curve with equation

$$y = \frac{x}{y + \ln y},$$

the gradient is 2.

The point P is close to the point with coordinates $(-0.3, 0.3)$.

a) Show that the y coordinate of P is a solution of the equation

$$y = \mathrm{e}^{-\frac{1}{2}(4y+1)}. \qquad \textbf{(6)}$$

b) By using an iterative formula based on the equation of part **(a)**, determine the coordinates of P correct to three decimal places. **(4)**

Question 9

$$f(x) \equiv \frac{16x^2 + 3x - 2}{x^2(3x-2)}, \; x \in \mathbb{R}, \; |x| < \frac{2}{3}, \; x \neq 0.$$

a) Determine the value of each of the constants A, B and C given that

$$f(x) \equiv \frac{A}{x^2} + \frac{B}{x} + \frac{C}{(3x-2)}. \qquad \textbf{(4)}$$

b) Find the binomial series expansion of $\frac{1}{3x-2}$, up and including the term in x^3. **(3)**

c) Hence, or otherwise, show that if x is numerically small

$$\frac{16x^2 + 3x - 2}{(3x-2)} \approx 1 - 8x^2 - 12x^3 - 18x^4 - 27x^5. \qquad \textbf{(4)}$$

Question 10

It is given that

$$\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta .$$

a) Prove the validity of the above trigonometric identity. **(4)**

b) By differentiating both sides of the above identity with respect to θ, show that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta .$$ **(4)**

c) Hence show that

$$\tan 3\theta \equiv \frac{3\tan\theta\sec^2\theta - 4\tan^3\theta}{4 - 3\sec^2\theta} .$$ **(3)**

d) Deduce that

$$\tan 3\theta \equiv \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} .$$ **(2)**

Question 11

The curve C is given parametrically by the equations

$$x = \cos^3 t ,\ y = \sin^3 t ,\ 0 < t < \frac{\pi}{2} .$$

a) Show that an equation of the normal to C at the point where $t = \theta$ is

$$x\cos\theta - y\sin\theta = \cos 2\theta .$$ **(5)**

The normal to C at the point where $t = \theta$ meets the coordinate axes at the points A and B.

b) Given that O is the origin, show further that the area of the triangle AOB is

$$\cos 2\theta \cot 2\theta .$$ **(4)**
