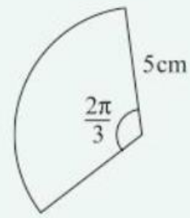
1 **a** Find the exact values of x for which $\tan x = \sqrt{3}$ in the interval $-180° \le x \le 180°$ **[3 marks]**

b Sketch the graph of $y = \tan 2x$ for $-90° \le x \le 90°$. State the equations of the asymptotes. **[3]**

2 A sector of a circle of radius 5 cm has an angle of $\frac{2\pi}{3}$ at its centre.
Calculate the exact value of



a The area of the sector, **[2]**

b The arc length of the sector. **[2]**

3 A sector of a circle with radius 3 cm has an area of 5.4 cm^2

a Calculate the angle subtended at the centre of the circle. **[2]**

b Calculate the perimeter of the sector. **[3]**

4 **a** State the exact value of **i** $\sec\left(\frac{\pi}{4}\right)$ **ii** $\cot\left(\frac{5\pi}{6}\right)$ **[4]**

b Sketch the graph of $y = \operatorname{cosec} x$ for the interval $-\pi \le x \le \pi$
Write the equations of the asymptotes. **[3]**

c State the range of $y = \operatorname{cosec} x$ **[1]**

5 Solve these equations for $0 \le x \le 360°$. Show your working.

a $\operatorname{cosec} x = 2$ **[3]** **b** $\sec x = -\sqrt{2}$ **[3]**

6 **a** On the same set of axes, sketch the graphs of $y = \sin x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ and $y = \arcsin x$ for $-1 \le x \le 1$ **[3]**

b Describe the geometric relationship between the two curves. **[1]**

c Work out the value of these expressions giving your answers as exact multiples of π

i $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$ **ii** $\arcsin\left(\cos\frac{\pi}{3}\right)$ **[2]**

7 The velocity of a particle at time t seconds is given by $v = 5\cos\left(\frac{t}{2} - \frac{\pi}{3}\right)$ m s^{-1}.

a State the maximum speed of the particle. **[1]**

b Give the times at which this maximum speed occurs for $0 \le t < 15$ **[3]**

8 Given that $\arccos\left(x + \frac{1}{2}\right) = \frac{\pi}{4}$, find the exact value of x **[3]**

9 **a** Use the identity $\sin^2\theta + \cos^2\theta \equiv 1$ to show that $\sec^2\theta - \tan^2\theta \equiv 1$ **[3]**

b Given that $\tan\theta = \sqrt{5}$, find the exact value of

i $\sec\theta$ **ii** $\cos\theta$ **[5]**

10 By using the formula $\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$, find the exact value of

a $\cos 75°$ **[3]** **b** $\cos 15°$ **[2]**

11 Given that $\sin(A+B)\equiv\sin A\cos B+\sin B\cos A$, show that $\sin 2x\equiv 2\sin x\cos x$ **[2]**

12 Write $6\sin\theta+8\cos\theta$ in the form $r\sin(\theta+\alpha)$, where $r>0$ and $0<\alpha<90°$ **[4]**

13 For $\theta=0.05$ radians, state the approximate value of

a $\sin\theta$ **[1]** **b** $\cos\theta$ **[1]** **c** $\tan\theta$ **[1]**

14 a Sketch the graphs of these equations for $0\le x\le 2\pi$. Label the x- and y-intercepts with their exact values.

i $y=\sin 3x$ **ii** $y=\cos\left(x+\frac{\pi}{3}\right)$ **[6]**

b Solve the equation $\sin 3x=\frac{\sqrt{2}}{2}$ for $0\le x\le\pi$. Show your working and give your answers in terms of π **[5]**

15 A triangle ABC has $AB=10\text{ cm}$, $BC=13\text{cm}$ and $\angle BCA=\frac{\pi}{4}$

a Calculate the possible lengths of AC **[5]**

b What is the minimum possible area of the triangle? **[3]**

16 The area of an isosceles triangle is 100 cm^2. Calculate the perimeter of the triangle, given that one of the angles is $\frac{\pi}{6}$ rad. **[7]**

17 The population (in thousands) of a particular species of insect around a lake t weeks after a predator is released is modelled by

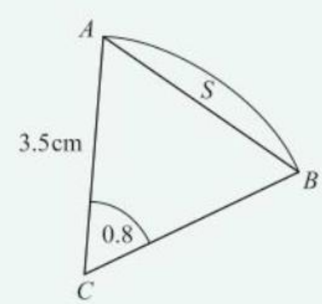
$P=6.5-4.1\sin\left(\frac{\pi t}{2.3}\right)$

a What was the initial population? **[2]**

b State the maximum possible population of the insect. **[1]**

c When does this maximum first occur? Give your answer to the nearest day. **[3]**

18 AB is the arc of a circle with radius 3.5 cm and centre C as shown.



a Calculate the

i Area of the sector, **ii** Perimeter of the sector. **[5]**

The segment S is bounded by the arc and the chord AB

b Calculate the

i Area of S **ii** Perimeter of S **[6]**

19 a Sketch these graphs for $0 \le x \le 2\pi$. Label the x- and y-intercepts and any asymptotes in terms of π

i $y = 2\operatorname{cosec} x$ **ii** $y = \sec\frac{x}{2}$ **[6]**

b State the range of each of the graphs in part **a**. **[3]**

20 $f(x) = \cot(x - 30°)$

a Sketch the graph of $y = f(x)$ for x in the interval $-180° \le x \le 180°$
Label the x- and y-intercepts and give the equations of any asymptotes. **[4]**

b Solve the equation $\cot(x - 30°) = 0.2$ for $-180° \le x \le 180°$. Show your working. **[3]**

21 Solve these equations for $0 \le x \le 2\pi$. Show your working and give your solutions as exact multiples of π

i $\sec(x + \pi) = 2$ **ii** $\operatorname{cosec}\left(x - \frac{\pi}{8}\right) = \sqrt{2}$ **[8]**

22 $f(x) = \arccos(x - 1)$

a Sketch the graph of $y = f(x)$ for $0 \le x \le 2$ **[3]**

b State the range of $f(x)$ **[2]**

c Work out the inverse of $f(x)$ and state its domain and range. **[4]**

23 Find the exact value of x for which

a $\arctan(2x - 1) = \frac{\pi}{3}$ **[2]** **b** $\operatorname{arccot}(x - 5) = \frac{2\pi}{3}$ **[2]**

24 a Show that the equation $2\tan^2 x = \sec x - 1$ can be written as $2\sec^2 x - \sec x - 1 = 0$ **[3]**

b Hence solve the equation $2\tan^2 x = \sec x - 1$ for x in the interval $0 \le x \le 2\pi$, showing your working. **[4]**

25 a Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$ **[3]**

b Solve the equation $\operatorname{cosec}^2\theta - 3\cot\theta - 1 = 0$ for $0 \le \theta < 360°$, showing your working. **[6]**

26 a Show that $\sec^4 x - \tan^4 x \equiv \sec^2 x + \tan^2 x$ **[2]**

b Find the values of x in the range $-\pi \le x \le \pi$ that satisfy $\sec^4 x - \tan^4 x = 5 + \tan^2 x$. Show your working. **[4]**

27 a By writing $\cos 3x$ as $\cos(2x + x)$, show that $\cos 3x = 4\cos^3 x - 3\cos x$ **[5]**

b Hence solve the equation $8\cos^3 x - 6\cos x = \sqrt{3}$ for x in the interval $0 \le x \le 2\pi$

Show your working and give your answers as exact multiples of π **[5]**

28 a Use the identity $\cos(A + B) \equiv \cos A\cos B - \sin A\sin B$ to show that $\cos 2x \equiv 2\cos^2 x - 1$ **[3]**

b Hence solve the equation $\cos 2x + 3\cos x + 2 = 0$ for $0 \le \theta < 360°$, showing your working. **[5]**

29 Prove by counter-example that $\cos(A+B) \neq \cos A + \cos B$ **[2]**

30 $f(x) = 8\cos x + 4\sin x$

a Write $f(x)$ in the form $r\cos(x-\alpha)$, where $r>0$ and $0<\alpha<\frac{\pi}{2}$ **[4]**

b Hence solve the equation $8\cos x + 4\sin x = \sqrt{5}$ for $0 < x \leq 2\pi$. Show your working. **[4]**

31 Sketch these graphs for $0 \leq x \leq 360°$

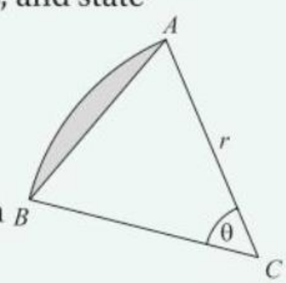
a $y = 2\cos(x+60°)$ **[4]** **b** $y = -\sin\left(\frac{x}{2}\right)$ **[2]**

32 $g(x) = \tan\left(\frac{\pi}{3} - x\right)$, for $0 \leq x \leq 2\pi$

a Sketch the graph of $y = g(x)$, clearly labelling the x- and y-intercepts, and state the equations of the asymptotes. **[4]**

b Solve the equation $g(x) = \frac{\sqrt{3}}{3}$. Show your working and give your solutions in terms of π **[4]**

33 The area of sector ABC is 56.7 cm² and the length of the arc AB is 12.6 cm

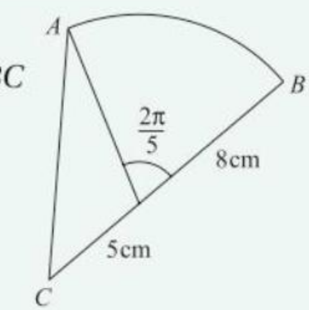


Calculate the area of the shaded segment. **[8]**

34 The shape ABC is formed from a sector and a triangle as shown in the diagram.

a Find the length of AC **[2]**

b Calculate **i** The area of ABC **ii** The perimeter of ABC **[7]**



35 a Sketch the graphs of

i $y = \cot(2x-60)$ for $0 \leq x \leq 180°$

ii $y = 1 - \text{cosec}\left(\frac{x}{2}\right)$ for $0 \leq x \leq 360°$

Give the equations of the asymptotes. **[7]**

b Solve the equation $\sec(3x+20) = \sqrt{2}$ for x in the interval $0 \leq x \leq 180°$. Show your working. **[5]**

36 a Solve the equation $\cot^2\theta = 5$ for $0 \leq \theta \leq 2\pi$ **[4]**

b Find the exact solutions of $\sec^2\left(\theta + \frac{\pi}{6}\right) = 2$ for $0 \leq \theta \leq 2\pi$. Show your working. **[5]**

37 Find the coordinates of the points of intersection of the curves $y = \text{cosec}\, x$ and $y = \sin x$ in the interval $0 \leq x \leq 360°$ **[4]**

38 a Sketch the graph of $y = |\arcsin x|$ for $-1 \leq x \leq 1$ **[2]**

b Sketch the graph of $y = 1 + 2\arccos x$ for $-1 \leq x \leq 1$ and label the y-intercept with its exact value. **[3]**

39 Show that the curve with Cartesian equation $\frac{x^2}{25}-\frac{y^2}{9}=1$ has parametric equations $x=5\sec\theta,\ y=3\tan\theta$ **[2]**

40 Solve the equation $3\operatorname{cosec}^2 x-\cot x=7$ for x in the interval $0\le x\le 360°$. Show your working. **[8]**

41 Find all the solutions of $2\tan^2 2\theta+\sec 2\theta-4=0$ for $0\le\theta\le\pi$. Show your working. **[9]**

42 Prove that $\cos\theta\cot\theta-\sin\theta\equiv\operatorname{cosec}\theta-2\sin\theta$ **[4]**

43 Solve the inequality $\cot^2 x>1+\operatorname{cosec} x$ for x in the interval $0\le x\le\pi$. Show your working. **[6]**

44 **a** **i** Prove that $\frac{\cos x}{\sin x}-\frac{\sin x}{1-\cos x}\equiv-\operatorname{cosec} x$

ii For what values of x is this identity valid? **[5]**

b Solve the equation $\frac{\cos x}{\sin x}-\frac{\sin x}{1-\cos x}=3$ for $0\le x\le 2\pi$. Show your working. **[4]**

45 **a** Prove that $\cos x\equiv 1-2\sin^2\left(\frac{x}{2}\right)$ **[3]**

b Find all the solutions of $\cos x+5\sin^2\left(\frac{x}{2}\right)=3$ for $0<x<360°$. Show your working. **[6]**

46 The height of a tide (in metres) in a harbour t hours after midnight is given approximately by the equation $h=2.8+\sqrt{3}\sin\left(\frac{t}{2}\right)-3\cos\left(\frac{t}{2}\right)$

A particular boat requires a depth of at least 3.5 m in order to safely leave or enter the harbour. The owners of the boat wish to depart the harbour in the afternoon.

Work out the earliest and latest times they can leave. **[8]**

47 Solve the equation $\sin 2x+\sin x=0$ for x in the interval $0<x<360°$. Show your working. **[7]**

48 Given that $\sin\theta+2\cos\theta$ can be written in the form $r\sin(\theta+\alpha)$ where $r>0$ and $0<\alpha<90°$,

a Find the value of r and the value of α **[4]**

b Calculate the minimum value of $\frac{1}{(\sin\theta+2\cos\theta)^2}$ and the smallest positive value of θ for which this minimum occurs, **[4]**

c Find the maximum value of $\frac{1}{3+\sin\theta+2\cos\theta}$ and express it in the form $a+b\sqrt{c}$.

Find also the smallest positive value of θ for which this maximum occurs. **[5]**