

1. A particle  $P$  of mass  $0.6\text{ kg}$  is released from rest and slides down a line of greatest slope of a rough plane. The plane is inclined at  $30^\circ$  to the horizontal. When  $P$  has moved  $12\text{ m}$ , its speed is  $4\text{ m s}^{-1}$ . Given that friction is the only non-gravitational resistive force acting on  $P$ , find

(a) the work done against friction as the speed of  $P$  increases from  $0\text{ m s}^{-1}$  to  $4\text{ m s}^{-1}$ , (4)

(b) the coefficient of friction between the particle and the plane. (4)

(Total 8 marks)

2. A block of mass  $10\text{ kg}$  is pulled along a straight horizontal road by a constant horizontal force of magnitude  $70\text{ N}$  in the direction of the road. The block moves in a straight line passing through two points  $A$  and  $B$  on the road, where  $AB = 50\text{ m}$ . The block is modelled as a particle and the road is modelled as a rough plane. The coefficient of friction between the block and the road is  $\frac{4}{7}$

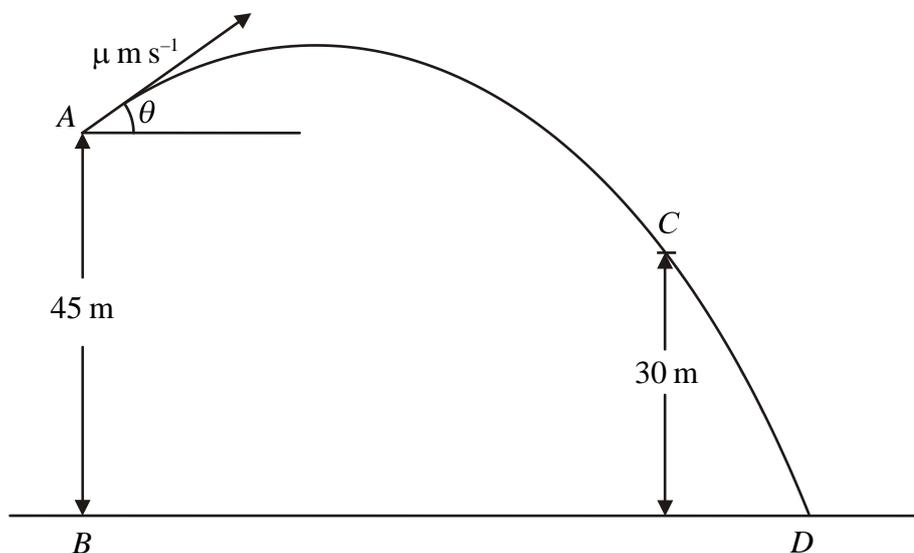
(a) Calculate the work done against friction in moving the block from  $A$  to  $B$ . (4)

The block passes through  $A$  with a speed of  $2\text{ m s}^{-1}$ .

(b) Find the speed of the block at  $B$ . (4)

(Total 8 marks)

3.



A particle  $P$  is projected from a point  $A$  with speed  $u \text{ m s}^{-1}$  at an angle of elevation  $\theta$ , where  $\cos \theta = \frac{4}{5}$ . The point  $B$ , on horizontal ground, is vertically below  $A$  and  $AB = 45 \text{ m}$ .

After projection,  $P$  moves freely under gravity passing through a point  $C$ , 30 m above the ground, before striking the ground at the point  $D$ , as shown in Figure 3.

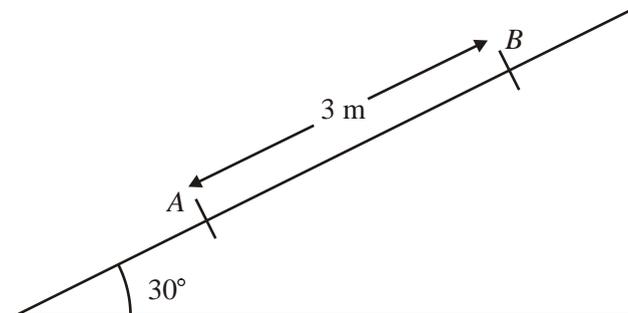
Given that  $P$  passes through  $C$  with speed  $24.5 \text{ m s}^{-1}$ ,

(a) using conservation of energy, or otherwise, show that  $u = 17.5$ , (4)

(b) find the size of the angle which the velocity of  $P$  makes with the horizontal as  $P$  passes through  $C$ , (3)

(c) find the distance  $BD$ . (7)  
(Total 14 marks)

4.



A particle  $P$  of mass 2 kg is projected from a point  $A$  up a line of greatest slope  $AB$  of a fixed plane. The plane is inclined at an angle of  $30^\circ$  to the horizontal and  $AB = 3$  m with  $B$  above  $A$ , as shown in the diagram above. The speed of  $P$  at  $A$  is  $10 \text{ m s}^{-1}$ .

Assuming the plane is smooth,

- (a) find the speed of  $P$  at  $B$ .

(4)

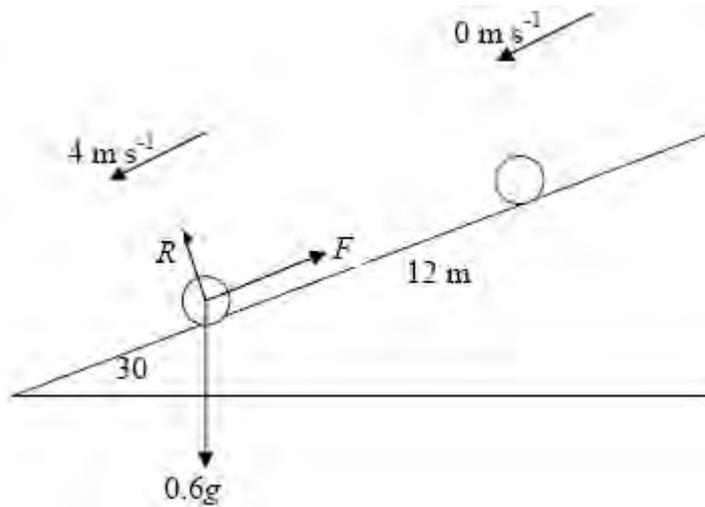
The plane is now assumed to be rough. At  $A$  the speed of  $P$  is  $10 \text{ m s}^{-1}$  and at  $B$  the speed of  $P$  is  $7 \text{ m s}^{-1}$ . By using the work-energy principle, or otherwise,

- (b) find the coefficient of friction between  $P$  and the plane.

(5)

(Total 9 marks)

1. (a)



$$\text{K.E. gained} = \frac{1}{2} \times 0.6 \times 4^2$$

$$\text{P.E. lost} = 0.6 \times g \times (12 \sin 30)$$

$$\text{Change in energy} = \text{P.E. lost} - \text{K.E. gained}$$

$$= 0.6 \times g \times 12 \sin 30 - \frac{1}{2} \times 0.6 \times 4^2 = 0.6 \times g \times 12 \sin 30 - \frac{1}{2} \times 0.6 \times 4^2 \quad \text{M1 A1 A1}$$

$$= 30.48$$

$$\text{Work done against friction} = 30 \text{ or } 30.5 \text{ J}$$

A1 4

(b)  $R(\uparrow) \quad R = 0.6g \cos 30$

B1

$$F = \frac{30.48}{12}$$

B1ft

$$F = \mu R$$

$$\mu = \frac{30.48}{12 \times 0.6g \cos 30}$$

M1

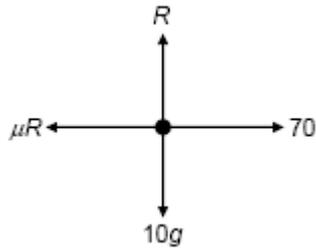
$$\mu = 0.4987$$

$$\mu = 0.499 \text{ or } 0.50$$

A1 4

[8]

2. (a)



$R(\uparrow): R = 10g$  B1

$F = \mu R \Rightarrow F = \frac{4}{7}(10g) = 56$  B1

$\therefore$  WD against friction  $= \frac{4}{7}(10g)(50)$  M1

2800(J) A1 4

(b)  $70(50) - "2800" = \frac{1}{2}(10)v^2 - \frac{1}{2}(10)(2)^2$  M1 \*

A1ft

$700 = 5v^2 - 20, 5v^2 = 720 \Rightarrow v^2 = 144$  d \* M1

Hence,  $v = \underline{12}$  (m s<sup>-1</sup>) A1 cao 4

Or N2L( $\rightarrow$ ):  $70 - \frac{4}{7}R = 10a$  M1 \*

$70 - \frac{4}{7} \times 10g = 10a, (a = 1.4)$  A1ft

AB ( $\rightarrow$ ):  $v^2 = (2)^2 + 2(1.4)(50)$  d \* M1

Hence,  $v = \underline{12}$  (m s<sup>-1</sup>) A1 cao 4

[8]

3. (a) Energy  $\frac{1}{2}m(24.5^2 - u^2) = mg \times 15$  M1A1=A1

$u^2 = 24.5^2 - 30g = 306.25$

$u = \sqrt{306.25} = 17.5$  \* cso A1 4

Alternative

$\rightarrow u_x = u \cos \theta = 0.8u, \uparrow u_y = u \sin \theta = 0.6u$

$v_y^2 = 0.36u^2 + 2 \times 9.8 \times 15 = 0.36u^2 + 294$

$24.5^2 = u_x^2 + v_y^2 = 0.64u^2 + 0.36u^2 + 294$  M1 A1,A1

$u^2 = 306.25 \Rightarrow u = 17.5$  \* cso A1 4

(b)  $\rightarrow u_x = u \cos \theta = 17.5 \times 0.8 = 14$  B1  
 $\psi = \arccos \frac{14}{24.5} \approx 55^\circ$  accept 55.2° M1A1 3  
 (0.96 rads; or 0.963 rads)

Alternative

$\rightarrow u_x = u \cos \theta = 17.5 \times 0.8 = 14$  B1  
 $\uparrow v_y^2 = u^2 \sin^2 \theta + 2 \times 9.8 \times 15 = 404.25$   
 $\psi = \arctan \frac{\sqrt{404.25}}{14} \approx 55^\circ$  accept 55.2° M1A1 3

(c)  $\uparrow u_y = u \sin \theta = 17.5 \times 0.6 = 10.5$  B1  
 $s = ut + \frac{1}{2} at^2 \Rightarrow -45 = 10.5t - 4.9t^2$  M1 A1  
 leading to  $t = 4.3$ . awrt  $t = 4.3$  or  $t = 4\frac{2}{7}$  A1  
 $\rightarrow BD = 14 \times 4\frac{2}{7}$  (14 × t) ft their t M1 Alft  
 $= 60$  (m) only A1 7

Alternative

Use of  $y = x \tan \theta - \frac{g \sec^2 \theta}{2u^2} x^2$  M1  
 $-45 = \frac{3}{4}x - \frac{g}{2 \times 17.5^2} \times \frac{25}{16} x^2$  B1,A1  
 $x^2 - 30x - 1800 = 0$  o.e. A1  
 Factors or quadratic formula M1 Alft  
 $BD = 60$  (m) A1

[14]

4. (a)  $\frac{1}{2} \times 2 \times 10^2 - \frac{1}{2} \times 2 \times v^2 = 2g \cdot 3 \sin 30^\circ$  M1 A1 A1  
 $v = 8.4 \text{ m s}^{-1}$  (8.40 m s<sup>-1</sup>) A1 4  
 Or  $(a = -g \sin 30^\circ)$   
 $v^2 = 10^2 - 2g \sin 30^\circ \times 3$  M1 A1 A1  
 $v = 8.4 \text{ m s}^{-1}$  (8.40 m s<sup>-1</sup>) A1 (4)

(b) $R = 2g \cos 30^\circ$	B1
$3F; \frac{1}{2} \times 2 \times 10^2 - \frac{1}{2} \times 2 \times 7^2; 2g \times 3 \sin 30^\circ$	B2 (-1 e.e.o.o)
$3\mu R = \frac{1}{2} \times 2 \times 10^2 - \frac{1}{2} \times 2 \times 7^2 - 2g \times 3 \sin 30^\circ$	M1
$\mu = 0.42(4)$	A1    5
OR $R = 2g \cos 30^\circ$	B1
$a = \frac{(7^2 - 10^2)}{2 \times 3} = \frac{17}{2}; -F; -2g \sin 30^\circ$	B2 (-1 e.e.o.o)
$-\mu R - 2g \sin 30^\circ = -\frac{17}{2} \times 2$	M1
$\mu = 0.42(4)$	A1    (5)

**[9]**

1. This question proved to be straightforward for well-prepared candidates.

In part (a) it was pleasing to see many candidates tackling this using the work-energy method, and there was less evidence this year of candidates double counting by including both the change in GPE and the work done against the weight, but candidates sometimes confused work done with just potential energy lost, or just kinetic energy gained. The alternative method using *suvat* to find the acceleration and then using  $F = ma$  was also common. In the final answers there was considerable confusion between work done against friction and the frictional force. Many lost the final A mark by leaving the answer as 30.48 despite having used  $g = 9.8$ .

In part (b) candidates frequently did not make the connection with part (a) and proceeded to start again from scratch. In this case, a common but expensive error was to omit the component of the weight from their equation of motion.

2. Few candidates had problems finding the frictional force in part (a), but once again many candidates were insecure about finding work done. Many candidates found the net work done by the horizontal force and against friction, rather than simply the work done against friction.

As usual the most popular approach in part (b) was to find the acceleration of the block and then the velocity after 50 m using  $v^2 = u^2 + 2as$ . A significant proportion of candidates who attempted to use the work-energy principle missed one or more terms. However, many of those candidates who misinterpreted part (a) were able to use their net work done successfully to find  $v$  using this method.

3. This proved to be the most challenging question. Although it looked as though some candidates had run out of time to complete this question satisfactorily, others had sufficient time to make multiple attempts. Some candidates failed to appreciate the nature of velocity direction and tried to use equations for constant acceleration without taking direction into consideration. The most successful way to answer the first part was to use the method suggested – conservation of energy. In (b) some failed to appreciate that the velocity at C was at an angle (not necessarily equal to  $\theta$ ) and that the horizontal velocity at A was constant throughout. Few candidates answered this concisely. The greatest difficulties were encountered in (c). A significant number of candidates failed to appreciate the use of displacement in constant acceleration equations and broke up the problem, quite unnecessarily, into sections where the particle was travelling up and then travelling down, making the solution of the problem much more difficult than it needed to be. There are several possible approaches to this question, some of them producing pleasingly concise solutions.

4. The question was accessible to almost all candidates, most of whom scored some marks, especially in part (a). Methods varied with about half using constant acceleration equations and  $F = ma$ , a third using work/energy and the rest using different methods in each part (and sometimes in the same part). The main errors were mistakes in sign, particularly with the acceleration in part (a), mixing methods in the same part, and in part (b), including force  $\times$  distance twice – once as PE and once as work against gravity. A few tried treating the first part as a projectile problem. There were relatively few fully correct solutions to the second part.