Year 2 Pure

Chapter 9 - DIFFERENTIATION

SIN and COS

$\sin f(x) \rightarrow f'(x)\cos x$

$\cos f(x) \rightarrow -f'(x)\sin x$

Exponents and Logs

$e^{f(x)} \rightarrow f'(x)e^{f(x)}$

$a^{f(x)} \rightarrow f'(x)a^{f(x)}\ln a$

$\ln f(x) \rightarrow \dfrac{f'(x)}{f(x)}$

Chain Rule (function within a function)

$y = (3x^4+x)^5$

$3x^4+x$	u^5
$12x^3+1$ ×	$5u^4$

$$\frac{dy}{dx} = (12x^3+1)(5u^4)$$
$$= (12x^3+1)(5(3x^4+x)^4)$$
$$= 5(12x^3+1)(3x^4+x)^4$$

Product Rule (product of two functions)

$y = x^2 \sin x$

x^2	$\sin x$
$2x$	$\cos x$

crossmultiply and add

$$\frac{dy}{dx} = x^2\cos x + 2x\sin x$$

Quotient Rule (division of two functions)

$y = \dfrac{2x}{x-3}$

$$\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$2x$	$x-3$
2	1

$$\frac{dy}{dx} = \frac{2(x-3) - 2x(1)}{(x-3)^2}$$

$$\frac{dy}{dx} = \frac{-6}{(x-3)^2}$$

More Trigonometric Functions (in the formula booklet)

$\tan kx \rightarrow k\sec^2 kx$

$\sec kx \rightarrow k\sec kx \tan kx$

$\cot kx \rightarrow -k\operatorname{cosec}^2 kx$

$\operatorname{cosec} kx \rightarrow -k\operatorname{cosec} kx \cot kx$

Parametric Equations

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

reciprocal of $\frac{dx}{dt}$

when using trig function you may need to use identities:

$1 + \tan^2 t = \sec^2 t$

$\cos^2 t + \sin^2 t = 1$

$1 + \cot^2 t = \operatorname{cosec}^2 t$

$\sin 2t = 2\sin t\cos t$

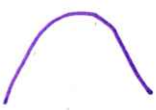
Implicts

Step 1. Differentiate with respect to y

Step 2. multiply by $\frac{dy}{dx}$

$$\frac{d}{dx} y^2 \rightarrow 2y\frac{dy}{dx}$$

Second Derivative

f(x) concave  when $f''(x) \leq 0$

f(x) convex  when $f''(x) \geqslant 0$

Point of Inflection  when $f''(x) = 0$

Rates of Change

$\frac{dA}{dx} \times \frac{dx}{dt}$

A = Area t = time $\frac{dA}{dt}$ Area is changing with time

r = radius t = time $\frac{dr}{dt}$ radius is changing with time

$\frac{dr}{dx} \times \frac{dx}{dt}$