IYGB - MMS PAPER R - QUESTION 1

a) USING A STATISTICAL CALCULATOR WE OBTAIN

$$r = 0.755$$

b) FROM THE CALCULATOR

$$y = a + bx \implies c = a + bm$$
$$\implies c = 2.8 + 1.1m$$

c) I) $C_{8.8} = 2.8 + 1.1 \times 8.8 \approx 12.48$ IE AROUND 12 CARS

AS THE P.M.C.C IS FAIRLY HIGH, AND 8.8 (£8,800) LIES WITHIN THE RANGE OF VALUES OF m WHICH WAS USED FOR THE REGRESSION LINE, THE ESTIMATE SHOULD BE RELIABLE (INTERPOLATION)

II) $C_{20} = 2.8 + 1.1 \times 20 \simeq 24.8$ IE AROUND 25 CARS

AS $m = 20$ IS "WAY ABOVE" THE LARGEST VALUE OF m, WHICH WAS USED TO PRODUCE THE REGRESSION LINE, THE ESTIMATE COULD NOT BE RELIABLE (EXTRAPOLATION)

d)
- $a = 2.8$ ("y INTERCEPT")

  THE NUMBER OF CARS EXPECTED TO BE SOLD IF NO MONEY WAS SPENT ON ADVERTISING

- $b = 1.1$ ("GRADIENT")

  THE NUMBER OF EXTRA CARS EXPECTED TO BE SOLD PER £1000 SPENT ON ADVERTISING

IYGB - MMS PAPER R - QUESTION 2

a)

"WEIGHTS" kg	FREQUENCY f	MIDPOINTS x	$y = \frac{x - 662.5}{25}$	fy	fy^2
$600 < m \leqslant 625$	11 $_{11}$	612.5	−2	−22	44
$625 < m \leqslant 650$	14 $_{25}$	637.5	−1	−14	14
$650 < m \leqslant 675$	28 $_{53}$	662.5	0	0	0
$675 \leqslant m \leqslant 700$	7	687.5	1	7	7
$700 \leqslant m \leqslant 725$	5	712.5	2	10	20
$725 < m \leqslant 750$	2	737.5	3	6	18
$750 < m \leqslant 775$	1	762.5	4	4	16

$\Sigma fy = -9$ $\quad$ $\Sigma fy^2 = 119$

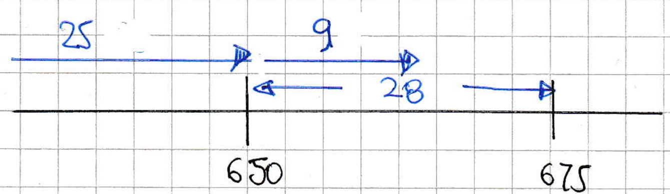
$$\bar{y} = \frac{\Sigma fy}{\Sigma f} = \frac{-9}{68}$$

$$\sigma_y = \sqrt{\frac{\Sigma fy^2}{\Sigma f} - \bar{y}^2} = \sqrt{\frac{119}{68} - \left(-\frac{9}{68}\right)^2} \simeq 1.316238\ldots$$

$$\bar{x} = (\bar{y} \times 25) + 662.5 \simeq 659.2$$

$$\sigma_x = 1.316238\ldots \times 25 \simeq 32.9$$

b) Q_2 IS $\frac{1}{2} \times 68 = 34^{\text{TH}}$ OBSERVATION WHICH LIES IN $650 < m \leqslant 675$



$$\therefore Q_2 = 650 + \frac{9}{28} \times 25 \simeq 658.0$$

IYGB - MMS PAPER R - QUESTION 3

a) X = NUMBER OF HVD VOTERS

$X \sim B(20, 0.4)$

$H_0 : p = 0.4$

$H_1 : p < 0.4$, WHERE p IS THE PROPORTION OF HVD VOTERS IN GENERAL

BECAUSE OF "ONLY 4", OTHERWISE $p \neq 0.4$

TESTING AT 5% SIGNIFICANCE, ON THE BASIS THAT $x = 4$

$P(X \leq 4) = 0.05095...$

$= 5.10\%$

$> 5\%$

THERE IS NO SIGNIFICANCE TO SUGGEST THAT $p < 0.4$, SO H.V.D CLAIM IS JUSTIFIED

THERE INSUFFICIENT EVIDENCE TO REJECT H_0

b) WE NOW REQUIRE FOR $X \sim B(n, 0.4)$ THAT $P(X \leq 0) < 0.01$

BY TRIAL & IMPROVEMENT OR LOGARITHMS

$\Rightarrow \binom{n}{0}(0.4)^0(0.6)^n < 0.01$

$\Rightarrow 1 \times 1 \times 0.6^n < 0.01$

$\Rightarrow 0.6^n < 0.01$

$\Rightarrow \log(0.6^n) < \log(0.01)$

$\Rightarrow n\log(0.6) < \log(0.01)$

DIVIDING BY A NEGATIVE QUANTITY

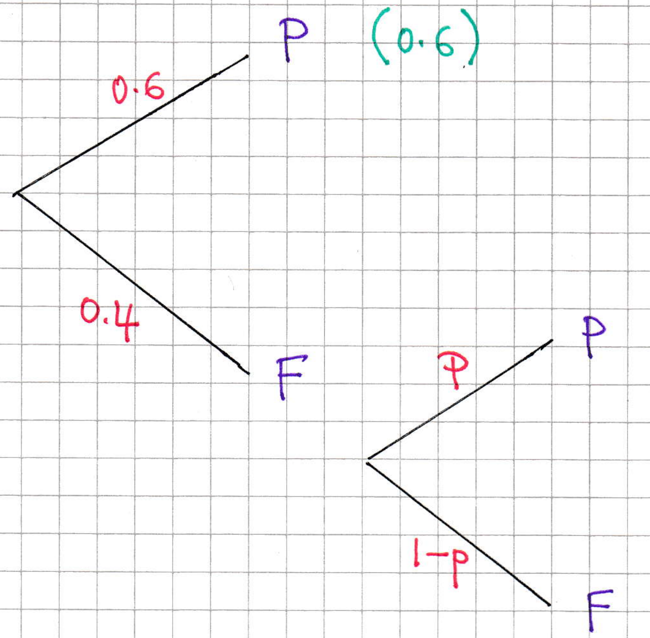
$\Rightarrow n > \dfrac{\log(0.01)}{\log(0.6)}$

$\Rightarrow n > 9.015...$

$\therefore n = 10$

IYGB – MMS PAPER R – QUESTION 4

DRAWING A DIAGRAM – LET "p" BE THE PROBABILITY OF PASSING 2^{ND} TIME



$\Longrightarrow$ P(passing) = 0.76

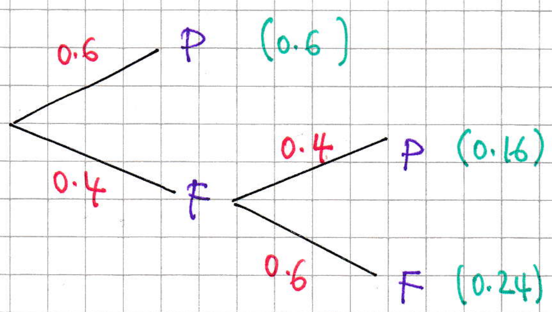
$\Longrightarrow$ P(passing 1st time) + P(passing 2nd time) = 0.76

$\Longrightarrow 0.6 + 0.4p = 0.76$

$\Longrightarrow 0.4p = 0.16$

$\Longrightarrow p = 0.4$

HENCE THE DIAGRAM CAN BE FULLY DRAWN



$$P(\text{FAILED 1ST ATTEMPT} \mid \text{PASSED TEST}) = \frac{P(\text{FAILED FIRST} \cap \text{PASSED TEST})}{P(\text{PASSED TEST})}$$

$$= \frac{0.16}{0.76} = \frac{16}{76} = \frac{4}{19}$$

IYGB - MMS PAPER R - QUESTION 5

a) • ONE ROCK HAVING (OR NOT HAVING) FOSSILS IS INDEPENDENT OF ANOTHER ROCK HAVING (OR NOT HAVING) FOSSILS

• CONSTANT RATE OF 10% OF A ROCK CONTAINING FOSSILS

X = NUMBER OF ROCKS CONTAINING FOSSILS

$X \sim B(20, 0.1)$

b) $P(X=1) = \binom{20}{1}(0.1)^1(0.9)^{19} = 0.2702$

c) $P(X \geqslant 1) = 1 - P(X=0) = 1 - \binom{20}{0}(0.1)^0(0.9)^{20}$

$= 1 - 0.12157... = 0.8784$

d) MODEL AS $Y \sim B(n, 0.1)$

$\Rightarrow P(W \geqslant 1) \geqslant 0.95$

$\Rightarrow 1 - P(W=0) \geqslant 0.95$

$\Rightarrow -P(W=0) \geqslant -0.05$

$\Rightarrow P(W=0) \leqslant 0.05$

$\Rightarrow \binom{n}{0}(0.1)^0(0.9)^n \leqslant 0.05$

$\Rightarrow 1 \times 1 \times 0.9^n \leqslant 0.05$

BY TRIAL & IMPROVEMENT OR LOGARITHMS

$\Rightarrow \log(0.9^n) \leqslant \log(0.05)$

$\Rightarrow n\log(0.9) \leqslant \log(0.05)$

$\Rightarrow n \geqslant \frac{\log(0.05)}{\log(0.9)}$ ← $\log(0.9) < 0$

$\Rightarrow n \geqslant 28.433...$

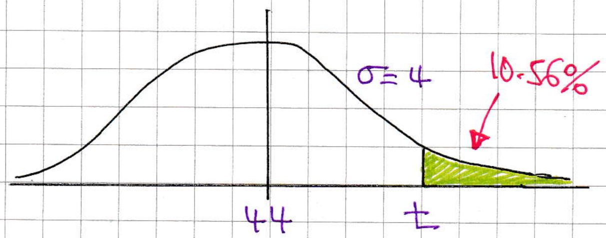
$\therefore n = 29$

IYGB - MMS PAPER R - QUESTION 6

X = TIME TO MANUFACTURE A SHIRT

$X \sim N(44, 4^2)$

a) LOOKING AT THE DIAGRAM OPPOSITE



$$\Rightarrow P(X > t) = 0.1056$$

$$\Rightarrow P(X < t) = 0.8944$$

$$\Rightarrow P\left(Z < \frac{t-44}{4}\right) = 0.8944$$

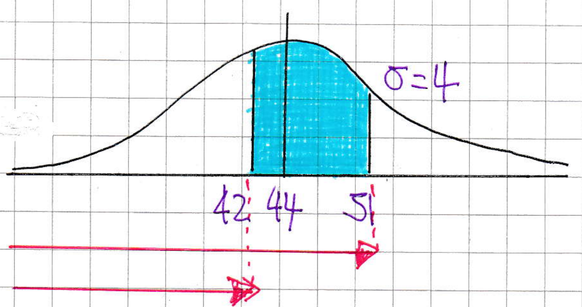
$\downarrow$ INVERSION

$$\Rightarrow \frac{t-44}{4} = +\Phi(0.8944)$$

$$\Rightarrow \frac{t-44}{4} = 1.25$$

$$\Rightarrow t = 49$$

b) LOOKING AT THE DIAGRAM BELOW



$$P(42 < X < 51)$$

$$= P(X < 51) - P(X < 42)$$

$$= P(X < 51) - [1 - P(X > 42)]$$

$$= P(X<51) + P(X>42) - 1$$

$$= P\left(Z < \frac{51-44}{4}\right) + P\left(Z > \frac{42-44}{4}\right) - 1$$

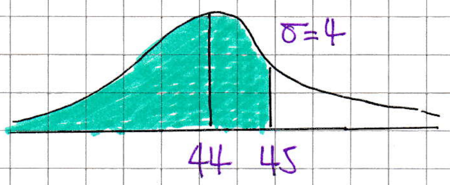
$$= \Phi(1.75) + \Phi(-0.5) - 1$$

$$= 0.6915 + 0.9599 - 1$$

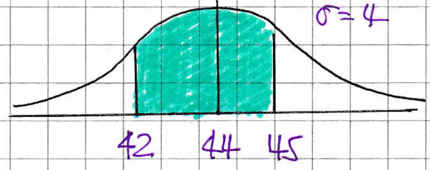
$$= 0.6514$$

IYGB – MMS PAPER R – QUESTION 6

c) **LOOKING AT TWO SEPARATE DIAGRAMS**



(DENOMINATOR)



(NUMERATOR)

$$P(X<45) = P\left(z < \frac{45-44}{4}\right)$$
$$= \Phi(0.25)$$
$$= 0.5987$$

$$P(42<X<45)$$
$$= P(X<45) - P(X<42)$$
$$= P(X<45) - [1 - P(X>42)]$$
$$= P(X>45) + P(X>42) - 1$$

BOTH CALCULATED ALREADY

$$= 0.5987 + 0.6915 - 1$$
$$= 0.2912$$

$$\underline{P(X>42 \mid X<45)} = \frac{P(42<X<45)}{P(X<45)} = \frac{0.2912}{0.5987} \simeq \underline{0.4847}$$

d) **SETTING HYPOTHESES**

$H_0: \mu = 44$

$H_1: \mu > 44$

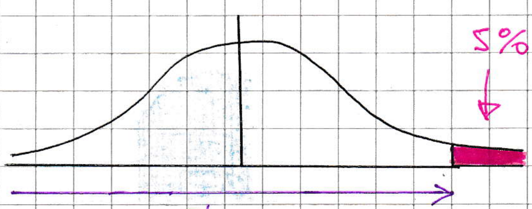
μ IS THE MEAN TIME OF ALL SHIRTS PRODUCED IN THIS FACTORY

$\bar{x} = 47$

$\sigma = 4$

$n = 4$

5% SIGNIFICANCE



$$\Phi^{-1}(0.95) = +1.6449$$

- $z\text{-STATISTIC} = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{47-44}{4/\sqrt{4}}$

$$= 1.5$$

- AS $1.5 < 1.6449$, THERE IS NO SIGNIFICANT EVIDENCE THAT μ IS GREATER THAN 44

<u>NOT SUFFICIENT EVIDENCE TO REJECT H_0</u>

IYGB – MMS PAPER R – QUESTION 7

$P(A) = x \quad \bullet \quad P(B) = y \quad \bullet \quad P(A \cup B) = 0.6 \quad \bullet \quad P(B|A) = 0.2$

a) PROCEED AS FOLLOWS.

- $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$

$$0.2 = \frac{P(B \cap A)}{x}$$

$$P(B \cap A) = 0.2x$$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$0.6 = x + y - 0.2x$$

$$0.6 = 0.8x + y$$

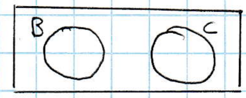
$\times 5$

$$3 = 4x + 5y$$

$$4x + 5y = 3$$

AS REQUIRED

b) MUTUALLY EXCLUSIVE $\Rightarrow$ $P(B) + P(C) = P(B \cup C)$



$$\Rightarrow y + (x + y) = 0.9$$

$$\Rightarrow x + 2y = 0.9$$

$$\Rightarrow x = 0.9 - 2y$$

SUB INTO THE EQUATION OF PART (a)

$$\Rightarrow 4(0.9 - 2y) + 5y = 3$$

$$\Rightarrow 3.6 - 8y + 5y = 3$$

$$\Rightarrow 0.6 = 3y$$

$$\Rightarrow y = 0.2$$

&

$$\Rightarrow x = 0.5$$

IYGB

c) USING $P(B|A) = 0.2$

$$P(B|A) = P(B) = y = 0.2$$

EVENTS ARE INDEPENDENT

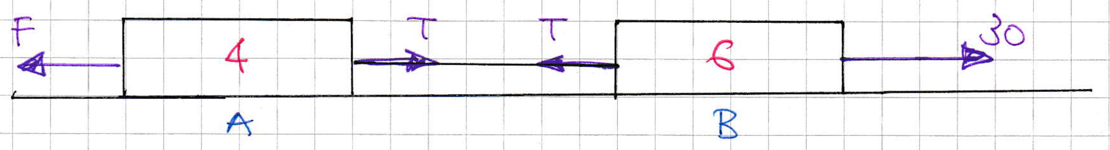
ALTERNATIVE

$$\left.\begin{array}{lll} P(A) = x & = & 0.5 \\ P(B) = y & = & 0.2 \\ P(A \cap B) = 0.2x & = & 0.1 \end{array}\right\} \quad \begin{aligned} P(A) \times P(B) &= 0.5 \times 0.2 \\ &= 0.1 \\ &= P(A \cap B) \end{aligned}$$

EVENTS ARE INDEPENDENT

IYGB - MMS PAPER R - QUESTION 8

DRAWING A DIAGRAM IGNORING "VERTICAL" FORCES AS THEY ARE IN EQUILIBRIUM



THERE ARE TWO CASES TO CONSIDER

- IF $a = 2\,\text{ms}^{-2}$ TO THE "RIGHT"

$(A): \; T - F = 4 \times 2$

$(B): \; 30 - T = 6 \times 2$

$T - F = 8$

$30 - T = 12$

$\therefore \; T = 18\,\text{N}$

$F = 10\,\text{N}$

- IF $a = 2\,\text{ms}^{-2}$ TO THE "LEFT"

$(A): \; F - T = 4 \times 2$

$(B): \; T - 30 = 6 \times 2$

$F - T = 8$

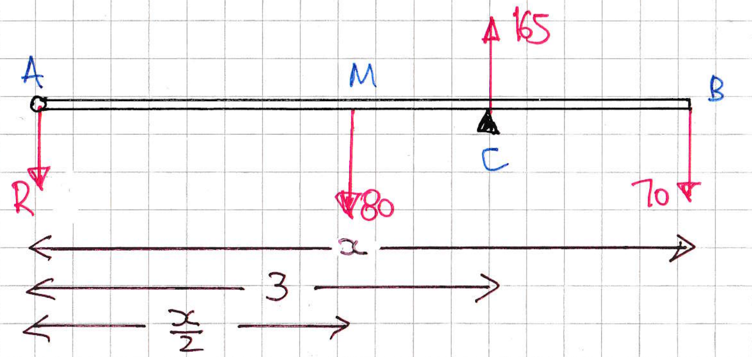
$T - 30 = 12$

$\therefore \; T = 42\,\text{N}$

$F = 50\,\text{N}$

IYGB - MMS PAPER R - QUESTION 9

a) LOOKING AT THE DIAGRAM



$\overset{\curvearrowright}{A}$: $80 \times \frac{x}{2} + 70x = 165 \times 3$

$40x + 70x = 495$

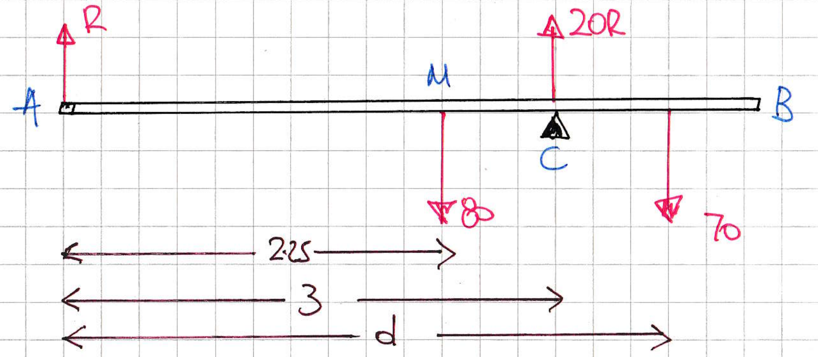
$110x = 495$

$x = 4.5\text{ m}$

b) I) "UNIFORM ROD" ... A RIGID ONE DIMENSIONAL OBJECT, WHOSE MASS ACTS AT ITS CENTRE

"PARTICLE" ... ROCK CAN BE PLACED EXACTLY AT THE POINT B

c) LOOKING AT A NEW DIAGRAM



IF THE REACTION AT A IS "UPWARDS"

- $20R + R = 80 + 70$

  $21R = 150$

  $R = \frac{50}{7}$

- $\overset{\curvearrowright}{A}$ $80 \times 2.25 + 70 \times d = 20R \times 3$

  $180 + 70d = 60R$

  $180 + 70d = 60 \times \frac{50}{7}$

  $70d = \frac{1740}{7}$

  $d \simeq 3.55\text{ m}$

  $\frac{174}{49}$

IYGB - MMS PAPER R - QUESTION 9

NOTE THAT IF THE REACTION AT A IS POINTING DOWNWARDS

- $R + 70 + 80 = 20R$

$$150 = 19R$$

$$R = \frac{150}{19}$$

- A: $80 \times 2.25 + 70d = 20R \times 3$

$$180 + 70d = 60R$$

$$18 + 7d = 6R$$

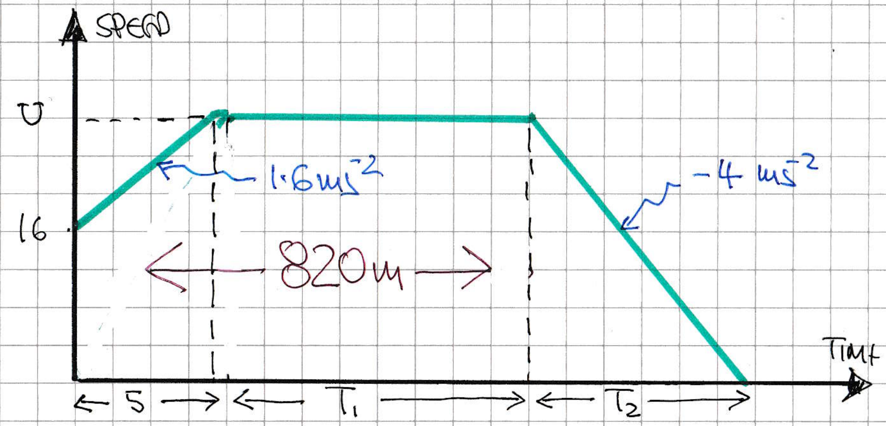
$$18 + 7d = 6 \times \frac{150}{19}$$

$$7d = \frac{558}{19}$$

$$d = 4.20 \quad \frac{558}{133}$$

IYGB - MMS PAPER R - QUESTION 10

RECORDING THE INFORMATION GIVEN



LOOKING AT THE "FIRST" SECTION

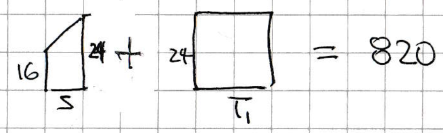
$\Rightarrow$ ACCELERATION = GRADIENT = $\frac{\Delta v}{\Delta t}$

$\Rightarrow 1.6 = \frac{v-16}{5}$

$\Rightarrow 8 = v - 16$

$\Rightarrow v = 24$

LOOKING AT THE FIRST 820m



$\frac{16+24}{2} \times 5 + 24T_1 = 820$

$100 + 24T_1 = 820$

$24T_1 = 720$

$T_1 = 30$

LOOKING AT THE LAST SECTION

DECELERATION = GRADIENT = $\frac{\Delta v}{\Delta t}$

$-4 = \frac{-24}{T_2}$

$T_2 = 6$

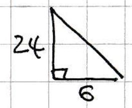
$\therefore$ TOTAL TIME $= 5 + T_1 + T_2$

$= 5 + 30 + 6$

$= 41\ s$

IYGB - MMS PAPER R - QUESTION 10

FINALLY THE DISTANCE COVERED IN THE LAST SECTION



$= \frac{1}{2} \times 6 \times 24 = 72\text{ m}$

$\therefore$ TOTAL DISTANCE IS $820 + 72 = 892\text{ m}$

IYGB - MMS PAPER R - QUESTION 11

- START BY OBTAINING AN EXPRESSION FOR THE VELOCITY

$$\Rightarrow a = 8 - 2t$$

$$\Rightarrow \frac{dv}{dt} = 8 - 2t$$

$$\Rightarrow 1\,dv = (8-2t)\,dt$$

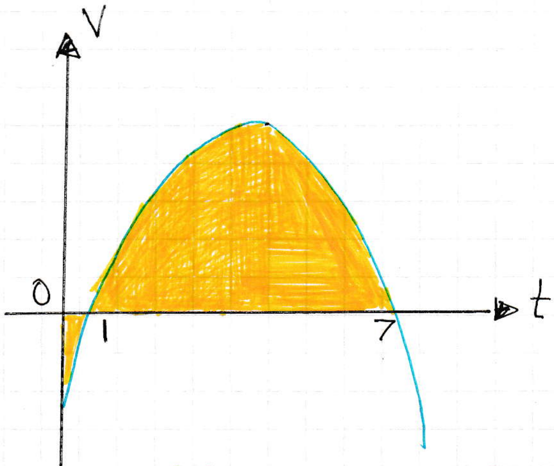
$$\Rightarrow \int_{v=-7}^{v} 1\,dv = \int_{t=0}^{t} 8-2t\,dt$$

$$\Rightarrow \Big[v\Big]_{-7}^{v} = \Big[8t - t^2\Big]_0^t$$

$$\Rightarrow v+7 = (8t-t^2) - 0$$

$$\Rightarrow v = -7 + 8t - t^2$$

- SKETCHING THE CURVE $v = f(t)$



$$-7+8t-t^2=0$$
$$t^2-8t+7=0$$
$$(t-7)(t-1)=0$$

$$\text{DISTANCE} = \left|\int_0^1 -7+8t-t^2\,dt\right| + \int_1^7 -7+8t-t^2\,dt$$

$$= \int_1^0 -7+8t-t^2\,dt + \int_1^7 -7+8t-t^2\,dt$$

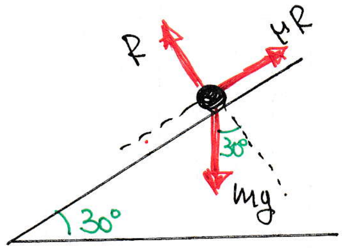
$$= \Big[-7t+4t^2-\tfrac{1}{3}t^3\Big]_1^0 + \Big[-7t+4t^2-\tfrac{1}{3}t^3\Big]_1^7$$

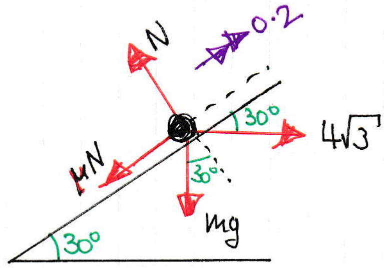
$$= 0-(-7+4-\tfrac{1}{3}) + (-49+196-\tfrac{343}{3}) - (-7+4-\tfrac{1}{3})$$

$$= \frac{118}{3}$$

$$= 39\tfrac{1}{3}\text{ m}$$

IYGB – MMS PAPER R – QUESTION 12





- START WITH PARTICLE IN LIMITING EQUILIBRIUM

$(\perp)$: $R = mg\cos 30°$

$(\parallel)$: $\mu R = mg\sin 30°$

- DIVIDING THE EQUATIONS

$$\Rightarrow \frac{\mu \cancel{R}}{\cancel{R}} = \frac{\cancel{mg}\sin 30}{\cancel{mg}\cos 30}$$

$$\Rightarrow \mu = \tan 30$$

$$\Rightarrow \mu = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

- NEXT THE PARTICLE IS ACCELERATING UP (NOTE THE NORMAL REACTION IS DIFFERENT)

$(\perp)$: $N = 4\sqrt{3}\sin 30° + mg\cos 30°$

$(\parallel)$: $\underbrace{4\sqrt{3}\cos 30 - \mu N - mg\sin 30 = m \times 0.2}_{\text{"}F = ma\text{"}}$

- SIMPLIFYING THE ABOVE EQUATIONS

$$\left.\begin{aligned} N &= 2\sqrt{3} + mg\frac{\sqrt{3}}{2} \\ 6 - \frac{\sqrt{3}}{3}N - \frac{1}{2}mg &= \frac{1}{5}m \end{aligned}\right\} \Rightarrow$$

- SUB THE FIRST INTO THE SECOND

$$\Rightarrow 6 - \frac{\sqrt{3}}{3}\left[2\sqrt{3} + mg\frac{\sqrt{3}}{2}\right] - \frac{1}{2}mg = \frac{1}{5}m$$

$$\Rightarrow 6 - 2 - \frac{1}{2}mg - \frac{1}{2}mg = \frac{1}{5}m$$

$$\Rightarrow 4 - mg = \frac{1}{5}m$$

$$\Rightarrow 20 - 5mg = m$$

$$\Rightarrow 20 = m + 5mg$$

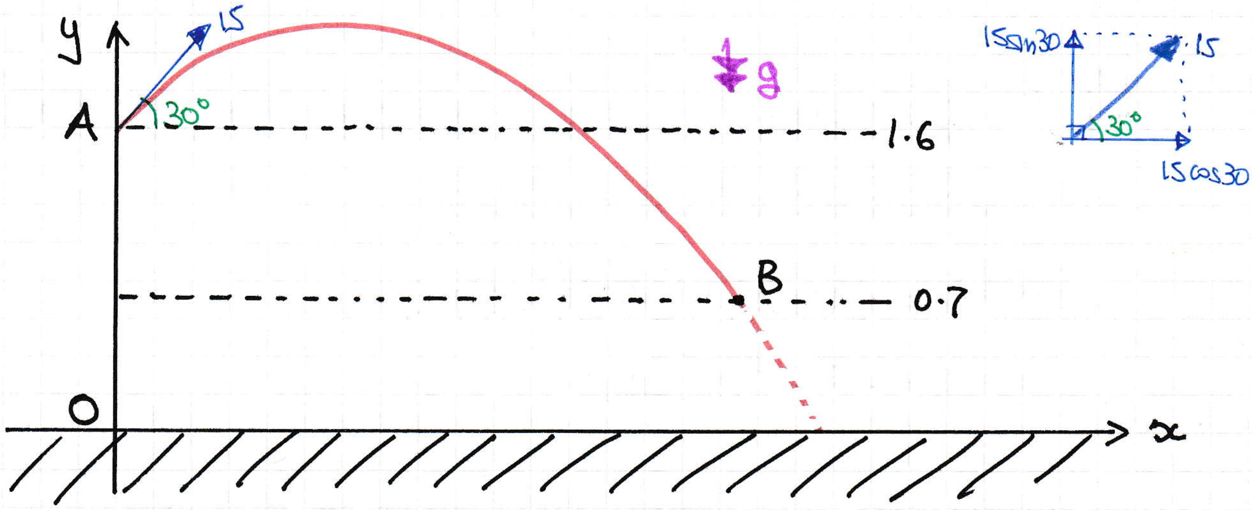
$$\Rightarrow 20 = m(1 + 5g)$$

$$\Rightarrow 20 = 50m$$

$$\Rightarrow m = \frac{2}{5} = 0.4 \text{ kg}$$

IYGB - MMS PAPER R - QUESTION 13

a)



- LOOKING AT THE JOURNEY FROM A TO B, IN THE VERTICAL

$\Rightarrow s = s_0 + ut + \frac{1}{2}at^2$

$\Rightarrow 0.7 = 1.6 + (15\sin 30°)t + \frac{1}{2}(-9.8)t^2$

$\Rightarrow 0.7 = 1.6 + 7.5t - 4.9t^2$

$\Rightarrow 4.9t^2 - 7.5t - 0.9 = 0$

$\Rightarrow 49t^2 - 75t - 9 = 0$

- BY THE QUADRATIC FORMULA WE OBTAIN

$\Rightarrow t = \frac{75 \pm \sqrt{7389}}{2 \times 49} = \begin{cases} -\cancel{0.11\ldots} \\ 1.64244\ldots \end{cases}$ ← FLIGHT TIME

- LOOKING AT THE HORIZONTAL MOTION WE HAVE

$\Rightarrow$ "DISTANCE = "SPEED" × "TIME"

$\Rightarrow x = (15\cos 30°) \times 1.64244\ldots$

$\Rightarrow x = 21.3359\ldots \simeq 21.34 \text{ m}$

(2 d.p)

IYGB - MMS PAPER R - QUESTION 13

b) • NOW USING "$v = u + at$" WE CAN FIND THE VERTICAL VELOCITY OF THE BALL AT B

$$\Rightarrow V = (15\sin 30) + (-9.8)(1.6424\ldots)$$

$$\Rightarrow V = -8.5959\ldots \text{ ms}^{-1}$$

• THE MAXIMUM SPEED WILL OCCUR AT B, IE AT THE LOWEST POINT OF THE PATH

AS THE HORIZONTAL SPEED IS CONSTANT, WE HAVE

$$\Rightarrow |V_{MAX}| = \sqrt{(-8.5959\ldots)^2 + (15\cos 30)^2}$$

$$\Rightarrow |V_{MAX}| = 15.58 \text{ ms}^{-1}$$

(2 d.p.)

ALTERNATIVE BY ENERGIES (NOT ALLOWED HERE)

• TAKING THE GROUND AS THE ZERO POTENTIAL LEVEL

$$\Rightarrow KE_A + P.E_A = KE_B + PE_B$$

$$\Rightarrow \tfrac{1}{2}\cancel{m}u^2 + \cancel{m}gH = \tfrac{1}{2}\cancel{m}v^2 + \cancel{m}gh$$

$$\Rightarrow u^2 + 2gH = v^2 + 2gh$$

$$\Rightarrow 15^2 + 2g(1.6) = v^2 + 2g(0.7)$$

$$\Rightarrow v^2 = 242.64$$

$$\Rightarrow |v| = 15.58 \text{ ms}^{-1}$$

(2 d.p.)

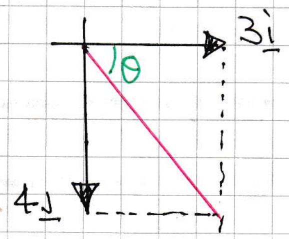
IYGB - MMS PAPER R - QUESTION 14

a) USING $\underline{r} = \underline{r}_0 + \underline{v}t$

$$4\underline{i} - 2\underline{j} = (-5\underline{i} + 10\underline{j}) + \underline{v} \times 3$$
$$9\underline{i} - 12\underline{j} = 3\underline{v}$$
$$\underline{v} = 3\underline{i} - 4\underline{j}$$

$\therefore$ SPEED $= |\underline{v}| = |3\underline{i} - 4\underline{j}| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = 5\ \text{ms}^{-1}$

b) USING THE VELOCITY VECTOR



$$\tan\theta = \frac{4}{3}$$
$$\theta = 53.13°$$

$\therefore$ BEARING IS

$90 + 53.13$

$= 143°$

c) USING AGAIN $\underline{r} = \underline{r}_0 + \underline{v}t$

$$\underline{r} = (-5\underline{i} + 10\underline{j}) + (3\underline{i} - 4\underline{j})t$$
$$\underline{r} = (3t-5)\underline{i} + (10-4t)\underline{j}$$

d) LET $B(10,-10)$ & $R(3t-5,\ 10-4t)$

$$\Rightarrow |BR| = \sqrt{[(3t-5)-10]^2 + [(10-4t)-(-10)]^2}$$
$$\Rightarrow 15 = \sqrt{(3t-15)^2 + (20-4t)^2}$$
$$\Rightarrow 225 = (3t-15)^2 + (20-4t)^2$$

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$$\Rightarrow \cancel{225} = 9t^2 - 90t + \cancel{225} + 400 - 160t + 16t^2$$

$$\Rightarrow 0 = 25t^2 - 250t + 400$$

$$\Rightarrow t^2 - 10t + 16 = 0$$

$$\Rightarrow (t-2)(t-8) = 0$$

$$\Rightarrow t = \begin{cases} 2 \\ 8 \end{cases}$$