1 If $\frac{3x^4+x^3-8x^2+3x+1}{x+2} \equiv Ax^3+Bx^2+Cx+D+\frac{E}{x+2}$ find the constants A, B, C, D and E **[6 marks]**

2 A function is defined by $f(x)=x^2-2$, $x \in \mathbb{R}$, $x \geq 0$

a State the range of $f(x)$ **[1]**

b Write an expression for the inverse function $f^{-1}(x)$, stating its domain. **[3]**

c Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same set of axes. **[4]**

d Find the value of x for which $f(x)=f^{-1}(x)$ **[3]**

3 Functions $f(x)$ and $g(x)$ are defined by

$$f(x)=\frac{x}{x-3},\ x \in \mathbb{R},\ x \neq 3, \text{ and } g(x)=\frac{5x-2}{x},\ x \in \mathbb{R},\ x \neq 0$$

a Work out an expression for the inverse function $f^{-1}(x)$ **[2]**

b Work out an expression for the composite function $gf(x)$ **[3]**

c Solve the equation $f^{-1}(x)=gf(x)$. Show your working. **[3]**

4 a Sketch the graph of $y=|2x-15|$ **[2]**

b Solve the equation $|2x-15|=3$ **[2]**

c Solve the inequality $|2x-15| \leq 3$ **[2]**

5 Determine which of the following functions are one-to-one, and which are many-to-one. Justify your answers.

A $y=3x+2$, $x \in \mathbb{R}$

B $y=x^2-5$, $x \in \mathbb{R}$

C $y=\frac{1}{x-3}$, $x \in \mathbb{R}$, $x \neq 3$

D $y=\sin x$, $x \in \mathbb{R}$ **[8]**

6 Use proof by contradiction to prove that, if n is an integer, and n^n is odd, then n is odd. **[5]**

7 The function $f(x)$ is defined by $f(x)=\frac{3x-7}{x^2-3x-4}-\frac{1}{x-4}$

a Show that $f(x)=\frac{2}{x+1}$ **[4]**

b What is the largest possible domain of $f(x)$? **[1]**

c Work out an expression for the inverse function $f^{-1}(x)$, stating its domain. **[2]**

d Solve the equation $f(x)=f^{-1}(x)$. Show your working. **[3]**

8 Decide which of the following statements are true and which are false. For those that are true, prove that they are true. For those that are false, give a counter example in each case.

A For $x \neq -1$, $\frac{4x}{(x+1)^2} \leq 1$

B $n!+1$ is prime for all positive integers, n

C The product of three consecutive odd integers is always a multiple of 15

D n^3-n is divisible by 6 for all positive integers, n **[13]**

9 Solve the equation $\frac{3}{x-2}-\frac{4}{x+1}=2$. Show your working. **[3]**

10 Write the Cartesian equation of the curve that is given parametrically by $x = \frac{1}{2t+1}$; $y = \frac{2}{3-t}$, $t > 3$. **[7]**

11 $f(x) = |3x|$, $x \in \mathbb{R}$, $g(x) = 2x - 1$, $x \in \mathbb{R}$

a Sketch the graph of $y = f(x)$ **[2]** **b** Sketch the graph of $y = gf(x)$ **[2]**

c Describe the transformation from $f(x)$ to $gf(x)$ **[2]**

12 If $\frac{6x^4 + 5x^3 - 4x^2 - 3x + 1}{2x+3} \equiv Ax^3 + Bx^2 + Cx + D + \frac{E}{2x+3}$, find the constants A, B, C, D and E **[6]**

13 Functions $f(x)$ and $g(x)$ are defined by

$$f(x) = e^{2x},\ x \in \mathbb{R}, \quad \text{and} \quad g(x) = \ln(3x-2),\ x \in \mathbb{R},\ x > \frac{2}{3}$$

a Write an expression for $fg(x)$ **[3]**

b Solve the equation $fg(x) = x^2$. Show your working. **[4]**

c Work out an expression for $f^{-1}(x)$ **[2]**

d Solve the equation $f(x) = 5$. Show your working. **[2]**

14 a Work out the values of the constants A and B for which $\frac{3x-5}{(x-2)(x-1)} \equiv \frac{A}{x-2} + \frac{B}{x-1}$ **[4]**

b Hence show that $\frac{3x-5}{(x-2)(x-1)}$ is a decreasing function for $x > 2$ **[2]**

15 a Prove that if a is an integer, and a^2 is a multiple of three, then a is also a multiple of three. **[4]**

b Use the method of proof by contradiction to prove that $\sqrt{3}$ is irrational. **[7]**

16 A curve, C, is given parametrically by $x = \sqrt{\sin t}$, $y = 3\sin t \cos t$, $0° \le t \le 90°$

a Show that a Cartesian equation for C is $y = 3x^2\sqrt{1-x^4}$ **[4]**

b Explain why there is no point on the curve for which $y = 2$ **[5]**

17 a Express each of these in partial fractions.

i $\frac{4x+1}{(x+1)(x-2)}$ **ii** $\frac{15-9x}{(x-1)(x-2)}$ **[4]**

b Hence solve the equation $\frac{4x+1}{(x+1)(x-2)} + \frac{15-9x}{(x-1)(x-2))} = 1$ **[3]**