Bronze.

A = $\begin{bmatrix} 1 & 2 \\ 1 & k \end{bmatrix}$ **B** = $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(a) Find the value of k for which matrix **A** is singular.

(1)

(b) Describe the transformation represented by matrix **B**.

(1)

(c) (i) Given that **A** and **B** are both non-singular, verify that $\mathbf{A}^{-1}\mathbf{B}^{-1} = (\mathbf{BA})^{-1}$.

(4)

(ii) Prove the result $\mathbf{M}^{-1}\mathbf{N}^{-1} = (\mathbf{NM})^{-1}$ for all non-singular matrices **M** and **N**.

(4)

(Total 10 marks)

SILVER.

The 3 × 3 matrices **A** and **B** satisfy

$$\mathbf{AB} = \begin{bmatrix} k & 8 & 1 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix}, \text{ where } \mathbf{A} = \begin{bmatrix} k & 6 & 8 \\ 0 & 1 & 2 \\ -3 & 4 & 8 \end{bmatrix}$$

and k is a constant.

(a) Show that **AB** is non-singular.

(1)

(b) Find $(\mathbf{AB})^{-1}$ in terms of k.

(5)

(c) Find $\mathbf{B}^{-1}$.

(4)

(Total 10 marks)

GOLD.

The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(a) Given that $\mathbf{A}^2 = \begin{bmatrix} p & -2 & -4 \\ 5 & 6 & 4 \\ 10 & q & 9 \end{bmatrix}$, find the value of p and the value of q.

(2)

(b) Given that $\mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I} = \mathbf{0}$, prove that

$$\mathbf{A}^{-1} = \frac{1}{6}(\mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I})$$

(2)

(c) Given that $\mathbf{A}^{-1} = \frac{1}{6}\begin{bmatrix} r & -2 & 2 \\ -1 & 5 & -2 \\ -2 & s & 2 \end{bmatrix}$, find the value of r and the value of s.

(2)

(d) Hence, or otherwise, find the solution of the system of equations

$$\begin{aligned} x \quad\quad - z &= k \\ x + 2y + z &= 5 \\ 2x + 2y + 3z &= 7 \end{aligned}$$

giving your answers in terms of k.

(3)

(Total 9 marks