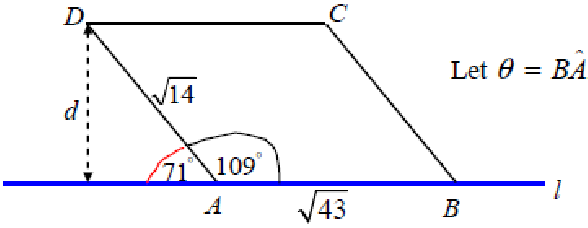
C4 Vectors Questions ANSWERS (77 marks)

Jan 2012

7.	$\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}\,,\ \overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}\,,\ \left\{\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}\right\}\ \ \&\ \ \overrightarrow{OD} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$		
(a)	$\overrightarrow{AB} = = \pm((5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k}));= 3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$		M1; A1 **[2]**
(b)	$l: \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$	See notes	M1 A1ft **[2]**
	 Let $\theta = B\hat{A}D$	Let d be the shortest distance from C to l.	
(c)	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$		M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{AD}\rvert} = \dfrac{\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}$	Applies dot product formula between their $\left(\overrightarrow{AB} \text{ or } \overrightarrow{BA}\right)$ and their $\left(\overrightarrow{AD} \text{ or } \overrightarrow{DA}\right)$.	M1
	$\cos\theta = \pm\left(\dfrac{-9+6-5}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}\right)$	Correct followed through expression or **equation**.	A1 $\sqrt{\ }$
	$\cos\theta = \dfrac{-8}{\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544... = 109 \text{ (nearest}^\circ)$	awrt 109	A1 cso AG **[4]**
(d)	$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{AB} = (-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$ $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{AD} = (5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) + (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$		M1
	So, $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$		A1 **[2]**
(e)	Area $ABCD = \left(\tfrac{1}{2}(\sqrt{43})(\sqrt{14})\sin 109^\circ\right); \times 2 = 23.19894905$	awrt 23.2	M1; dM1 A1 **[3]**
(f)	$\dfrac{d}{\sqrt{14}} = \sin 71$ or $\sqrt{43}\,d = 23.19894905...$		M1
	$\therefore\ d = \sqrt{14}\sin 71^\circ = 3.537806563...$	awrt 3.54	A1

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7. (a)

M1: Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$.

Can be implied by two out of three components correct in $3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ or $-3\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$

A1: $3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$

(b)

M1: An expression of the form $(3 \text{ component vector}) \pm \lambda(3 \text{ component vector})$

A1ft: $\mathbf{r} = \overrightarrow{OA} + \lambda(\text{their } \pm\overrightarrow{AB})$ or $\mathbf{r} = \overrightarrow{OB} + \lambda(\text{their } \pm\overrightarrow{AB})$.

Note: Candidate must begin writing their line as $\mathbf{r} =$ or $l = \ldots$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \ldots$ So, Line = ... would be A0.

(c)

M1: An attempt to find either the vector $\overrightarrow{AD}$ or $\overrightarrow{DA}$.

Can be implied by two out of three components correct in $-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ or $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, respectively.

M1: Applies dot product formula between their $(\overrightarrow{AB} \text{ or } \overrightarrow{BA})$ and their $(\overrightarrow{AD} \text{ or } \overrightarrow{DA})$.

A1ft: Correct followed through expression or **equation**. The dot product must be correctly followed through correctly and the square roots although they can be un-simplified must be followed through correctly.

A1: Obtains an angle of awrt 109 *by correct solution only*.

Award the final A1 mark if candidate achieves awrt 109 by either taking the dot product between:

(i) $\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or (ii) $\begin{pmatrix} -3 \\ -3 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly.

Award A0, cso for those candidates who take the dot product between:

(iii) $\begin{pmatrix} -3 \\ -3 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or (iv) $\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$.

They will usually find awrt 71 and apply 180 – awrt 71 to give awrt 109. If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso. If still in doubt, here, send to review.

(d)

M1: Applies either $\overrightarrow{OD}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OB}$ + their $\overrightarrow{AD}$.

This mark can be implied by two out of three correctly followed through components in their $\overrightarrow{OD}$.

A1: For $2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$.

(e)

M1: $\frac{1}{2}(\text{their } AB)(\text{their } CB)\sin(\text{their } 109^\circ \text{ or } 71^\circ \text{ from (b)})$. Awrt 11.6 will usually imply this mark.

dM1: Multiplies this by 2 for the parallelogram. Can be implied.

Note: $\frac{1}{2}((\text{their } AB + \text{their } AB))(\text{their } CB)\sin(\text{their } 109^\circ \text{ or } 71^\circ \text{ from (b)})$

A1: awrt 23.2

(f)

M1: $\dfrac{d}{\text{their } AD} = \sin(\text{their } 109^\circ \text{ or } 71^\circ \text{ from (b)})$ or $(\text{their } AB)\, d = (\text{their Area } ABCD)$

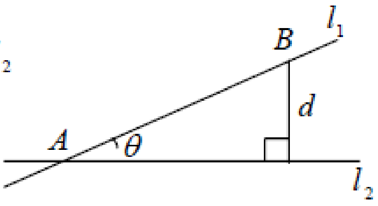
Award M0 for $(\text{their } AB)$ in part (f), if the area of their parallelogram in part (e) is $(\text{their } AB)(\text{their } CB)$.

Award M0 for $\dfrac{d}{\text{their } \sqrt{43}} = \sin 71$ or $(\text{their } \sqrt{14})d = 23.19894905\ldots$

A1: awrt 3.54

Note: Some candidates will use their answer to part (f) in order to answer part (e).

June 2011

6.	(a) **i**: $6-\lambda=-5+2\mu$		
	j: $-3+2\lambda=15-3\mu$ Any two equations	M1	
	leading to $\lambda=3,\quad \mu=4$	M1 A1	
	$\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+3\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}-5\\15\\3\end{pmatrix}+4\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$	M1 A1	
	k: LHS $=-2+3(3)=7$, RHS $=3+4(1)=7$	B1	**(6)**
	(As LHS = RHS, lines intersect)		
	Alternatively for B1, showing that $\lambda=3$ and $\mu=4$ both give $\begin{pmatrix}3\\3\\7\end{pmatrix}$		
	(b) $\begin{pmatrix}-1\\2\\3\end{pmatrix}.\begin{pmatrix}2\\-3\\1\end{pmatrix}=-2-6+3=\sqrt{14}\sqrt{14}\cos\theta$ $(\theta\approx110.92°)$	M1 A1	
	Acute angle is $69.1°$ awrt 69.1	A1	**(3)**
	(c) $\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+1\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}5\\-1\\1\end{pmatrix}$ $(\Rightarrow B$ lies on $l_1)$	B1	**(1)**
	(d) Let d be shortest distance from B to l_2 		
	$\overrightarrow{AB}=\begin{pmatrix}5\\-1\\1\end{pmatrix}-\begin{pmatrix}3\\3\\7\end{pmatrix}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}$	M1	
	$\lvert\overrightarrow{AB}\rvert=\sqrt{\left(2^2+(-4)^2+(-6)^2\right)}=\sqrt{56}$ awrt 7.5	A1	
	$\frac{d}{\sqrt{56}}=\sin\theta$	M1	
	$d=\sqrt{56}\sin69.1°\approx6.99$ awrt 6.99	A1	**(4)**
			[14]

Jan 2011

Question Number	Scheme	Marks
4. **(a)**	$\overrightarrow{AB} = -2\mathbf{i}+2\mathbf{j}-\mathbf{k}-(\mathbf{i}-3\mathbf{j}+2\mathbf{k}) = -3\mathbf{i}+5\mathbf{j}-3\mathbf{k}$	M1 A1 (2)
(b)	$\mathbf{r} = \mathbf{i}-3\mathbf{j}+2\mathbf{k}+\lambda(-3\mathbf{i}+5\mathbf{j}-3\mathbf{k})$ or $\mathbf{r} = -2\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(-3\mathbf{i}+5\mathbf{j}-3\mathbf{k})$	M1 A1ft (2)
(c)	$\overrightarrow{AC} = 2\mathbf{i}+p\mathbf{j}-4\mathbf{k}-(\mathbf{i}-3\mathbf{j}+2\mathbf{k})$ $= \mathbf{i}+(p+3)\mathbf{j}-6\mathbf{k}$ or $\overrightarrow{CA}$ $\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 1 \\ p+3 \\ -6 \end{pmatrix}.\begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} = 0$ $-3+5p+15+18=0$ Leading to $p=-6$	 B1 M1 M1 A1 (4)
(d)	$AC^2 = (2-1)^2+(-6+3)^2+(-4-2)^2 \quad (=46)$ $AC = \sqrt{46}$ accept awrt 6.8	M1 A1 (2) **[10]**

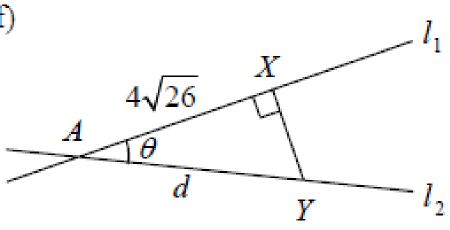
June 2010

7.	(a) **j** components $3+2\lambda=9 \quad \Rightarrow \lambda=3$ $\quad (\mu=1)$	M1 A1	
	Leading to $C:(5,9,-1)$ accept vector forms	A1	**(3)**
	(b) Choosing correct directions or finding $\overrightarrow{AC}$ and $\overrightarrow{BC}$	M1	
	$\begin{pmatrix}1\\2\\1\end{pmatrix}.\begin{pmatrix}5\\0\\2\end{pmatrix}=5+2=\sqrt{6}\sqrt{29}\cos\angle ACB$ use of scalar product	M1 A1	
	$\angle ACB=57.95°$ awrt 57.95°	A1	**(4)**
	(c) $A:(2,3,-4) \quad B:(-5,9,-5)$		
	$\overrightarrow{AC}=\begin{pmatrix}3\\6\\3\end{pmatrix}, \quad \overrightarrow{BC}=\begin{pmatrix}10\\0\\4\end{pmatrix}$		
	$AC^2=3^2+6^2+3^2 \quad \Rightarrow \quad AC=3\sqrt{6}$	M1 A1	
	$BC^2=10^2+4^2 \quad \Rightarrow \quad BC=2\sqrt{29}$	A1	
	$\triangle ABC=\frac{1}{2}AC\times BC\sin\angle ACB$		
	$=\frac{1}{2}3\sqrt{6}\times2\sqrt{29}\sin\angle ACB\approx33.5$ $\quad 15\sqrt{5}$, awrt 34	M1 A1	**(5)**
			[12]

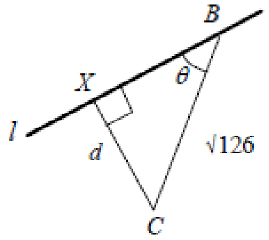
Alternative method for (b) *and* (c)

(b) $A:(2,3,-4) \quad B:(-5,9,-5) \quad C:(5,9,-1)$		
$AB^2=7^2+6^2+1^2=86$		
$AC^2=3^2+6^2+3^2=54$		
$BC^2=10^2+0^2+4^2=116$ Finding all three sides	M1	
$\cos\angle ACB=\frac{116+54-86}{2\sqrt{116}\sqrt{54}} \quad (=0.53066\ \ldots)$	M1 A1	
$\angle ACB=57.95°$ awrt 57.95°	A1	**(4)**
If this method is used some of the working may gain credit in part (c) and appropriate marks may be awarded if there is an attempt at part (c).		

Jan 2010

Q4	(a) A: $(-6, 4, -1)$	Accept vector forms	B1	(1)
	(b) $\begin{pmatrix}4\\-1\\3\end{pmatrix}.\begin{pmatrix}3\\-4\\1\end{pmatrix}=12+4+3=\sqrt{4^2+(-1)^2+3^2}\sqrt{3^2+(-4)^2+1^2}\cos\theta$		M1 A1	
	$\cos\theta=\frac{19}{26}$	awrt 0.73	A1	(3)
	(c) X: $(10, 0, 11)$	Accept vector forms	B1	(1)
	(d) $\overrightarrow{AX}=\begin{pmatrix}10\\0\\11\end{pmatrix}-\begin{pmatrix}-6\\4\\-1\end{pmatrix}$	Either order	M1	
	$=\begin{pmatrix}16\\-4\\12\end{pmatrix}$	cao	A1	(2)
	(e) $\lvert\overrightarrow{AX}\rvert=\sqrt{16^2+(-4)^2+12^2}$		M1	
	$=\sqrt{416}=\sqrt{16\times26}=4\sqrt{26}$ ✱	Do not penalise if consistent incorrect signs in (d)	A1	(2)
	(f) 	Use of correct right angled triangle	M1	
		$\frac{\lvert\overrightarrow{AX}\rvert}{d}=\cos\theta$	M1	
		$d=\frac{4\sqrt{26}}{\frac{19}{26}}\approx 27.9$ awrt 27.9	A1	(3)
				[12]

June 2009

Q7	(a)	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}10\\14\\-4\end{pmatrix}-\begin{pmatrix}8\\13\\-2\end{pmatrix}=\begin{pmatrix}2\\1\\-2\end{pmatrix}$ or $\overrightarrow{BA}=\begin{pmatrix}-2\\-1\\2\end{pmatrix}$	M1	
		$\mathbf{r}=\begin{pmatrix}8\\13\\-2\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}10\\14\\-4\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ accept equivalents	M1 A1ft	(3)
	(b)	$\overrightarrow{CB}=\overrightarrow{OB}-\overrightarrow{OC}=\begin{pmatrix}10\\14\\-4\end{pmatrix}-\begin{pmatrix}9\\9\\6\end{pmatrix}=\begin{pmatrix}1\\5\\-10\end{pmatrix}$ or $\overrightarrow{BC}=\begin{pmatrix}-1\\-5\\10\end{pmatrix}$		
		$CB=\sqrt{\left(1^2+5^2+(-10)^2\right)}=\sqrt{(126)}\quad(=3\sqrt{14}\approx 11.2)$ awrt 11.2	M1 A1	(2)
	(c)	$\overrightarrow{CB}.\overrightarrow{AB}=\left\lvert\overrightarrow{CB}\right\rvert\left\lvert\overrightarrow{AB}\right\rvert\cos\theta$		
		$(\pm)(2+5+20)=\sqrt{126}\sqrt{9}\cos\theta$	M1 A1	
		$\cos\theta=\frac{3}{\sqrt{14}}\Rightarrow\theta\approx 36.7°$ awrt 36.7°	A1	(3)
	(d)			
		$\frac{d}{\sqrt{126}}=\sin\theta$	M1 A1ft	
		$d=3\sqrt{5}(\approx 6.7)$ awrt 6.7	A1	(3)
	(e)	$BX^2=BC^2-d^2=126-45=81$	M1	
		$! \; CBX=\frac{1}{2}\times BX\times d=\frac{1}{2}\times 9\times 3\sqrt{5}=\frac{27\sqrt{5}}{2}(\approx 30.2)$ awrt 30.1 or 30.2	M1 A1	(3)
				[14]
		Alternative for (e)		
		$! \; CBX=\frac{1}{2}\times d\times BC\sin\angle XCB$	M1	
		$=\frac{1}{2}\times 3\sqrt{5}\times\sqrt{126}\sin(90-36.7)°$ sine of correct angle	M1	
		≈ 30.2 $\frac{27\sqrt{5}}{2}$, awrt 30.1 or 30.2	A1	(3)