IYGB - MMS PAPER Q - QUESTION 1

a) 14   25   31   36   37   41   51   52   55   79   112

n = 11, n+1 RULE APPLY

$Q_1 = \frac{1}{4}(11+1) = 3^{RD}$ OBS $\Rightarrow Q_1 = 31$

$Q_2 = \frac{1}{2}(11+1) = 6^{TH}$ OBS $\Rightarrow Q_2 = 41$

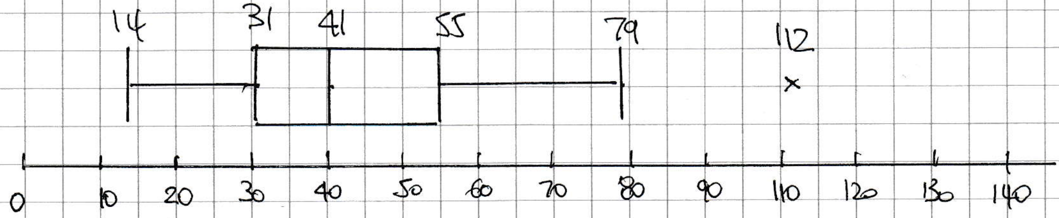
$Q_3 = \frac{3}{4}(11+1) = 9^{TH}$ OBS $\Rightarrow Q_3 = 55$

b) LOWER BOUND $= Q_1 - 1.5(Q_3 - Q_1) = 31 - 1.5(55-31) = -5$

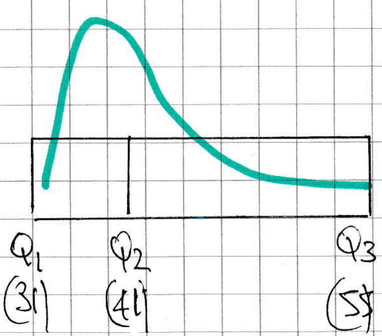
UPPER BOUND $= Q_3 + 1.5(Q_3 - Q_1) = 55 + 1.5(55-31) = 91$

$\therefore$ ONLY 112 IS AN OUTLIER

c)



d) USING THE QUARTILES



POSITIVE SKEW SINCE $Q_2 - Q_1 < Q_3 - Q_2$

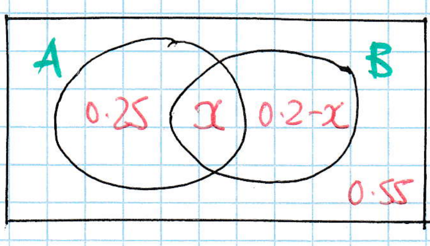
IYGB - MMS PAPER Q - QUESTION 2

$P(A \cap B') = 0.25 \qquad P(A) = 2P(B) \qquad P(A \cup B) = 0.45.$

$P(A' \cap B') = 1 - 0.45$

$P(A' \cap B') = 0.55$

FILL IN A VENN DIAGRAM (PARTLY)



LET $P(A \cap B) = x$

$\Rightarrow P(B \cap A') = 1 - 0.25 - 0.55 - x$

$\Rightarrow P(B \cap A') = 0.2 - x$

USING $P(A) = 2P(B)$

$\Rightarrow 0.25 + x = 2[x + 0.2 - x]$

$\Rightarrow 0.25 + x = 0.4$

$\Rightarrow x = 0.15$

$\therefore P(A \cap B) = 0.15$ //

USING THE "STANDARD FORMULA

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$\Rightarrow P(A \cup B') = (0.25 + x) + (\cancel{0.25} + 0.55) - \cancel{0.25}$

$\Rightarrow P(A \cup B') = 0.25 + 0.55 + x$

$\Rightarrow P(A \cup B') = 0.95$ //

IYGB - MMS PAPER Q - QUESTION 3

a) USING A STATISTICAL CALCULATOR WE OBTAIN

$$r = 0.789$$

b) IT WILL BE UNCHANGED, AS r IS NOT AFFECTED BY SCALING

c) SETTING HYPOTHESIS

$H_0 : \rho = 0$

$H_1 : \rho > 0$

, WHERE ρ IS THE P.M.C.C FOR THE POPULATION (NOT JUST THE SAMPLE OF 8)

THE CRITICAL VALUE AT 1%, $n = 8$ IS $0.7887 \simeq 0.789$

AS $0.789 \simeq 0.7887$, THE TEST IS INCONCLUSIVE, SO A TEST WITH A LARGER SAMPLE MIGHT BE APPROPRIATE

d) OBTAINING A REGRESSION LINE FROM A CALCULATOR

$$y = a + bx$$

$$y = 3.96 + 0.462x \quad \text{(COEFFICIENTS AT 3 sf)}$$

WHEN $x = 30$

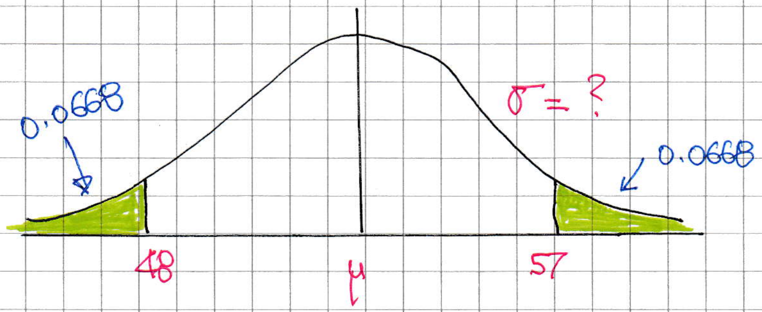
$$y = 3.96 + 0.462 \times 30$$

$$y \simeq 17.82$$

$$y \simeq 18$$

IYGB - MMS PAPER Q - QUESTION 4

$$\boxed{Y \sim N(\mu, \sigma^2)}$$



$P(Y<48) = P(Y>57) = 0.0668$

BY SYMMETRY

$$\mu = \frac{48+57}{2} = \frac{105}{2} = 52.5$$

USING $P(Y>57) = 0.0668$

$\Rightarrow P(Y<57) = 0.9332$

$\Rightarrow P\left(z < \frac{57-52.5}{\sigma}\right) = 0.9332$

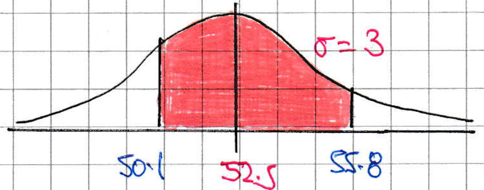
INVERSION

$\Rightarrow \frac{4.5}{\sigma} = +\Phi^{-1}(0.9332)$

$\Rightarrow \frac{4.5}{\sigma} = 1.5$

$\Rightarrow \sigma = 3$

TO "FINISH OFF"



$P(50.1 < Y < 55.8)$

$= P(Y<55.8) - P(Y<50.1)$

$= P(Y<55.8) - \left[1 - P(Y>50.1)\right]$

$= P(Y<55.8) + P(Y>50.1) - 1$

$= P\left(z < \frac{55.8-52.5}{3}\right) + P\left(z > \frac{50.1-52.5}{3}\right) - 1$

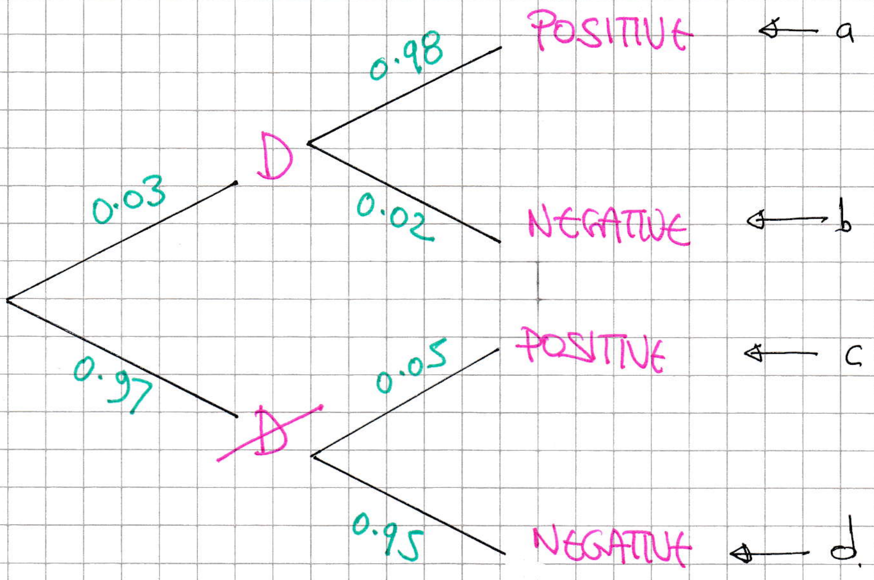
$= \Phi(1.1) + \Phi(-0.8) - 1$

$= 0.8643 + 0.7881 - 1$

$= 0.6524$

IYGB - MMS PAPER Q - QUESTION 5

a)



b) $P(\text{POSITIVE}) = (0.03 \times 0.98) + (0.97 \times 0.05) = 0.0779$

c) $P(D' \mid \text{POSITIVE}) = \dfrac{P(D' \cap \text{POSITIVE})}{P(\text{POSITIVE})} = \dfrac{0.97 \times 0.05}{0.0779} = 0.6226$

OR

$$\boxed{\frac{c}{a+c}}$$

d) TEST IS NOT EFFECTIVE AS IT PREDICTS A "HEALTHY PERSON" BEING ILL WITH PROBABILITY 0.6226 WHICH IS VERY HIGH

IYGB - MMS PAPER Q - QUESTION 6

X = No of "No Show"

$X \sim B(15, 0.15)$

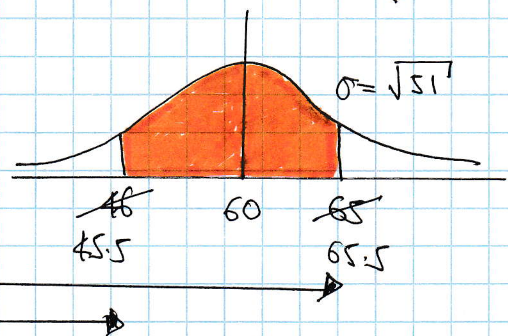
a) I) $P(X=2) = \binom{15}{2}(0.15)^2(0.85)^{13} = 0.2856$

II) $P(X \geqslant 2) = 1 - P(X \leqslant 1) = 1 - 0.3186 = 0.6814$

b) $X \sim B(400, 0.15)$

- $E(X) = np = 400 \times 0.15 = 60$
- $Var(X) = np(1-p) = 60 \times 0.85$
  $= 51 > 5$

APPROXIMATE BY $Y \sim N(60, 51)$



$P(45 < X \leqslant 65)$

$= P(46 \leqslant X \leqslant 65)$

$= P(45.5 < Y < 65.5)$

$= P(Y < 65.5) - P(Y < 45.5)$

$= P(Y < 65.5) - [1 - P(Y > 45.5)]$

$= P(Y < 65.5) + P(Y > 45.5) - 1$

$= P\left(z < \frac{65.5-60}{\sqrt{51}}\right) + P\left(> \frac{45.5-60}{\sqrt{51}}\right) - 1$

$= \Phi(0.770154\ldots) + \Phi(-2.030406\ldots) - 1$

$= 0.77934\ldots + 0.97884 - 1$

$= 0.7582$

IYGB - MMS PAPER Q - QUESTION 6

c) $X \sim B(20, 0.15)$

$H_0 : p = 0.15$

$H_1 : p \neq 0.15$, WHERE p REPRESENTS THE PROPORTION OF MISSED APPOINTMENTS IN GENERAL

CRITICAL REGION AT 10% SIGNIFICANCE IS REQUIRED, AT 5% EACH TAIL

- $P(X \leq 0) = 0.0388 = 3.88\% < 5\%$
- $P(X \leq 1) = 0.1756 = 17.56\% > 5\%$

$\vdots$

- $P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9327 = 0.0673 = 6.73\% > 5\%$
- $P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.9781 = 0.0219 = 2.19\% < 5\%$

$\therefore$ CRITICAL REGION $= \{0\} \cup \{7, 8, 9, \ldots 20\}$

d) LOOKING AT ABOVE "TABLES"

- $P(X \leq 0) = 3.88\%$ ← CLOSER
- $P(X \leq 1) = 17.56\%$

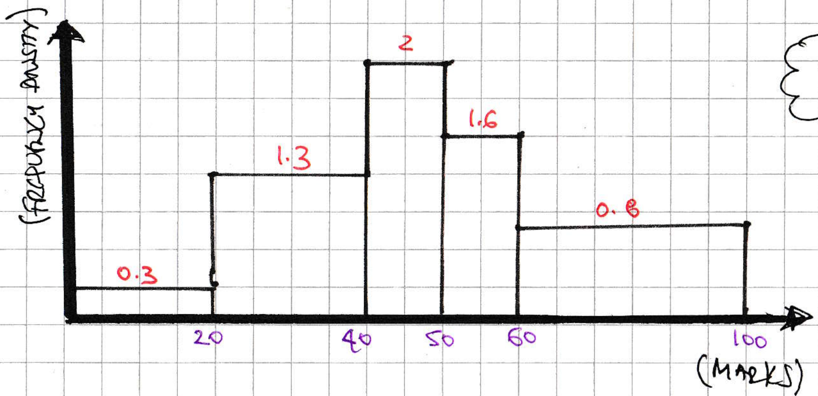
$\vdots$

- $P(X \geq 6) = 6.73\%$ ← CLOSER
- $P(X \geq 7) = 2.19\%$

$\therefore$ C.R $= \{0\} \cup \{6, 7, 8, \ldots, 20\}$

IYGB – MMS PAPER Q – QUESTION 7

a)

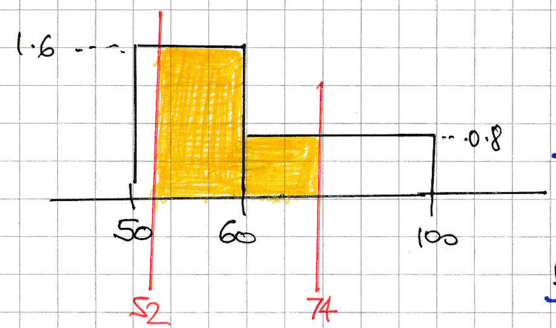


250 STUDENTS

- START BY DETERMINING THE SCALE FACTOR OF AREA TO FREQUENCY
- TOTAL AREA $= (20\times0.3)+(20\times1.3)+(10\times2)+(10\times1.6)+(40\times0.8) = 100$
- FREQUENCY : AREA

  250 : 100

  2.5 : 1   i.e. SCALE FACTOR $\times 2.5$

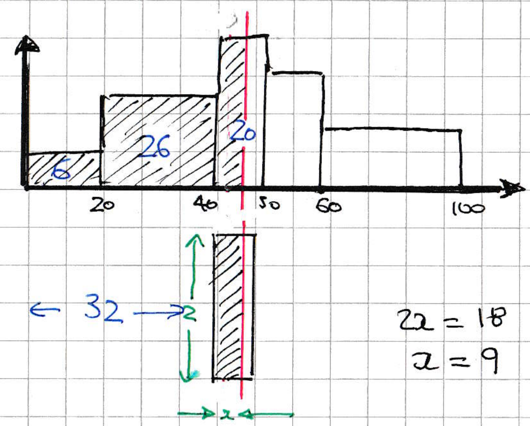
- 

AREA $= (8\times1.6)+(14\times0.8)$

$= 24$

REQUIRED FREQUENCY $= 24\times2.5 = 60$

b) WE USE AREA INSTEAD OF FREQUENCY



$2x = 18$

$x = 9$

$\therefore Q_2 = 49$

IYGB – MMS PAPER Q – QUESTION 7

c) RECONSTRUCTING A FREQUENCY TABLE WITH MIDPOINTS

MIDPOINTS	FREQUENCY (ADJUSTED FROM AREA)
10	$20 \times 0.3 \times 2.5 = 15$
30	$20 \times 1.3 \times 2.5 = 65$
45	$10 \times 2 \times 2.5 = 50$
55	$10 \times 1.6 \times 2.5 = 40$
80	$40 \times 0.8 \times 2.5 = 80$
	250

FROM CALCULATOR IN STAT MODE

$$\Sigma x = 12950, \quad \Sigma x^2 = 794250, \quad n = 250$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{12950}{250} = 51.8$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{794250}{250} - 51.8^2} \approx 22.2$$

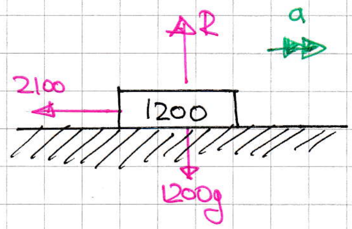
IYGB - MMS PAPER Q - QUESTION 8

a) STARTING WITH DYNAMICS ("F = ma") TO FIND THE ACCELERATION

$\Rightarrow$ "$F = ma$"

$\Rightarrow$ $-2100 = 1200a$

$\Rightarrow$ $a = -1.75 \text{ ms}^{-2}$



NOW KINEMATICS, FROM THE INSTANT THE BRAKES ARE APPLIED UNTIL THE CAR STOPS

$u = 28 \text{ ms}^{-1}$
$a = -1.75 \text{ ms}^{-2}$
$s =$
$t = ?$
$v = 0 \text{ ms}^{-1}$

$\Rightarrow$ $v = u + at$

$\Rightarrow$ $0 = 28 - 1.75t$

$\Rightarrow$ $1.75t = 28$

$\Rightarrow$ $t = 16 \text{ s}$

USING THE "QUANTITIES FROM ABOVE"

$s = ut + \frac{1}{2}at^2$

$s = 28 \times 16 + \frac{1}{2}(-1.75) \times 16^2$

$s = 448 - 224$

$s = 224 \text{ m}$

OR

$s = \frac{u+v}{2} \times t$

$s = \frac{28+0}{2} \times 16$

$s = 14 \times 16$

$s = 224 \text{ m}$

OR

$v^2 = u^2 + 2as$

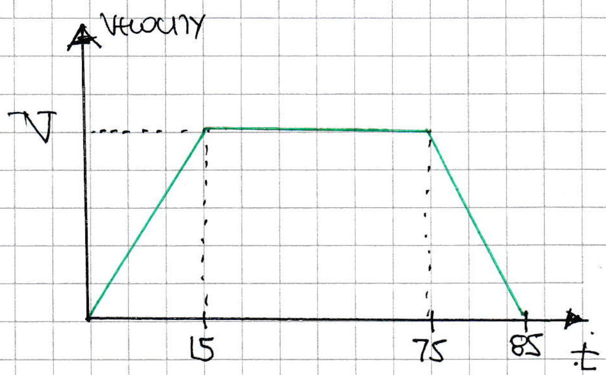
$0^2 = 28^2 + 2(-1.75)s$

$3.5s = 784$

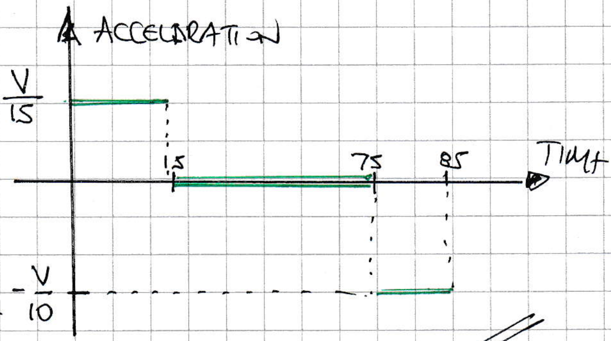
$s = 224 \text{ m}$

IYGB – MMS PAPER Q – QUESTION 9

a) VELOCITY (SPEED) TIME GRAPH



ACCELERATION – TIME GRAPH



b) DISTANCE = AREA UNDER GRAPH

$\Longrightarrow$ "AREA OF TRAPEZIUM" $= 1015$

$\Longrightarrow \frac{85+60}{2} \times V = 1015$

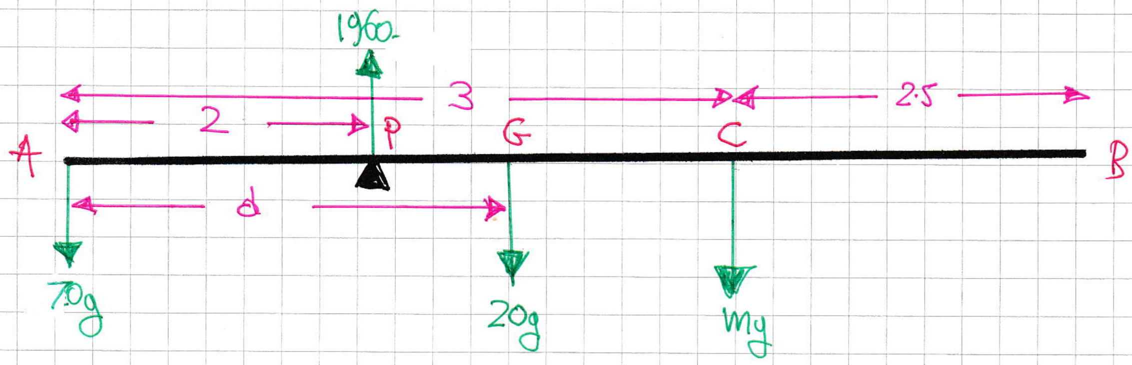
$\Longrightarrow \frac{145}{2} V = 1015$

$\Longrightarrow 145V = 2030$

$\Longrightarrow V = 14$

IYGB - MMS PAPER Q - QUESTION 10

STARTING WITH A DIAGRAM



RESOLVING FORCES VERTICALLY

$\Rightarrow 70g + 20g + mg = 1960$

$\Rightarrow 90g + mg = 200g$

$\frac{1960}{9.8} = 200$

$\Rightarrow 90 + m = 200$

$\Rightarrow m = 110 \text{ kg}$

NOW TAKING MOMENTS ABOUT A

$\Rightarrow 1960 \times 2 = 20g \times d + mg \times 3$

$\Rightarrow 200g \times 2 = 20gd + 330g$

$\div g$

$\Rightarrow 400 = 20d + 330$

$\Rightarrow 70 = 20d$

$\Rightarrow d = 3.5 \text{ m}$

IYGB, MMS PAPER Q – QUESTION 11

a) USING "$\underline{v} = \underline{u} + \underline{a}t$"

$$\underline{v} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} t$$

$$\underline{v} = \begin{pmatrix} 2-t \\ t \end{pmatrix}$$

WHEN $t = 8$

$$\underline{v}_8 = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$$

SPEED IS |VELOCITY VECTOR|

$$|\underline{v}_8| = \left| \begin{matrix} -6 \\ 8 \end{matrix} \right| = \sqrt{(-6)^2 + 8^2} = \sqrt{36+64} = 10 \text{ ms}^{-1}$$

b) USING $\underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$$\underline{r} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} t + \frac{1}{2} \begin{pmatrix} -1 \\ 1 \end{pmatrix} t^2$$

WHEN $t = 8$ WE OBTAIN

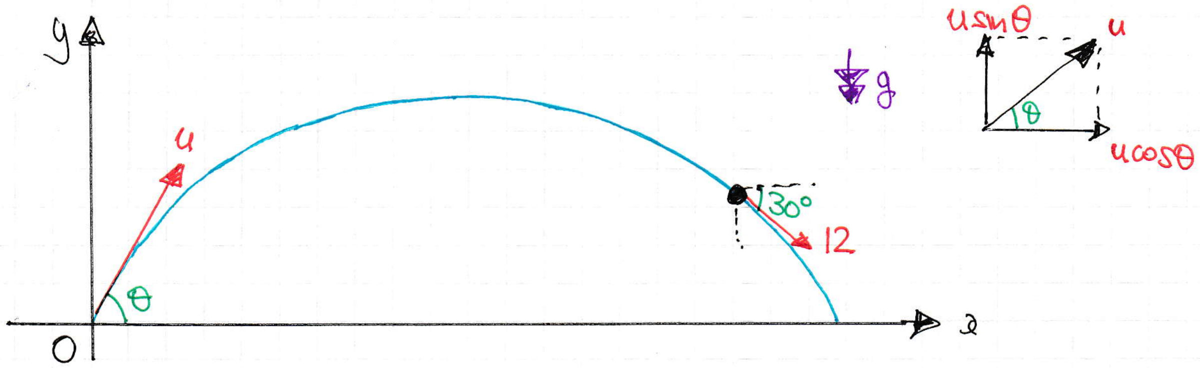
$$\underline{r}_8 = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + 8 \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \frac{1}{2} \times 8^2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0+16-32 \\ -2+0+32 \end{pmatrix} = \begin{pmatrix} -16 \\ 30 \end{pmatrix}$$

DISTANCE FROM O IS THE MODULUS OF THE ABOVE POSITION VECTOR

$$\text{DISTANCE} = \left| \begin{matrix} -16 \\ 30 \end{matrix} \right| = \sqrt{(-16)^2 + 30^2} = \sqrt{256+900} = \sqrt{1156} = 34 \text{ m}$$

NOTE THAT INTEGRATION & CONDITIONS CAN ALSO BE USED TO SOLVE THIS PROBLEM

IYGB - MMS PAPER Q - QUESTION 12

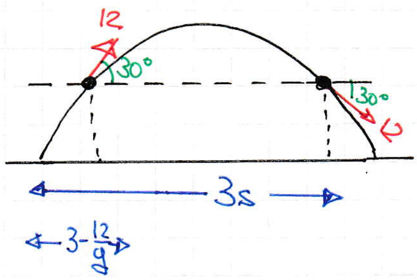


- LOOKING AT VERTICAL, AND CONSIDERING VELOCITY IN THAT DIRECTION

$$\text{"}v = u + at\text{"}$$
$$-12\sin 30 = u\sin\theta - g \times 3$$
$$-6 = u\sin\theta - 3g$$
$$u\sin\theta = 3g - 6$$

- REPEAT, LOOKING FOR VERTICAL VELOCITY +6

$$\text{"}v = u + at\text{"}$$
$$6 = u\sin\theta - gt$$
$$6 = 3g - 6 - gt$$
$$gt = 3g - 12$$
$$t = 3 - \frac{12}{g}$$

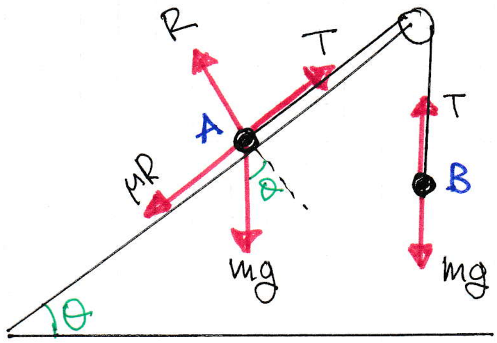


- HENCE THE REQUIRED TIME IS

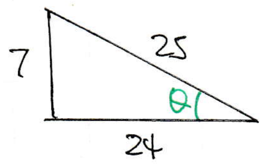
$$3 - \left(3 - \frac{12}{g}\right) = \frac{12}{g} = \frac{60}{49}$$

$$\approx 1.22\text{s}$$

IYGB - MMS PAPER Q - QUESTION 13



$\sin\theta = \frac{7}{25}$



$\therefore \cos\theta = \frac{24}{25}$

- LOOKING AT B (EQUILIBRIUM)

$$T = mg \quad - \text{I}$$

- LOOKING AT A

$$(\perp): \quad R = mg\cos\theta \quad - \text{II}$$
$$(\parallel): \quad T = \mu R + mg\sin\theta \quad - \text{III}$$

- SUBSTITUTE (I) & (II) INTO (III)

$$\Rightarrow mg = \mu(mg\cos\theta) + mg\sin\theta$$
$$\Rightarrow \cancel{mg} = \mu \cancel{mg}\cos\theta + \cancel{mg}\sin\theta$$
$$\Rightarrow 1 = \mu\cos\theta + \sin\theta$$
$$\Rightarrow 1 = \mu \times \frac{24}{25} + \frac{7}{25}$$
$$\Rightarrow 25 = 24\mu + 7$$
$$\Rightarrow 18 = 24\mu$$
$$\Rightarrow \mu = \frac{3}{4}$$

IYGB - MMS PAPER Q - QUESTION 14

PROCEED AS FOLLOWS

$$\Rightarrow \quad \underline{F} = \underline{F}_1 + \underline{F}_2$$

$$\Rightarrow \lambda(3\underline{i} - 2\underline{j}) = (2\underline{i} + 7\underline{j}) + (4\underline{i} + k\underline{j})$$

ACTS PARALLEL TO THIS VECTOR (RESULTANT FORCE ACTS IN THE DIRECTION OF $\underline{a}$)

$$\Rightarrow 3\lambda\underline{i} - 2\lambda\underline{j} = 6\underline{i} + (k+7)\underline{j}$$

- $3\lambda = 6$
  $\lambda = 2$
- $-2\lambda = k+7$
  $-4 = k+7$
  $k = -11$

NOW THE EQUATION OF MOTION GIVES IF $k=-11$

$$\Rightarrow \underline{F} = m\underline{a}$$

$$\Rightarrow (2\underline{i} + 7\underline{j}) + (4\underline{i} - 11\underline{j}) = m\underline{a}$$

$$\Rightarrow 6\underline{i} - 4\underline{j} = m\underline{a}$$

$$\Rightarrow |6\underline{i} - 4\underline{j}| = m|\underline{a}|$$

$$\Rightarrow 2|3\underline{i} - 2\underline{j}| = m \times 5\sqrt{13}$$

$$\Rightarrow 2\sqrt{3^2 + (-2)^2} = 5m\sqrt{13}$$

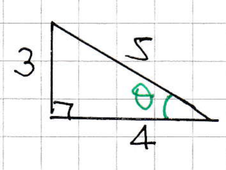
$$\Rightarrow 2\cancel{\sqrt{13}} = 5m\cancel{\sqrt{13}}$$

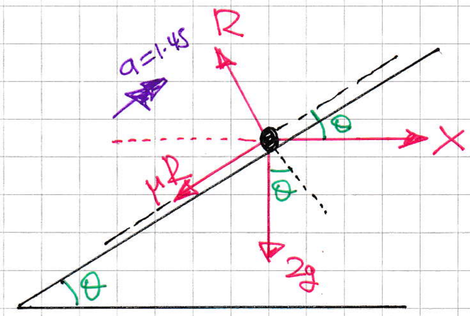
$$\Rightarrow m = \frac{2}{5}$$

IYGB - MMS PAPER Q - QUESTION 15

STARTING WITH A DIAGRAM AND DRAWING THE PUSHING FORCE AS A PULLING FORCE

$\tan\theta = \frac{3}{4}$





$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

$\mu = \frac{1}{2}$

RESOLVING PERPENDICULAR & PARALLEL TO THE PLANE

$(\perp)$: $R = X\sin\theta + 2g\cos\theta$ (EQUILIBRIUM)

$(\parallel)$ $X\cos\theta - \mu R - 2g\sin\theta = 2a$ ("F = ma")

BY SUBSTITUTING "R = ..." INTO THE SECOND EQUATION

$\Rightarrow X\cos\theta - \mu(X\sin\theta + 2g\cos\theta) - 2g\sin\theta = 2a$

$\Rightarrow \frac{4}{5}X - \frac{1}{2}\left(\frac{3}{5}X + 2g \times \frac{4}{5}\right) - 2g \times \frac{3}{5} = 2 \times 1.45$

$\Rightarrow \frac{4}{5}X - \frac{3}{10}X - \frac{4}{5}g - \frac{6}{5}g = 2.9$

$\Rightarrow \frac{1}{2}X - 2g = 2.9$

$\Rightarrow X - 4g = 5.8$

$\Rightarrow X = 45\text{ N}$

IYGB – MMS PAPER Q – QUESTION 16

USING INTEGRATION TO OBTAIN A VELOCITY EXPRESSION

$$a = \frac{dv}{dt} = 16 - 6t$$

$$v = \int 16 - 6t \, dt$$

$$v = 16t - 3t^2 + A$$

USING $t=1$, $v=1$

$$\Rightarrow \quad 1 = 16 - 3 + A$$

$$\Rightarrow \quad \underline{A = -12}$$

$$\Rightarrow \quad \boxed{v = -3t^2 + 16t - 12}$$

INTEGRATE AGAIN TO GET THE DISPLACEMENT

$$x = \int -3t^2 + 16t - 12 \, dt$$

$$x = -t^3 + 8t^2 - 12t + B$$

USING $t=1$, $x=-5$

$$\Rightarrow \quad -5 = -1 + 8 - 12 + B$$

$$\Rightarrow \quad -5 = -5 + B$$

$$\Rightarrow \quad \underline{B = 0}$$

$$\Rightarrow \quad \boxed{x = -t^3 + 8t^2 - 12t}$$

IYGB - MMS PAPER Q - QUESTION 16

NOW SOLVING $x=0$ ("PASSES THROUGH THE ORIGIN")

$$\Rightarrow 0 = -t^3 + 8t^2 - 12t$$

$$\Rightarrow t^3 - 8t^2 + 12t = 0$$

$$\Rightarrow t(t^2 - 8t + 12) = 0$$

$$\Rightarrow t(t-2)(t-6) = 0$$

$$t = \begin{cases} \cancel{0} \\ 2 \\ 6 \end{cases}$$

FINALLY WE CAN FIND THE VELOCITY USING $v = -3t^2 + 16t - 12$

- $t=2$

$V_2 = -3(2)^2 + 16 \times 2 - 12$

$V_2 = -12 + 32 - 12$

$\underline{V_2 = 8}$

- $t=6$

$V_6 = -3 \times 6^2 + 16 \times 6 - 12$

$V_6 = -108 + 96 - 12$

$\underline{V_6 = -24}$

$\therefore$ THE REQUIRED SPEEDS ARE $8\,\text{ms}^{-1}$ & $24\,\text{ms}^{-1}$ //