CM

A Level Maths Question Countdown

7 days until the 1st exam

Information

- Each of the ten sheets will contain five pure questions and two applied questions.

Pure questions

- Two of the pure questions will be 'standard'.
- Two of the pure questions will be 'problems'.
- The last pure question will involve modelling.

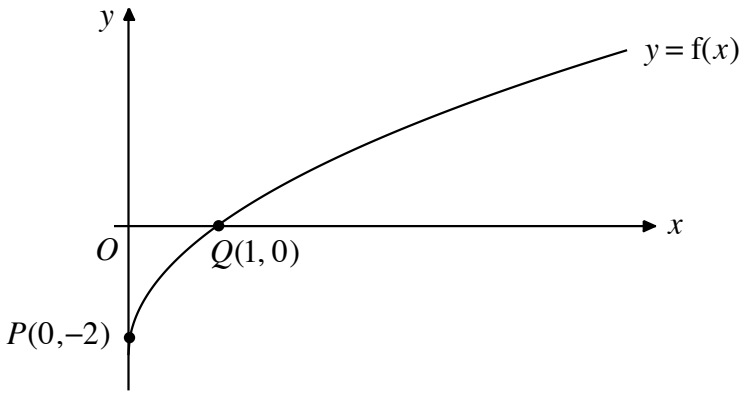
Applied questions

- One of the questions will focus on statistics.
- One of the questions will focus on mechanics.
- On alternate days, the statistics question will look at the large data set. Note that these questions may be brief as opposed to full length exam questions.

Notes to self

Pure questions – standard

1



The diagram above shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, $x \geqslant 0$.

The curve C meets the coordinate axes at $P(0, -2)$ and $Q(1, 0)$, as shown.

(a) On separate axes, sketch the curves with equation

(i) $y = \mathrm{f}(2x) + 2$

(ii) $y = -\mathrm{f}(x + 1)$

(iii) $y = |\mathrm{f}(x)|$

(b) With reference to the diagram, explain why the function f has an inverse.

2 Solve the following differential equations according to the given initial conditions. In each case, give y in terms of x and simplifying where possible.

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 \mathrm{e}^{-2y}$, $\quad x = 0, y = 1$

(b) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{xy}{1 + x^2}$, $\quad x = 0, y = 2$

(c) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3\mathrm{e}^{x-y}$, $\quad x = 0, y = \ln 3$

Pure questions – problems

3 A curve C has parametric equations

$$x = t + \frac{2}{t}, \quad y = t - \frac{2}{t}, \quad t > 0$$

(a) Show that the Cartesian equation of C can be expressed as

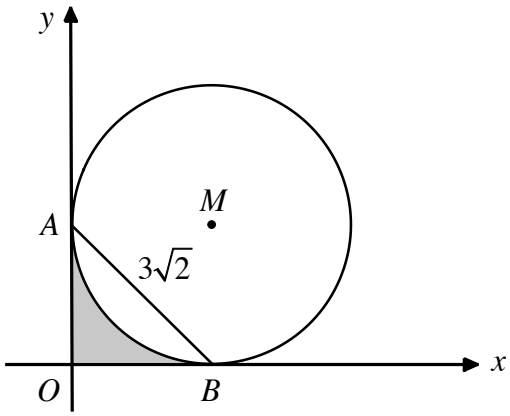
$$Px^2 + Qy^2 = R$$

where P, Q and R are integers to be found.

(b) Find, in terms of t, an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(c) Deduce that there are no stationary points.

4



The diagram above shows a sketch of the circle C, which touches the x axis at the point A and touches the y axis at the point B. The point M is the centre of C.

Given that the chord AB has length $3\sqrt{2}$,

(a) find the coordinates of A and B.

(b) Hence, write down the equation of C, giving your answer in the form

$$(x - a)^2 + (y - b)^2 = k$$

where a, b and k are integers to be found.

The region shown shaded in the diagram is bounded by C and the coordinate axes.

(c) Showing your working clearly, find the area of the shaded region.

Pure questions – modelling

5 A pipe is connected to an empty tank of height 5 m.

At time $t = 0$, water begins to flow into the tank through the pipe.

The height of water in a tank, H metres, is modelled to vary according to

$$D = k(e^{0.2t} - 1), \quad t \geqslant 0$$

where t is time in minutes since the tank started to fill and k is a constant.

(a) Explain why k must be positive in the model.

Given that it takes the two minutes for water in the tank to reach a height of 1.5 m,

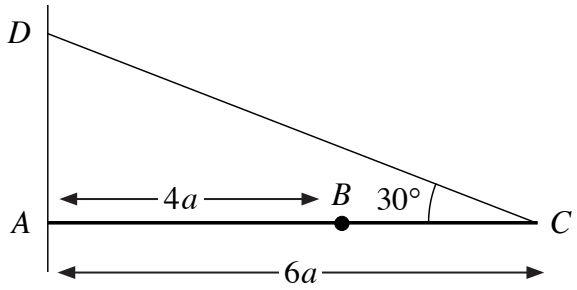
(b) find the value of k.

(c) Hence calculate the time taken for the tank to completely fill with water.

(d) Suggest how the model can be refined if the tank is not initially empty.

Applied questions – mechanics

6



A horizontal rod AC has mass m and length $6a$. The end A rests against a rough vertical wall. A particle of mass $2m$ is attached to the rod at B, where $AB = 4a$. One end of a string is attached to the rod at C and the other end is attached to a point D on the wall, where D is vertically above A. The rod is held in equilibrium in a vertical plane perpendicular to the wall. The string is inclined at an angle of 30° the horizontal, as shown in the diagram.

The string is modelled as light and inextensible, the rod is modelled as uniform and coefficient of friction between the rod and the wall is μ.

Using the model,

(a) find the tension in the string,

(b) show that the horizontal component of the force exerted by the wall on the rod has magnitude $\dfrac{11\sqrt{3}}{6}mg$.

(c) Find the range of values of possible values of μ.

Applied questions – statistics

7 The times taken, t minutes, for individuals to complete a quiz are summarised in the following grouped frequency table.

Time (t minutes)	**Frequency**
0 – 5	20
5 – 10	18
10 – 15	21

(a) Use linear interpolation to find an estimate for the value of the median.

(b) Show that the estimated value for the lower quartile is 3.69 to three significant figures.

(c) Estimate the interquartile range for this distribution.

The mid-point of each class was represented by x and its corresponding frequency by f giving

$$\sum xf = 447.5 \qquad \sum x^2 f = 4418.75$$

(d) Calculate, to three significant figures, estimates for the mean and standard deviation.

The person timing the quiz made an error and the individuals each took 5 minutes longer than the times recorded above. The table below summarises the actual times.

Time (t minutes)	**Frequency**
5 – 10	20
10 – 15	18
15 – 20	21

(e) Without further calculations, explain the effect this would have on each of your estimates to parts (a), (b), (c) and (d).