IYGB – FS1 PAPER N – QUESTION 1

x	0	1	2	3	4
$P(X=x)$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	a	$\frac{1}{24}$

a) $\sum P(X=x) = 1$

$\frac{3}{8} + \frac{1}{3} + \frac{1}{4} + a + \frac{1}{24} = 1$

$1 + a = 1$

$a = 0$

b) $E(X) = \sum x P(X=x)$

$\Rightarrow E(X) = \left(0 \times \frac{3}{8}\right) + \left(1 \times \frac{1}{3}\right) + \left(2 \times \frac{1}{4}\right) + (3 \times 0) + \left(4 \times \frac{1}{24}\right)$

$\Rightarrow E(X) = 0 + \frac{1}{3} + \frac{1}{2} + 0 + \frac{1}{6}$

$\Rightarrow E(X) = 1$

c) $E(X^2) = \sum x^2 P(X=x)$

$\Rightarrow E(X^2) = \left(0^2 \times \frac{3}{8}\right) + \left(1^2 \times \frac{1}{3}\right) + \left(2^2 \times \frac{1}{4}\right) + \left(3^2 \times 0\right) + \left(4^2 \times \frac{1}{24}\right)$

$\Rightarrow E(X^2) = 0 + \frac{1}{3} + 1 + 0 + \frac{2}{3}$

$\Rightarrow E(X^2) = 2$

$\text{Var}(X) = E(X^2) - [E(X)]^2$

$\text{Var}(X) = 2 - 1^2$

$\text{Var}(X) = 1$

IYGB - FS1 PAPER N - QUESTION 2

SETTING HYPOTHESES

H_0: Data could be modelled by $B(5, 0.2)$

H_1: Data could not be modelled by $B(5, 0.2)$

ANALYSING THE DATA TO OBTAIN EXPECTED FREQUENCIES & CONTRIBUTIONS

NUMBER x	FREQUENCY O_i	EXPECTED FREQUENCIES $E_i = P(X=x) \times 80$	CONTRIBUTIONS $\frac{(O_i - E_i)^2}{E_i}$
0	15	$0.32768 \times 80 = 26.2144$	4.797
1	36	$0.4096 \times 80 = 32.768$	0.319
2	17	$0.2048 \times 80 = 16.384$	
3	10	$0.0512 \times 80 = 4.096$	
4	1	$0.0064 \times 80 = 0.512$	
5	1	$0.00032 \times 80 = 0.0256$	
	80		

Rows 2–5 combined: frequency 29, expected frequency 21.0176, contribution 3.032

- $\nu = 3 - 1 = 2$
- $\chi^2_2(5\%) = 5.991$
- $\sum_{i=1}^{3} \frac{(O_i - E_i)^2}{E_i} = 8.148$

AS $8.148 > 5.991$ IT APPEARS THAT THE DATA COULD NOT BE MODELLED BY $B(5, 0.2)$

SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - FS1 PAPER N - QUESTION 3

a) MODELLING WITH A CONTINUOUS UNIFORM DISTRIBUTION (RECTANGULAR)

$$f(x) = \begin{cases} \frac{1}{180} & 0 \leq x \leq 180 \\ 0 & \text{OTHERWISE} \end{cases}$$

I) $P(X<70) = \frac{70}{180} = \frac{7}{18}$

II) USING STANDARD FORMULA $E(X) = \frac{a+b}{2}$

$$E(X) = \frac{0+180}{2} = 90$$

III) USING STANDARD FORMULA $Var(X) = \frac{(b-a)^2}{12}$

$$Var(X) = \frac{(180-0)^2}{12} = 2700$$

$\therefore$ STANDARD DEVIATION $= \sqrt{2700} \simeq 52.0$

b) MODEL AS FOLLOWS

$$P(\text{shorter piece is AT MOST } 70 \text{ cm}) = P(X<70) + P(X>110)$$

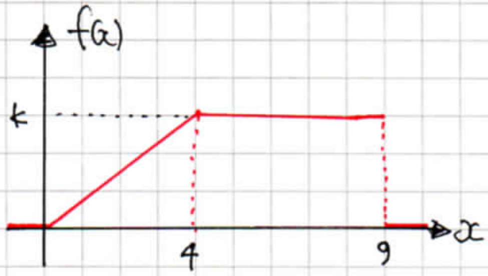
$$= \frac{7}{18} + \frac{7}{18}$$

$$= \frac{7}{9}$$

IYGB - FS1 PAPER N - QUESTION 4

LOOKING AT THE DIAGRAM OPPOSITE

$$f(x) = \begin{cases} mx & 0 \le x \le 4 \\ k & 4 < x \le 9 \\ 0 & \text{OTHERWISE} \end{cases}$$



USING "$y = mx$" WE OBTAIN

$$\Rightarrow k = m \times 4$$
$$\Rightarrow k = 4m$$
$$\Rightarrow m = \frac{1}{4}k$$

ALSO WE HAVE LOOKING AT THE AREA

$$\left(\frac{1}{2} \times 4 \times k\right) + (5 \times k) = 1$$
$$2k + 5k = 1$$
$$7k = 1$$
$$k = \frac{1}{7} \qquad \therefore \ m = \frac{1}{28}$$

FINALLY THE EXPECTATION CAN BE FOUND

$$E(X) = \int_a^b x f(x)\, dx = \int_0^4 x\left(\frac{1}{28}x\right) dx + \int_4^9 x\left(\frac{1}{7}\right) dx$$

$$= \int_0^4 \frac{1}{28}x^2\, dx + \int_4^9 \frac{1}{7}x\, dx = \left[\frac{1}{84}x^3\right]_0^4 + \left[\frac{1}{14}x^2\right]_4^9$$

$$= \left(\frac{64}{84} - 0\right) + \left(\frac{81}{14} - \frac{16}{14}\right) = \frac{16}{21} + \frac{65}{14}$$

$$= \frac{227}{42} \approx 5.40$$

IYGB - FS1 PAPER N - QUESTION 5

a)

$\sum x = 235$	$\sum x^2 = 7853$	$\sum xy = 4904$
$\sum y = 152$	$\sum y^2 = 3214$	$n = 8$

CALCULATE THE VALUES OF S_{xx} S_{yy} S_{xy}

- $S_{xx} = \sum x^2 - \frac{\sum x \sum x}{n} = 7853 - \frac{235 \times 235}{8} = 949.875$
- $S_{yy} = \sum y^2 - \frac{\sum y \sum y}{n} = 3214 - \frac{152 \times 152}{8} = 326$
- $S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 4904 - \frac{235 \times 152}{8} = 439$

FIND THE P.M.C.C

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{439}{\sqrt{949.875 \times 326}} = 0.7889009725\ldots$$

$$\simeq 0.789$$

b) THE P.M.C.C WILL BE UNCHANGED AT 0.789, AS IT IS NOT AFFECTED BY SCALING

IYGB - FS1 PAPER N - QUESTION 6

a) $P(X > 0.5) = 1 - P(X < 0.5) = 1 - F(0.5)$

$= 1 - \frac{1}{5}(0.5^2)(6 - 0.5^2) = 1 - \frac{23}{80}$

$= \frac{57}{80}$

b) USING $f(x) = \frac{d}{dx}\left[F(x)\right]$

$$\frac{d}{dx}\left[\frac{1}{5}x^2(6-x^2)\right] = \frac{d}{dx}\left(\frac{6}{5}x^2 - \frac{1}{5}x^4\right) = \frac{12}{5}x - \frac{4}{5}x^3$$

$$= \frac{4}{5}x\left(3 - x^2\right)$$

$$\therefore f(x) = \begin{cases} \frac{4}{5}x(3-x^2) & 0 \leqslant x \leqslant 1 \\ 0 & \text{OTHERWISE} \end{cases}$$

c) $\text{Var}(X) = E(X^2) - \left[E(X)\right]^2$

$\text{Var}(X) = \frac{7}{15} - \left(\frac{16}{25}\right)^2 = \frac{107}{1875} = 0.057066\ldots$

$\therefore$ STANDARD DEVIATION $= \sqrt{0.057066\ldots} \simeq 0.238886\ldots$

$\therefore$ MODE BY DIFFERENTIATION

$$\frac{d}{dx}\left(f(x)\right) = \frac{d}{dx}\left(\frac{12}{5}x - \frac{4}{5}x^3\right) = \frac{12}{5} - \frac{12}{5}x^2$$

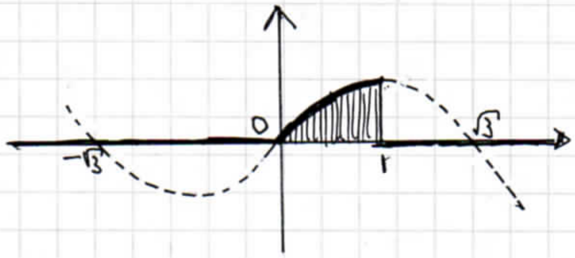
SET TO ZERO

$\frac{12}{5} - \frac{12}{5}x^2 = 0$

$\frac{12}{5} = \frac{12}{5}x^2$

$x^2 = 1$

$x = +1$



$\therefore$ MODE IS 1

IYGB - FS1 PAPER N - QUESTION 6

FINALLY

$$\frac{\text{MEAN} - \text{MODE}}{\text{STANDARD DEVIATION}} = \frac{0.64 - 1}{0.238886\ldots} \simeq -1.507$$

IYGB - FS1 PAPER N - QUESTION 7

OBTAIN SUMMARY STATISTICS FOR A-G

$\sum x = 254 \qquad \sum x^2 = 9906 \qquad n = 7$

$\sum y = 209 \qquad \sum xy = 7865$

TEST 1 → x
TEST 2 → y

CALCULATE S_{xx} & S_{xy}

$$S_{xx} = \sum x^2 - \frac{\sum x \sum x}{n} = 9906 - \frac{254 \times 254}{7} = \frac{4826}{7} \approx 689.43$$

$$S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 7865 - \frac{254 \times 209}{7} = \frac{1969}{7} \approx 281.29$$

OBTAIN THE GRADIENT

$$b = \frac{S_{xy}}{S_{xx}} = \frac{1969/7}{4826/7} = 0.407998\ldots$$

OBTAIN THE EQUATION

$$\left.\begin{array}{l} \bar{x} = \frac{\sum x}{n} = \frac{254}{7} \\ \bar{y} = \frac{\sum y}{n} = \frac{209}{7} \end{array}\right\} \Longrightarrow a = \bar{y} - b\bar{x}$$

$$\Longrightarrow a = \frac{209}{7} - 0.407998\ldots \times \frac{254}{7}$$

$$\Longrightarrow a = \frac{286}{19} \approx 15.0526\ldots$$

$$\therefore \quad y = 0.408x + 15.05$$

FINALLY USING THE ABOVE EQUATION WITH $x = 40$

$$y = 0.408 \times 40 + 15.05$$

$$y \approx 31.3726\ldots$$

$$\therefore k = 31$$

IYGB - FS1 PAPER N - QUESTION 8

a) X = NUMBER OF BOTTLES PER 10 METRES

$X \sim Po\left(\frac{250}{1000} \times 10\right)$

$X \sim Po(2.5)$

$P(X>4) = P(X \geq 5) = 1 - P(X \leq 4) = \ldots$ tables/calculator

$= 1 - 0.8912$

$= \underline{0.1088}$

b) Y = NUMBER OF CANS PER 30 METRES

$Y \sim Po\left(\frac{160}{1000} \times 30\right)$

$Y \sim Po(4.8)$

$$P(Y=5) = \frac{e^{-4.8} \times 4.8^5}{5} = \underline{0.1747}$$

c) W = NUMBER OF CANS PER 50 METRES

$W \sim Po\left(\frac{160}{1000} \times 50\right)$

$W \sim Po(8)$

$H_0 : \lambda = 8$ OR $H_0 : \mu = 160$

$H_1 : \lambda > 8$ $H_1 : \mu > 160$

(WHERE λ (OR μ) REFERS TO THE MEAN/RATE OF THE ENTIRE COASTLINE

TESTING AT THE 1% SIGNIFICANCE, ON THE BASIS THAT $W = 15$

$\Longrightarrow P(W \geq 15) = 1 - P(W \leq 14)$

IYGB - FS1 PAPER N - QUESTION 8

$$= 1 - 0.9827$$

$$= 0.0173$$

$$= 1.73\% > 1\%$$

THERE IS NO SIGNIFICANT EVIDENCE (AT 1% SIGNIFICANCE LEVEL) THAT THE NUMBER OF CANS PER km IS GREATER THAN 160.

THERE IS NOT SUFFICIENT EVIDENCE TO REJECT H_0

d) LET T = TOTAL CANS & BOTTLES PER 100 METRES

$$T \sim Po\left(\frac{250+160}{1000} \times 100\right)$$

$$T \sim Po(41)$$

APPROXIMATE BY $V \sim N(41, 41)$

$$P(T > 50) = P(T \geq 51)$$

$$= P(V > 50.5)$$

$$= 1 - P(V < 50.5)$$

$$= 1 - P\left(Z < \frac{50.5 - 41}{\sqrt{41}}\right)$$

$$= 1 - \Phi(1.48365...)$$

$$= 1 - 0.9310$$

$$= 0.0690$$