AS Level
Maths

Bronze Set C, Paper 1 (Edexcel version)

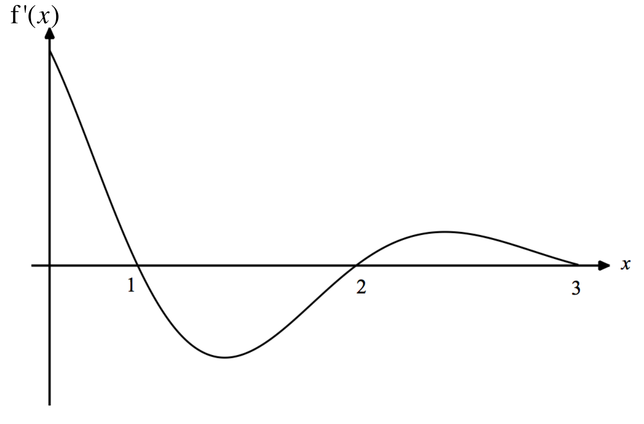
Question	Solution	Partial Marks	Guidance
1 (a)	$\sqrt{28}=\sqrt{4\times 7}=\sqrt{4}\sqrt{7}=2\sqrt{7}$ $\sqrt{343}=\sqrt{49\times 7}=\sqrt{49}\sqrt{7}=7\sqrt{7}$ Hence $\sqrt{28}+\sqrt{343}=2\sqrt{7}+7\sqrt{7}$ $=9\sqrt{7}$ (so $a=9$)	M1 A1 **[2]**	Complete method to decompose the surds and then attempts to add their decomposed surds At the very least, we need to see the correct simplified values of √28 and √343 Correct simplification **Answer only is 0/2**
1 (b)	$\frac{9+3\sqrt{7}}{9\sqrt{7}}=\frac{9+3\sqrt{7}}{9\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$ $=\frac{9\sqrt{7}+3(7)}{9(7)}$ $=\frac{21+9\sqrt{7}}{63}$ $=\frac{1}{3}+\frac{1}{7}\sqrt{7}$ (so $b=1/3$ and $c=1/7$)	M1* M1(dep*) A1 **[3]**	Replaces denominator with their (a). That is all that is required Attempts to rationalise denominator by multiplying by $k\sqrt{7}$ ***OR*** attempts to partition denominator and then rationalise the 9/9√7 term Correct simplified expression with sufficient working shown
1 (b) ALT	$\frac{9+3\sqrt{7}}{\sqrt{28}+\sqrt{343}}=\frac{9+3\sqrt{7}}{\sqrt{28}+\sqrt{343}}\times\frac{\sqrt{28}-\sqrt{343}}{\sqrt{28}-\sqrt{343}}$ $=\frac{9\sqrt{28}-9\sqrt{343}+3\sqrt{196}-3\sqrt{2401}}{28-343}$ $=\frac{9(2\sqrt{7})-9(7\sqrt{7})+3(14)-3(49)}{-315}$ $=\frac{-105-45\sqrt{7}}{-315}$ $=\frac{1}{3}+\frac{1}{7}\sqrt{7}$	M1* M1(dep*) A1 **[3]**	Multiplies numerator and denominator by ±(√28 – √343) ***OR*** ±(2√7 – 7√7) Obtains numerator of the form $\alpha+\beta\sqrt{7}$, $\alpha, \beta\neq 0$ Correct simplified expression with sufficient working shown

2 (a)	$\frac{OB}{\sin 50} = \frac{10}{\sin(180-53-50)}$ $\Rightarrow OB = \frac{10\sin 50}{\sin 77}$ So OB **= 7.86(1…) cm**	M1 A1 **[2]**	Complete method to find OB using the sine rule Awrt 7.86
2 (b)	sector OAB = ¼ of a circle with radius OB so area of sector OAB $\frac{1}{4}(\pi)(7.861...)^2 = 48.545...$ area of the triangle is $\frac{1}{2}(OB)(OA) = \frac{1}{2}(7.861...)(7.861...)$ = 30.905… So area of shaded region = 48.545… – 30.905… **= 17.6(4…) cm²**	 M1 M1 A1 **[3]**	 Correct method to find the area of the sector OAB using their (a) Correct method to find the area of the triangle OAB using their (a) Obtains correct area of the shaded region. Awrt 17.6
3 (a)	$\overrightarrow{AB} = (-2\mathbf{i}+\mathbf{j})-(5\mathbf{i}-3\mathbf{j}) = -7\mathbf{i}+4\mathbf{j}$ $\overrightarrow{OC} = \overrightarrow{OA} + 2\overrightarrow{AB}$ $= (5\mathbf{i}-3\mathbf{j})+2(-7\mathbf{i}+4\mathbf{j})$ $= -9\mathbf{i}+5\mathbf{j}$	M1* A1 M1(dep*) A1 **[4]**	Correct method to find $\overrightarrow{AB}$ Correct vector $\overrightarrow{AB}$ Employs correct method to find the position vector of C ALT: can use $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{AB}$ Correct position vector of C
3 (b)	$\lvert\overrightarrow{OC}\rvert = \sqrt{(-9)^2+5^2}$ = √(106) (or awrt 10.3)	M1 A1 **[2]**	Correct method to find the distance of C from O using their (a) Correct distance

4	$y = 3x^2 - x^3$ $\Rightarrow \frac{dy}{dx} = 6x - 3x^2$ When $x = 1$, $\frac{dy}{dx} = 6(1) - 3(1)^2 = 3$ So gradient of tangent at $x = 1$ is 3 Also when $x = 1, y = 2$ Hence equation of tangent given by $y - 2 = 3(x - 1)$ $\Rightarrow y = 3x - 1$	M1* A1 M1(dep*) A1 M1 A1 **[6]**	Complete method to find *dy*/*dx* Correct *dy*/*dx* Substitutes $x = 1$ into their *dy*/*dx* Correct gradient of the tangent Uses their gradient of the tangent and some value for *y* to write down the equation of the tangent Obtains correct equation of the tangent in the required form
5 (a) (i)	5 m^3	B1 **[1]**	Cao
5 (a) (ii)	$2 = 5e^{-0.14t}$ $\Rightarrow e^{-0.14t} = 0.4$ $\Rightarrow -0.14t = \ln 0.4$ $\Rightarrow t = -\frac{1}{0.14}\ln 0.4$ **= 6.54(4…) minutes**	M1 A1 **[2]**	Complete method to find the value of *t* at which $V = 2$ with use of ln(e) = 1 seen Correct time. Awrt 6.54
5 (b) (i)	e.g. replace 5 with 10 / use $V = 10e^{-0.14t}$	B1 **[1]**	Some comment that 5 should be replaced with 10 Do not allow vague descriptions, e.g. 'increase 5' Do not allow suggestions to use '$V = 5e^{-0.14t} + 5$' as this changes other factors about the model

5 (b) (ii)	e.g. replace 0.14 with a smaller number	B1 **[1]**	A suitable refinement
5 (c)	The volume of water in the tank never (physically) reaches zero / the tank never empties according to the model / cannot be used to predict how long it takes for the tank to empty / not realistic for large t	B1 **[1]**	A correct limitation
6 (a)	Radius of C_1 + radius of C_2 = distance of M from O Radius of $C_1 = 2$ Distance of M from $O = \sqrt{4^2+3^2} = 5$ So radius of $C_2 = 5 - 2 = 3$ (as required)	 B1 M1 A1 **[3]**	 States or implies correct radius of C_1 Employs a correct method to find the radius of C_2 Convincingly shows that the radius of C_2 is 3
6 (b)	$(x-4)^2 + (y-3)^2 = 9$ (so $a = 4$, $b = 3$, $k = 9$)	B1 B1 **[2]**	Correct LHS Correct RHS (allow 3^2)
6 (c)	(from the figure,) $N = (4, 0)$	B1 **[1]**	Correct coordinates of N
7 (a)	$\mathrm{f}(x) = \frac{10x^2}{\sqrt{x^7}} - \frac{5\sqrt{x}}{\sqrt{x^7}}$ $= \frac{10x^2}{x^{\frac{7}{2}}} - \frac{5x^{\frac{1}{2}}}{x^{\frac{7}{2}}}$ $= 10x^{-\frac{3}{2}} - 5x^{-3}$ (so $p = -3/2$, $q = -3$)	 B1 B1 **[2]**	 For $10x^{-3/2}$ or $p = -3/2$ For $-5x^{-3}$ or $q = -3$

7 (b)	$f'(x) = 10\left(-\frac{3}{2}\right)x^{-\frac{5}{2}} - 5(-3)x^{-4}$	M1	Attempts to differentiate f(*x*) ($x^n \to x^{n-1}$ on both terms)
	$= -15x^{-\frac{5}{2}} + 15x^{-4}$	A1	Correct f \`(*x*)
	Maximum point when $f'(x) = 0$ $\Rightarrow -15x^{-\frac{5}{2}} + 15x^{-4} = 0$ $\Rightarrow x^{-4} = x^{-\frac{5}{2}}$ $\Rightarrow x^{\frac{3}{2}} = 1$	M1	Sets their f \`(*x*) = 0 and solves for *x*
	$\Rightarrow x = 1$	A1	Obtains correct value of *x* at which maximum point occurs Allow steps omitted in solution for *x* as it is somewhat obvious that the solution has to be 1
	When *x* = 1, $y = 10(1)^{-\frac{3}{2}} - 5(1)^{-3} = 10 - 5 = 5$	A1 cso	Correct value of *y* at the maximum point
	So coordinates of maximum point at (1, 5)	**[5]**	**Coordinate form not necessary** **Answer only is 0/5**
7 (c)	Intersect once for $k \le 0$ or $k = 5$	M1	$k \le 0$ or $k =$ their 5 seen or implied Condone in terms of *y* for the M1. M0 if in terms of *x*
	Hence set of values of *k* for which *C* and *L* intersect once is $\{k \in \mathbb{R} : k \le 0\} \cup \{k \in \mathbb{R} : k = 5\}$	A1ft **[2]**	Correct set of values ft their 5 Allow omission of $\in \mathbb{R}$ Accept equivalent sets, e.g. $\{k : k \le 0 \ \cup \ k = 5\}$ and $\{k : k \le 0 \text{ or } k = 5\}$

8 (a)	$\left(1+\frac{x}{3}\right)^{10}=1^{10}+{}^{10}C_1(1)^9\left(\frac{x}{3}\right)^1+{}^{10}C_2(1)^8\left(\frac{x}{3}\right)^2+\ldots$ $=1+\frac{10}{3}x+5x^2+\ldots$	B1 M1 A1 A1 **[4]**	For 1 appearing in the expansion Two terms of the form ${}^{10}C_k\,(1)^k\,(x/3)^{10-k}$, $1\le k\le 9$ oe (allow factorial definition for ${}^{10}C_k$ terms) The two terms may be listed and not added for the M1 Correct linear term in expansion Correct quadratic term in expansion
8 (b)	$1+\frac{x}{3}=1.021\Rightarrow x=0.063$ So $(1.021)^{10}\approx 1+\frac{10}{3}(0.063)+5(0.063)^2$ **$\approx$1.2298**	M1* M1(dep*) A1 cao **[3]**	Sets $1 + x/3 = 1.021$ and solves for x Substitutes their 0.063 into their (a) Obtains correct estimate to 5 significant figures
8 (c)	Use more terms in the expansion	B1 **[1]**	Correct suggestion for how to improve accuracy of (b)
9		B1 B1 **[2]**	Correct shape of the graph – the graph should decay with increasing x. We need to see three intersections with the x axis and two turning points. The amplitude of the 2nd turning point should be less than the 1st (or at least not noticeably bigger). Ignore behaviour after third intersection. Condone y on vertical axis Suggestion of a turning point at $x = 0$ is B0 x intersections at 1, 2 and 3, clearly labelled. Note this is independent on the 1st B1
10 (a)	Mistake 1: uses $2\log_2 x = \log_2(2x)$ – should be $\log_2(x^2)$ Mistake 2: in line 4, 3^2 should be 2^3	B1 B1 **[2]**	Identifies mistake (in line 2) and states correction Identifies mistake (in line 4) and states correction

10 (b)	$\Rightarrow \dfrac{x^2+1}{x^2}=8$ $\Rightarrow x^2+1=8x^2$ $\Rightarrow 7x^2=1$ $\Rightarrow x=\sqrt{\dfrac{1}{7}}$	M1 A1 oe **[2]**	Obtains this equation oe and solves for x Obtains correct value of x (no others) Accept equivalent forms and awrt 0.378
11 (a)	$f(1) = 1^3 + 2(1)^2 – 1 –2$ $= 1 + 2 – 1 – 2$ $= 0$ Hence since f(1) = 0, (by the factor theorem), $(x – 1)$ is a factor of f(x)	M1 A1 **[2]**	Attempts to find f(1) with substitution seen **Use of long division or any other method is M0** Obtains f(1) = 0 **and** gives a conclusion, e.g. 'hence $(x – 1)$ is a factor', 'as required', 'qed', '□'
11 (b)	$x-1 \overline{)x^3+2x^2-x-2}$ quotient x^2+3x+2 $\underline{x^3-x^2}$ ↓ $3x^2-x$ $\underline{3x^2-3x}$ ↓ $+2x-2$ $\underline{+2x-2}$ 0 So $f(x) = (x – 1)(x^2 + 3x + 2)$ $= (x – 1)(x + 1)(x + 2)$	M1 A1 A1 **[3]**	Employs a correct method to find the quadratic factor of f(x) Two methods: 1) long division (scheme) – M1 for one iteration completed successfully, so obtaining x^2 in quotient and subtraction seen to obtain $3x^2$ 2) inspection – M1 for quadratic factor of the form $(x^2 + qx + 2)$, $q \neq 0$ Obtains correct quadratic factor Expresses f(x) as a product of three linear factors
11 (c)	possible values of a are 1, –1 or –2	B1 B1 **[2]**	Any one correct value of a All three correct values of a

12 (i)	Case 1: Suppose n is even, then $n = 2k$ for some integer k $\Rightarrow n(n+2)(n^2-1) = (2k)(2k+2)(4k^2-1)$ $= 4 \times k(k+1)(4k^2-1)$ which is divisble by 4 Hence true if n is even Case 2: Suppose n is odd, then $n = 2k + 1$ for some integer k $\Rightarrow n(n+2)(n^2-1) = (2k+1)(2k+1+2)((2k+1)^2-1)$ $= (2k+1)(2k+3)(4k^2+4k)$ $= 4 \times (2k+1)(2k+3)(k^2+k)$ which is divisble by 4 Hence true if n is odd Hence $n(n + 2)(n^2 – 1)$ is divisible by 4 for all integers n	M1 A1 M1 A1 A1 **[5]**	Considers the statement in the case when n is even by substituting in $n = 2k$ (or similar) Shows that statement is true in the case when n is even No need for a conclusion/statement here, they just need to extract a factor of 4 correctly Considers the statement in the case when n is odd by substituting in $n = 2k + 1$ (or similar) Shows that statement is true in the case when n is odd No need for a conclusion/statement here, they just need to extract a factor of 4 correctly Complete and convincing proof with final concluding statement Must also include specification that ‘k’ is an integer in their characterisations of the odd/even numbers for a convincing proof
12 (ii) (a)	$40^2 + 40 + 41 = 40(40+1) + 41$ $= 40(41) + 41$ $= 41 \times (40+1)$ $= 41 \times 41$ (as required)	B1 **[1]**	Shows the result convincingly Ignore candidates that compute both sides and show they are equal (rubric infringement)

12 (ii) (b)	When $p = 40$, $p^2 + p + 41 = 41 \times 41$ which is not prime Hence Jyoti's claim is false	B1 **[1]**	Shows the statement is false using a suitable counter-example Note that $p = 40$ is the smallest integer counter-example to this statement, so any proofs using $p < 40$ can be ignored. Some other values that work are $p = 41, 44, 49, 56$ – in these cases, require justification that the result is not prime
13 (a)	$\sqrt{(3--2)^2+(6-p)^2} = 5\sqrt{5}$ $\Rightarrow 5^2+(6-p)^2 = 125$ $\Rightarrow 25+36-12p+p^2 = 125$ $\Rightarrow p^2-12p-64 = 0$ $\Rightarrow (p+4)(p-16) = 0$ Since $p < 0$, $p = -4$	M1* M1(dep*) A1 **[3]**	Sets up correct equation in terms of p Obtains a 3TQ and employs a complete method to solve it Correct value of p
13 (b)	Midpoint of $AB = \left(\frac{3-2}{2}, \frac{6-4}{2}\right) = \left(\frac{1}{2}, 1\right)$ Gradient of $AB = \frac{-4-6}{-2-3} = \frac{-10}{-5} = 2$ So gradient of perp. bisector is $-1/2$ Hence eq. of perpendicular bisector is $y-1 = -\frac{1}{2}\left(x-\frac{1}{2}\right)$	B1ft M1 A1 M1 A1 **[5]**	Correct midpoint of AB ft their p Complete method to find the gradient of the <u>perpendicular bisector</u> using their p Correct gradient of the perpendicular bisector Attempts to find the equation using their midpoint and their gradient Correct equation in any form. Other correct forms are: $2x + 4y = 5$, $4y = 5 - 2x$ etc.
13 (c)	When $x = 0$, $4y = 5$ so crosses y axis at $(0, 5/4)$ When $y = 0$, $5 - 2x = 0$ so crosses x axis at $(5/2, 0)$ Hence area of the triangle is $\frac{1}{2}\left(\frac{5}{2}\right)\left(\frac{5}{4}\right)$ $= \frac{25}{8}$ units^2	M1* M1(dep*) A1 **[3]**	Complete method to find the coordinates of C and D using their (b) Uses their coordinates to find the area of the triangle Correct area of the triangle

14 (i)	Maximum value of h is 7 Occurs when $\sin x = -1 \Rightarrow x = -90^\circ$	B1 M1 A1 **[3]**	States correct maximum value of h States the maxima occur when $\sin x = -1$ Correct value of x at which they occur Ignore additional given values of x *outside* the domain of h **Correct answer only (max h = 7 at $x = -90^\circ$) is 3/3**
14 (ii)	$(\sin\theta - \cos\theta)(\sin\theta + 5\cos\theta) = 0$ $\Rightarrow \sin\theta - \cos\theta = 0$ or $\sin\theta + 5\cos\theta = 0$ $\Rightarrow \tan\theta = 1$ or $\tan\theta = -5$ $\tan\theta = 1 \Rightarrow \theta = 45$ Other value in range is 45 + 180 = 225 $\tan\theta = -5 \Rightarrow \theta = -78.690...$ Values in range are $-78.690 + 180$ and $-78.690 + 360$ = 101.31 = 281.31 So solutions are **θ = 45°, 101.3°, 225°, 281.3°**	M1* A1 A1 M1(dep*) A1 cao **[5]**	Correctly factorises the equation Obtains correct values of $\tan\theta$ Obtains θ = 45 **and** θ = 225 Correct method to solve $\tan\theta = -5$. So need to see them find the principal value and then use the CAST diagram/graph correctly to find the two values in range Obtains 101.3 and 281.3 (one dp necessary)

15	$y = \int f'(x)dx$ $= \int \left(2 - \frac{2}{\sqrt{x}}\right)dx$ $= 2x - \frac{2\sqrt{x}}{1/2} + c$ $= 2x - 4\sqrt{x} + c$ Curve passes through (1, –2), $-2 = 2 - 4 + c \Rightarrow c = 0$ So $y = 2x - 4\sqrt{x}$ Curve crosses the x axis when $2x = 4\sqrt{x} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$ $\therefore$ Signed area $= \int_0^4 (2x - 4\sqrt{x})dx$ $= \left[x^2 - \frac{8\sqrt{x^3}}{3}\right]_0^4$ $= 4^2 - \frac{8\sqrt{4^3}}{3} \quad \{-0\}$ $= -\frac{16}{3}$ Hence area is $\frac{16}{3}$ units^2	M1 M1* A1 M1(dep*) A1 A1ft M1**(dep*) M1(dep**) A1ft A1 **[10]**	**This is an overall process/strategy mark.** Award this once: 1) candidate has attempted to find y in terms of x by integration 2) and then attempted to use their y to find the shaded area using integration Attempts to integrate to find x $(x^n \to x^{n+1})$ Correct indefinite integration **including constant** Uses prescribed condition to find the value of the constant Obtains correct y in terms of x (this does not need to be stated explicitly and can be implied) Correct value for the curve crosses the x axis seen at any stage ft their y in terms of x Attempts to integrate their y in terms of x again (ignore limits, just looking for a second attempt at integration) Uses limits of 0 to 'their 4' and substitutes these into their indefinite integral in the correct order Obtains correct value for the area (up to sign) ft their y in terms of x Correct area
15 Further Notes	Special case: if candidates simply integrate f `($x$) between 0 and some upper limit can score SCM1 for a correct method seen to evaluate a definite integral. The method must be sound. **ALT:** $y$ can be found by $\int_y^{-2} dy' = \int_x^1 x'dx'$ ($x$` and y` are dummy variables, but condone x and y). 2nd M1 – correct method for indefinite integration seen on RHS $(x^n \to x^{n+1})$, 1st A1 – correct indefinite integration on both sides, 3rd M1 – correct substitution of the limits. Rest is as the scheme		