Mark schemes

Q1.

(a) Use of $(\sum\alpha)^2 = \sum\alpha^2 + 2\sum\alpha\beta$

M1

AG

A1

2

(b) $p = 0$, $q = 5 + 6i$

B1,B1

2

(c) (i) Substitute 3i for z **or** use $3i\beta\gamma = -r$

allow for $3i\beta\gamma = r$

M1

$-27i + 15i - 18 + r = 0$ **or** $\beta\gamma = 5 + 6i + \alpha^2$

any form

A1

$r = 18 + 12i$

one error

A1F

3

(ii) Cubic is $(z - 3i)(z^2 + 3iz - 4 + 6i)$

or use of $\beta\gamma$ and $\beta + \gamma$

clearly shown

M1A1

2

(iii) $f(-2) = 0$ or equate imaginary parts

M1

$\beta = -2$, $\gamma = 2 - 3i$

correct answers no working and no check B1 only

A1,A1F

3

[12]

Q2.

(a) (i) $\alpha + \beta + \gamma = 0$

B1 **1**

(ii) $\alpha\beta\gamma = -q$

B1 **1**

(b) $\alpha^3 + p\alpha + q = 0$

M1

$\sum\alpha^3 + p\sum\alpha + 3q = 0$

m1

$\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$
AG

A1

Alternative:

Use of $(\sum\alpha)^3 = (\sum\alpha^3) + 6\alpha\beta\gamma + 3(\sum\alpha\sum\alpha\beta - 3\alpha\beta\gamma)$

(M1)

Substitution of $\sum \alpha = 0$

(m1)

Result

(A1) **3**

(c) (i) $\beta = 4 - 7\text{i}$, $\gamma = -8$

B1,B1 **2**

(ii) Attempt at either p or q

M1

$p = 1$

A1F

$q = 520$
ft incorrect roots provided p and q are real

A1F **3**

(d) Replace z by $\frac{1}{z}$ in cubic equation

or $\sum \frac{1}{\alpha} = -\frac{p}{q}, \sum \frac{1}{\alpha\beta} = 0, \frac{1}{\alpha\beta\gamma} = -\frac{1}{q}$

ft on incorrect p and / or q

M1
A1F

$520z^3 + z^2 + 1 = 0$ coefficients must be integers

CAO

A1

3

[13]

Q3.

(a) $\alpha + \beta = -\frac{7}{2}$

B1

$\alpha\beta = 4$

B1

(b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(-\frac{7}{2}\right)^2 - 2(4)$

Using correct identity with ft or correct substitution

M1

$= \frac{49}{4} - 8 = \frac{17}{4}$

CSO AG. A0 if $\alpha + \beta$ has wrong sign

A1

2

(c) (Sum =)

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{17/4}{16} \left(= \frac{17}{64}\right)$

Writing $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ *in a correct suitable form with ft or correct substitution*

M1

$= \frac{17}{64}$

ft wrong value for $\alpha\beta$

A1F

(Product =) $\frac{1}{(\alpha\beta)^2} = \frac{1}{16}\left(=\frac{4}{64}\right)$

ft wrong value for $\alpha\beta$

B1F

$x^2 - Sx + P$ (= 0)

*Using correct general form of LHS of eqn **with** ft substitution of c's S and P values. PI*

M1

Eqn is $64x^2 - 17x + 4 = 0$

CSO Integer coefficients and '= 0' needed

A1 5

[9]

Q4.

(a) (i) ($\alpha\beta\gamma$ =) $-37 + 36i$

B1 1

(ii) ($\beta\gamma$ =) $(-2 + 3i)(1 + 2i) = -2 + 3i - 4i - 6$

correct unsimplified but must simplify i^2

M1

$(-8 - i)\,\alpha = -37 + 36i$

$\Rightarrow (8 + i)\,\alpha = 37 - 36i$

***AG** be convinced*

A1cso 2

(iii) $\Rightarrow \alpha = \frac{37-36i}{8+i} \times \frac{8-i}{8-i}$

M1

$= \frac{296 - 37i - 288i - 36}{65}$

correct unsimplified

A1

$= \frac{260 - 325i}{65} = 4 - 5i$

A1cao

3

Alternative:

(8 + i)(m + n i) = 37 − 36i

8m − n = 37; m + 8n = −36

(M1)

either m = 4 or n = −5

(A1)

$\alpha = 4 - 5i$

(A1)

(3)

(b) $\alpha + \beta + \gamma = -p$

−2 + 3i + 1 + 2i + 4 − 5i = 3

(⇒ p =) −3

B1

1

(c) $\alpha\beta + \beta\gamma + \gamma\alpha = q$

$q = \sum\alpha\beta$ and attempt to evaluate three products FT "their" α

(7 + 22i) + (−8 − i) + (14 + 3i) = q

M1

q = 13 + 24i

A1cao

2

[9]

Q5.

(a) (i) $\alpha + \beta + \gamma = 5$

B1

$\alpha\beta\gamma = 4$

B1

2

(ii) $\alpha\beta\gamma^2 + \alpha\beta^2\gamma + \alpha^2\beta\gamma = \alpha\beta\gamma(\alpha + \beta + \gamma)$

M1

= 5 × 4 = 20

FT their results from (a)(i)

A1√

2

(b) (i) If α, β, γ are all real then $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 \geq 0$

Hence α, β, γ cannot all be real

argument must be sound

E1

1

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha = k$

$\sum \alpha\beta = k$ *PI*

B1

$(\alpha\beta + \beta\gamma + \gamma\alpha)^2 = \sum \alpha^2\beta^2 + 2(\alpha\beta\gamma^2 + \alpha\beta^2\gamma + \alpha^2\beta\gamma)$

correct identity for $\left(\sum \alpha\beta\right)^2$

M1

$= -4 + 2(20)$

substituting their result from (a)(ii)

A1√

$k = \pm 6$

must see $k = \ldots$

A1 **cso**

4

[9]

Q6.

(a) $\alpha + \beta = -\frac{3}{2}$

OE

B1

$\alpha\beta = -3$

OE

B1

2

(b) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

Using correct identity for $\alpha^3 + \beta^3$ *in terms of* $\alpha + \beta$ *and* $\alpha\beta$.

M1

$= \left(-\frac{3}{2}\right)^3 - 3(-3)(-3/2)$

with ft / or correct substitution

A1F

$= -\frac{27}{8} - \frac{27}{2} = -\frac{135}{8}$

CSO AG. Correct evaluation of each of $(-1.5)^3$ and $-3(-3)(-1.5)$ must be seen before the printed answer is stated

A1

3

(c) Sum = $\alpha + \frac{\alpha}{\beta^2} + \beta + \frac{\beta}{\alpha^2} = \alpha + \beta + \frac{\alpha^3 + \beta^3}{(\alpha\beta)^2} = -\frac{3}{2} + \frac{-135/8}{9}$

Writing $\alpha + \frac{\alpha}{\beta^2} + \beta + \frac{\beta}{\alpha^2}$ in a suitable form with ft/or correct substitution

M1

Sum = $-\frac{27}{8}$

PI OE exact value eg − 3.375 (A0 if $\alpha\beta = 3$ used to get $(\alpha\beta)^2 = 9$)

A1

Product = $\alpha\beta + \frac{\beta}{\alpha} + \frac{\alpha}{\beta} + \frac{1}{\alpha\beta} = \alpha\beta + \frac{\alpha^2 + \beta^2}{\alpha\beta} + \frac{1}{\alpha\beta}$ (*)

Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ (= $\frac{9}{4}$ + 6)

(*) OE with correct identity for $\alpha^2 + \beta^2$ used in (c). Subst of values not required but PI by correct value of Product

M1

Product = $-3 - \frac{1}{3}\left(\frac{9}{4} + 6\right) - \frac{1}{3} = -\frac{73}{12}$

PI OE exact value

A1

$x^2 - Sx + P$ (= 0)

*Using correct general form of LHS of eqn **with** ft substitution of c's S and P values.*

M1

Eqn is $24x^2 + 81x - 146 = 0$

OE but integer coefficients and ' = 0' needed

A1

6

[11]

Q7.

(a) (i) $\alpha + \beta + \gamma = 4i$

B1 **1**

(ii) $\alpha\beta\gamma = 4 - 2i$

B1 **1**

(b) (i) $\alpha + \alpha = 4i, \quad \alpha = 2i$

AG

B1 **1**

(ii) $\beta\gamma = \dfrac{4-2i}{2i} = -2i - 1$

Some method must be shown, eg $\frac{2}{i} - 1$

M1

AG

A1 **2**

(iii) $q = \alpha\beta + \beta\gamma + \gamma\alpha$

M1

$= \alpha(\beta + \gamma) + \beta\gamma$

Or $\alpha^2 + \beta\gamma$ *, ie suitable grouping*

M1

= 2i.2i − 2i − 1 = −2i − 5

AG

A1 **3**

(c) Use of $\beta + \gamma = 2i$ and $\beta\gamma = -2i - 1$

M1

$z^2 - 2iz - (1 + 2i) = 0$

Elimination of say γ *to arrive at* $\beta^2 - 2i\beta - (1 + 2i) = 0$ *M1A0 unless also some reference to* γ *being a root*

AG

A1

2

(d) f (−1) = 1 + 2i − 1 − 2i = 0

For any correct method

M1

$\beta = -1$, $\gamma = 1 + 2\text{i}$

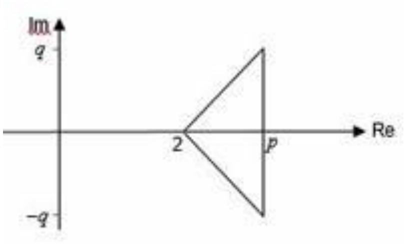
A1 for each answer

A1A1

3

[13]

Q8.

Marking Instructions	**AO**	**Marks**	**Typical Solution**
Writes β and γ in the form $p \pm q\text{i}$ (seen anywhere in the solution)	AO2.5	B1	Real coefficients $\Rightarrow \beta = p + q\text{i}$ and $\gamma = p - q\text{i}$
Uses "sum of the roots = −b/a" together with a conjugate pair to determine the real part (p) of β and γ	AO3.1a	M1	$\alpha + \beta + \gamma = 8$ $\Rightarrow 2 + p + q\text{i} + p - q\text{i} = 8$ $\Rightarrow 2 + 2p = 8$ $\Rightarrow p = 3$
Uses '(their p)' − 2 and the area of the triangle on an Argand diagram to determine the imaginary parts of β and γ	AO3.1a	M1	$(p - 2)q = 8$ $\Rightarrow q = 8$ 
Uses a correct method to find the value of c or d using 'their' values of $p \pm q\text{i}$	AO1.1a	M1	$\beta = 3 + 8\text{i}$ and $\gamma = 3 - 8\text{i}$
Obtains correct values for c and d. CAO	AO1.1b	A1	$d = -\alpha\beta\gamma = -146$ $c = \sum\alpha\beta = 85$
			Total 5 marks

Q9.

(a) $\alpha + \beta = -2$

B1

$\alpha\beta = -5$

B1

2

(b) $\alpha^2 + \beta^2 = (\alpha^2 + \beta^2)^2 - 2\alpha\beta = (-2)^2 - 2(-5)$

OE Using correct identity for $\alpha^2 + \beta^2$ with ft or correct substitution

M1

$= 14$

CSO A0 if $\alpha + \beta$ has wrong sign

A1

2

(c) $\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2)$

PI Seen at least once in part (c).

OE eg $\alpha^3\beta + \alpha\beta^3 = \alpha\beta[(\alpha + \beta)^2 - 2\alpha\beta]$

M1

S(um)= $\alpha^3\beta + \alpha\beta^3 + 2 = (-5)(14) + 2 = -68$

Correct or ft c's $\alpha\beta$ × c's [answer (b)] + 2

A1F

P(roduct) = $(\alpha\beta)^4 + \alpha^3\beta + \alpha\beta^3 + 1 = (-5)^4 + (-5)(14) + 1 = 556$

Correct or

ft [c's $\alpha\beta$]^4 + c's $\alpha\,\beta$ × c's [answer (b)] + 1

A1F

$x - Sx + P$ (=0)

Using correct general form of LHS of eqn ***with*** *ft substitution of c's S and P values.*

M1

Eqn.: $x^2 + 68x + 556 = 0$

CSO ACF

A1

5

[9]

Q10.

Marking Instructions	AO	Marks	Typical Solution
Writes the expression in terms of $\sum\alpha$ and $\sum\alpha\beta$ Award for correct expansion followed by use of $\sum\alpha^2 = (\sum\alpha)^2 - 2\sum\alpha\beta$	AO3.1a	M1	$(\alpha - \beta)^2 + (\gamma - \alpha)^2 + (\beta - \gamma)^2$ $= \alpha^2 - 2\alpha\beta + \beta^2 + \gamma^2 - 2\gamma\alpha + \alpha^2 + \beta^2 - 2\beta\gamma + \gamma^2$ $= 2\alpha^2 + 2\beta^2 + 2\gamma^2 - 2\alpha\beta - 2\gamma\alpha - 2\beta\gamma$ $= 2\sum\alpha^2 - 2\sum\alpha\beta$ $= 2((\sum\alpha)^2 - 2\sum\alpha\beta) - 2\sum\alpha\beta$
Substitutes $\pm m$ for $\sum\alpha$ **and** $\pm n$ for $\sum\alpha\beta$	AO1.1a	M1	$= 2(\sum\alpha)^2 - 6\sum\alpha\beta$ $= 2(-m)^2 - 6 \times n = 2m^2 - 6n$
Gives a reason for expression ≥ 0 Condone lack of reference to roots being real.	AO2.4	E1	But as α, β and γ are real then each of $(\alpha - \beta)^2$, $(\gamma - \alpha)^2$ and $(\beta - \gamma)^2$ must be non-negative. $\therefore (\alpha - \beta)^2 + (\gamma - \alpha)^2 + (\beta - \gamma)^2 \geq 0$
Completes fully correct proof to reach the required result. This mark is only available if all previous marks have been awarded. Lose this mark for sight of $\sum\alpha = m$	AO2.1	R1	$\therefore 2m^2 - 6n \geq 0$ $2m^2 \geq 6n$ $m^2 \geq 3n$ **AG**
			Total 4 marks

Q11.

(a) (i) $\alpha + \beta = 4$, $\alpha\beta = 13$

B1B1

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

M1

… $= 4^2 - 26 = -10$

convincingly shown (AG)

A1

2

(iii) The square of a real number is positive (or zero)

E1

The sum of two such squares is positive (or zero)

E1

2

(b) (i) $(\alpha + i) + (\beta + i) = 4 + 2i$

ft wrong value in (a)(i)

B1F

1

(ii) $(\alpha + i)(\beta + i) = 12 + 4i$

ditto

M1A1F

2

(c) Correct coeff of x or constant term

Using c's answers in (b)

M1

$x^2 - (4 + 2i)\,x + (12 + 4i) = 0$

ft wrong answers in (b)

A1F

2

[11]