Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

A level Further Mathematics
Core Pure Mathematics

Practice Paper 1

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **7** questions in this question paper. The total mark for this paper is **68**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Given that 4 and 2i − 3 are roots of the equation

$$x^3 + ax^2 + bx - 52 = 0$$

where a and b are real constants,

(a) write down the third root of the equation,

(1)

(b) find the value of a and the value of b.

(5)

(Total 6 marks)

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2. (a) Use de Moivre's theorem to show that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

(5)

(b) Hence solve for $0 \leq \theta \leq \frac{\pi}{2}$

$$64\cos^6\theta - 96\cos^4\theta + 36\cos^2\theta - 3 = 0$$

giving your answers as exact multiples of π.

(5)

(Total 10 marks)

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3. (a) A sequence of numbers is defined by

$$u_1 = 8$$

$$u_{n+1} = 4u_n - 9n, \; n \geq 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^n + 3n + 1$$

(5)

(b) Prove by induction that, for $m \in \mathbb{Z}^+$,

$$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^m = \begin{pmatrix} 2m+1 & -4m \\ m & 1-2m \end{pmatrix}$$

(5)

(Total 10 marks)

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4. A company plans to build a new fairground ride. The ride will consist of a capsule that will hold the passengers and the capsule will be attached to a tall tower. The capsule is to be released from rest from a point half way up the tower and then made to oscillate in a vertical line. The vertical displacement, x metres, of the top of the capsule below its initial position at time t seconds is modelled by the differential equation,

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + x = 200\cos t, \quad t \geqslant 0$$

where m is the mass of the capsule including its passengers, in thousands of kilograms.

The maximum permissible weight for the capsule, including its passengers, is 30 000N.

Taking the value of g to be 10 ms^{-2} and assuming the capsule is at its maximum permissible weight,

(a) (i) explain why the value of m is 3

(ii) show that a particular solution to the differential equation is

$$x = 40\sin t - 20\cos t$$

(iii) hence find the general solution of the differential equation.

(8)

(b) Using the model, find, to the nearest metre, the vertical distance of the top of the capsule from its initial position, 9 seconds after it is released.

(4)

(Total 12 marks)

5.

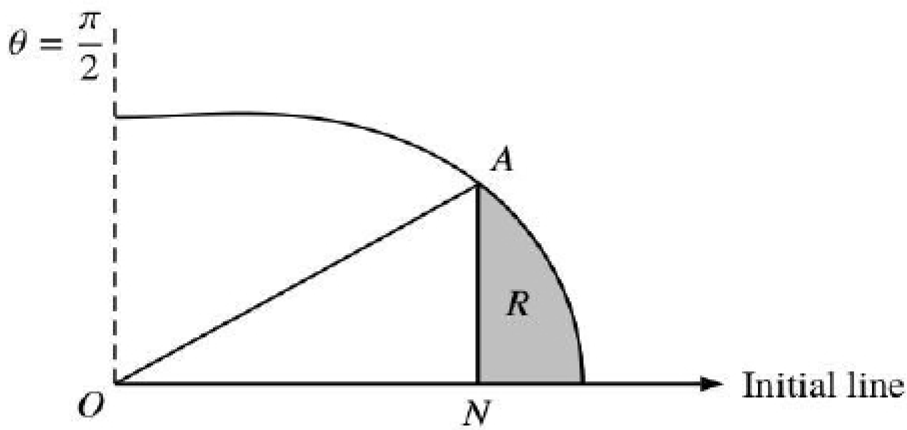

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 4 + \cos 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point A on C, the value of r is $\frac{9}{2}$

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line AN.

Find the exact area of the shaded region R, giving your answer in the form $p\pi + q\sqrt{3}$ where p and q are rational numbers to be found.

(Total 9 marks)

6. (i)

$$p\frac{dx}{dt} + qx = r \qquad \text{where } p,\ q \text{ and } r \text{ are constants}$$

Given that $x = 0$ when $t = 0$

(a) find x in terms of t

(4)

(b) find the limiting value of x as $t \to \infty$

(1)

(ii)

$$\frac{dy}{d\theta} + 2y = \sin\theta$$

Given that $y = 0$ when $\theta = 0$, find y in terms of θ

(7)

(Total 12 marks)

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7. Show that

(a) $\displaystyle\int_5^8 \frac{1}{x^2 - 10x + 34}\,dx = k\pi,$

giving the value of the fraction k,

(5)

(b) $\displaystyle\int_5^8 \frac{1}{\sqrt{(x^2 - 10x + 34)}}\,dx = \ln(A + \sqrt{n}),$

giving the values of the integers A and n.

(4)

(Total 9 marks)

TOTAL FOR PAPER: 68 MARKS