

CM

A Level Maths Question Countdown

2 days until the 1st exam

Information

- Each of the ten sheets will contain five pure questions and two applied questions.

Pure questions

- Two of the pure questions will be 'standard'.
- Two of the pure questions will be 'problems'.
- The last pure question will involve modelling.

Applied questions

- One of the questions will focus on statistics.
- One of the questions will focus on mechanics.
- On alternate days, the statistics question will look at the large data set. Note that these questions may be brief as opposed to full length exam questions.

Notes to self

Pure questions – standard

- 1 $f(x) = 2x^3 + ax^2 + 18x - 24$, where a is a constant.

Given that $(x - 2)$ is a factor of $f(x)$, show that the equation $f(x) = 0$ has only one real root.

- 2 (i) Consider the statement $|2a + 3b| = 2|a| + 3|b|$.

Using a suitable counterexample, **disprove** this statement.

- (ii) Prove that $3^n \geq 1 + 2n$ for all $n \in \mathbb{N}$, $n \leq 5$.

- (iii) (a) Prove that $\sqrt[3]{7}$ is irrational.

(b) Deduce that $m + n\sqrt[3]{7}$ is irrational for all rational numbers m and n , $n \neq 0$.

Pure questions – problems

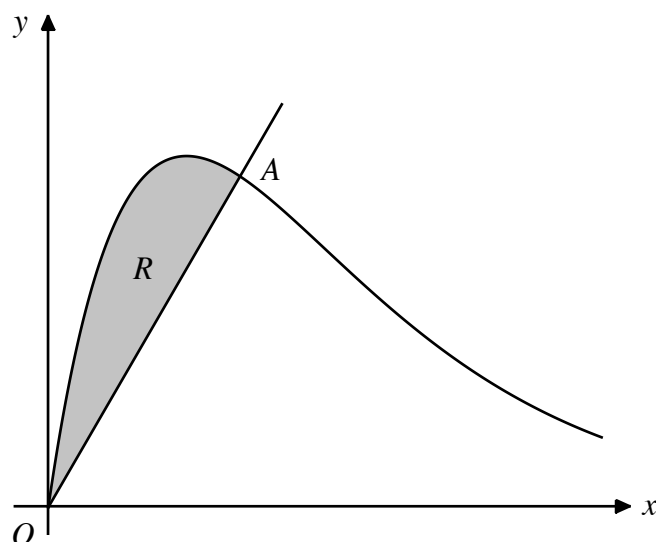
- 3 The curve C has the equation $y = f(x)$, where

$$f(x) = 6x - x^2 - 7$$

- (a) Express $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are constants to be found.
- (b) Sketch the curve with equation C , showing clearly the coordinates of the turning point and the intersection with the y axis.

The equation $f(x) = 0$ has a root α , where $1 < \alpha < 3$.

- (c) Explain why use of the Newton-Raphson process with first approximation 3 will **not** converge to α .
- (d) Use the Newton-Raphson process with first approximation 1 twice to find an approximation for α . Give your approximation to four decimal places.
- (e) Find the exact value of α .



The diagram above shows a sketch of the graph of $y = 8xe^{-2x}$ and the graph of $y = 2x$ for $x \geq 0$. The graphs intersect at the origin O and at the point A , as shown in the diagram.

The region R , shown shaded in the diagram, is bounded by the two graphs.

Showing your method clearly, find the **exact** area of the region R .

(Solutions based entirely on graphical or numerical methods are not acceptable.)

Pure questions – modelling

- 5 A group of biologists did an experiment with a bacterial colony.

A model to represent how the size of the colony varied during the experiment is given by

$$T = 112 - 0.4(n - 12)^2, \quad n \geq 0$$

where T is the number, in thousands, of bacteria in the colony n hours after the experiment started.

- Find the initial number of bacteria in the colony.
- Determine the length of time for which the population of bacteria is greater than 63 600.
- Calculate the change in the population of the colony in the third hour of the experiment.

During the experiment, the biologists add an antibiotic to the colony that causes the population of the colony to decrease to 0.

Theo looks at the model and assumes that the biologists add the antibiotic when $n = 12$.

- Explain, in relation to the model, the significance of this value of n .
- Using Theo's assumption, find the time taken for the antibiotic to eliminate the colony.
- Suggest **one** reason why Theo's assumption may not be correct.

Applied questions – mechanics

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The particles P and Q are used to model a trailer being towed by a caravan. In the model, P represents the trailer, which has mass 1000 kg, and Q represents the caravan, which has mass 2400 kg. The particles are modelled to be connected by a light horizontal rod. At time $t = 0$, a constant force \mathbf{F} of magnitude 5400 N is applied to Q in the direction PQ , as shown in the diagram. Starting from rest, the system moves under the influence of \mathbf{F} for 30 seconds. During this motion, the resistances to motion of P and Q are modelled as constant with magnitude 750 N and 500 N respectively.

Find

- (a) the acceleration of the system as it moves under the action of \mathbf{F} ,
- (b) the speed of the system at $t = 30$,
- (c) the tension in the rod as the system as it moves under the action of \mathbf{F} .

At time $t = 30$, \mathbf{F} is removed and the system starts to decelerate. The resistances to motion are unchanged.

- (d) Find the time taken for the system to come to rest from the instant when \mathbf{F} is removed.
- (e) Determine whether the rod is in tension or in thrust as the system decelerates.
- (f) Draw a speed-time graph to represent the motion of the system.
- (g) Give a physical interpretation of the force \mathbf{F} in the model.

Applied questions – statistics

7 Imogen is investigating the relationship between rainfall, r mm, and pressure, p hPa, in the UK.

For her investigation, she takes a simple random sample of 12 data points from Hurn 1987.

- (a) Describe how this sample can be taken.
- (b) Using your knowledge of the large data, explain why her sampled data may have to be cleaned.

Using her sample, Imogen plots a scatter graph for p against r and calculates the equation of the regression line to be

$$p = 1012 - 0.4136r$$

- (c) Interpret the figure -0.4136 in the equation of the regression line.

Imogen claims that

“the regression line suggests the average pressure in Hurn 1987 is close to 1012 hPa.”

- (d) Using your knowledge of the large data set,
 - (i) give one reason to support Imogen’s claim,
 - (ii) give one limitation of Imogen’s claim.