Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

A level Further Mathematics

Further Mechanics 1

Practice Paper 5

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are **7** questions in this question paper. The total mark for this paper is **75**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. A truck of mass 750 kg is moving with constant speed v m s^{-1} down a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{49}$. The resistance to motion of the truck is modelled as a constant force of magnitude 1200 N. The engine of the truck is working at a constant rate of 9 kW.

(a) Find the value of v.

(4)

On another occasion the truck is moving up the same straight road. The resistance to motion of the truck from non-gravitational forces is modelled as a constant force of magnitude 1200N. The engine of the truck is working at a constant rate of 9 kW.

(b) Find the acceleration of the truck at the instant when it is moving with speed 4.5 m s^{-1}.

(4)

(Total 8 marks)

[Mark scheme for Question 1]

[Examiner comment]

2.

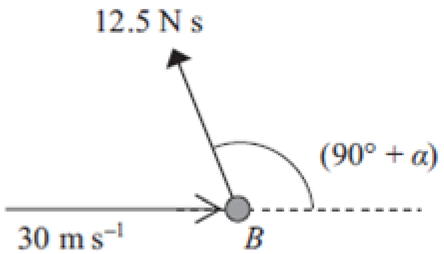

Figure 1

A small ball B of mass 0.25 kg is moving in a straight line with speed 30 m s^{-1} on a smooth horizontal plane when it is given an impulse. The impulse has magnitude 12.5 N s and is applied in a horizontal direction making an angle of $(90° + \alpha)$, where $\tan\alpha = \frac{3}{4}$, with the initial direction of motion of the ball, as shown in Figure 1.

(i) Find the speed of B immediately after the impulse is applied.

(ii) Find the direction of motion of B immediately after the impulse is applied.

(Total 6 marks)

[Mark scheme for Question 2]

[Examiner comment]

3.

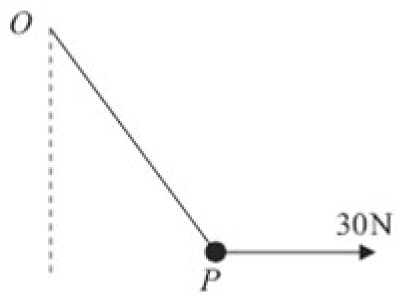

Figure 2

A particle P of weight 40 N is attached to one end of a light elastic string of natural length 0.5 m. The other end of the string is attached to a fixed point O. A horizontal force of magnitude 30 N is applied to P, as shown in Figure 2. The particle P is in equilibrium and the elastic energy stored in the string is 10 J.

Calculate the length OP.

(Total 10 marks)

[Mark scheme for Question 3]

[Examiner comment]

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4. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane*, **i** *being horizontal and* **j** *being vertically upwards*.]

A line of greatest slope of a fixed smooth plane is parallel to the vector $(-4\mathbf{i} - 3\mathbf{j})$.
A particle P falls vertically and strikes the plane. Immediately before the impact, P has velocity $-7\mathbf{j}$ m s^{-1}. Immediately after the impact, P has velocity $(-a\mathbf{i} + \mathbf{j})$ m s^{-1}, where a is a positive constant.

(*a*) Show that $a = 6$

(2)

(*b*) Find the coefficient of restitution between P and the plane.

(6)

(Total 8 marks)

[Mark scheme for Question 4]

[Examiner comment]

5.

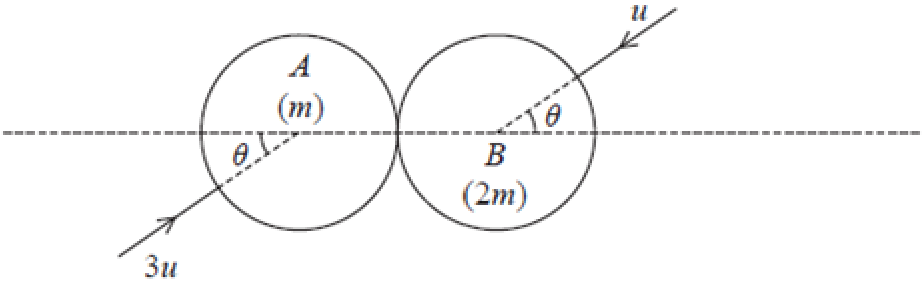

Figure 3

Two smooth uniform spheres A and B with equal radii have masses m and $2m$ respectively. The spheres are moving in opposite directions on a smooth horizontal surface and collide obliquely. Immediately before the collision, A has speed $3u$ with its direction of motion at an angle θ to the line of centres, and B has speed u with its direction of motion at an angle θ to the line of centres, as shown in Figure 3. The coefficient of restitution between the spheres is $\frac{1}{8}$.

Immediately after the collision, the speed of A is twice the speed of B.

Find the size of the angle θ.

(Total 12 marks)

[Mark scheme for Question 5]

[Examiner comment]

6. A light elastic string has natural length a and modulus of elasticity $\frac{3}{2}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A. The particle is released from rest at A and falls vertically. When P has fallen a distance $a + x$, where $x > 0$, the speed of P is v.

(*a*) Show that

$$v^2 = 2g(a + x) - \frac{3gx^2}{2a}.$$

(4)

(*b*) Find the greatest speed attained by P as it falls.

(4)

After release, P next comes to instantaneous rest at a point D.

(*c*) Find the magnitude of the acceleration of P at D.

(6)

(Total 14 marks)

Mark scheme for Question 6

Examiner comment

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7. A particle P of mass $2m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. A particle Q of mass $3m$ is moving with speed u in the same direction as P. The particles collide directly. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(*a*) Show that the speed of Q immediately after the collision is $\frac{8}{5}u$.

(5)

(*b*) Find the total kinetic energy lost in the collision.

(5)

After the collision between P and Q, the particle Q collides directly with a particle R of mass m which is at rest on the plane. The coefficient of restitution between Q and R is e.

(*c*) Calculate the range of values of e for which there will be a second collision between P and Q.

(7)

(Total 17 marks)

Mark scheme for Question 7

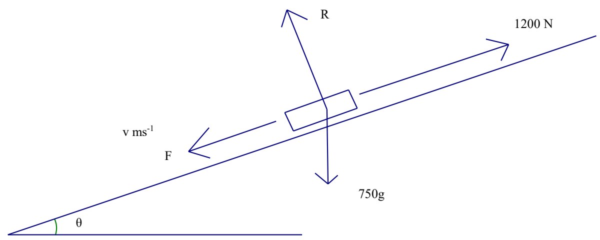
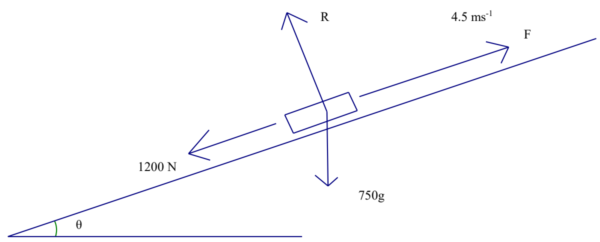
Examiner comment

TOTAL FOR PAPER: 75 MARKS

Further Mathematics – Further Mechanics 1– Practice Paper 05 –

Mark scheme –

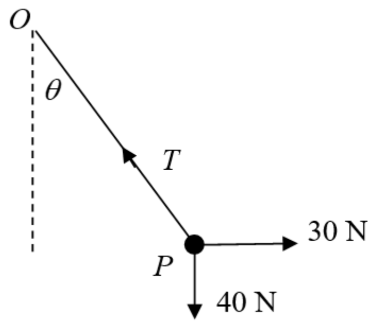
Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

Question	Scheme		Marks
1(a)			
	Motion down the plane:	Dimensionally correct. Condone sign errors and sin/cos confusion.	M1
	$F + 750g\sin\theta = 1200$	$(F + 450 = 1200)$	A1
	Use of $P = Fv$: $F = \frac{9000}{v}$	Award in (b) if not seen in (a)	B1
	$\frac{9000}{v} + 750g \times \frac{3}{49} = 1200$		
	$v = \frac{9000}{750} = 12$		A1
			(4)
(b)			
	$F = ma$: $F - (750g\sin\theta + 1200) = 750a$	Dimensionally correct. Condone sign errors and sin/cos confusion.	M1
	$\frac{9000}{4.5} - \left(750g \times \frac{3}{49} + 1200\right) = 750a$	Unsimplified equation with at most one error	A1
		Correct unsimplified equation	A1
	$a = 0.47$ (0.467) (m s^{-2})	2 or 3 sf only not $\frac{7}{15}$	A1
			(4)
			(8 marks)

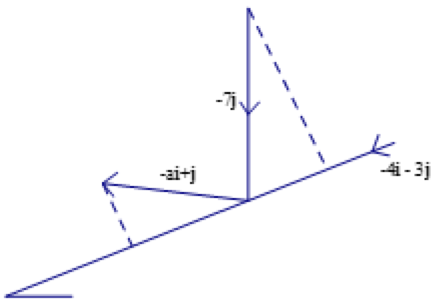
Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2(a)	$12.5\sin\alpha = \frac{1}{4}(v_1 - -30)$ or	
	$-12.5\sin\alpha = \frac{1}{4}(v_1 - 30)$ $(v_1 = 0)$	**M1A1**
	$12.5\cos\alpha = \frac{1}{4}(v_2 - 0)$ $(v_2 = 40)$	**M1A1**
	speed is 40 m s^{-1};	**A1**
	perpendicular to original direction	**A1**
		(6)
	Alternative	
	Using a vector triangle: $(\frac{1}{4}v)^2 = 7.5^2 + 12.5^2 - 2\text{x}7.5\text{x}12.5\cos(90^\circ - \alpha)$	**M1A1**
	$v = 40\text{m s}^{-1}$	**A1**
	$\frac{12.5}{\sin\theta} = \frac{7.5}{\sin\alpha}$	**M1A1**
	$\theta = 90^\circ$	**A1**
		(6)
		(6 marks)

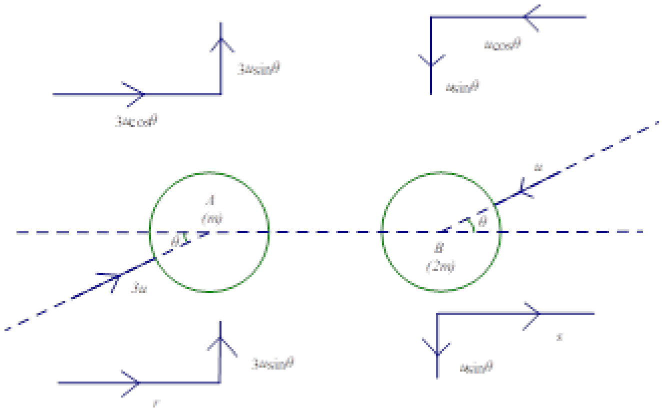
Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

Question	Scheme	Marks
3		
	↑ $T\cos\theta = 40$ M1 attempt at both equations	M1A1
	→ $T\sin\theta = 30$	A1
	leading to $T = 50$	M1A1
	$E = \frac{\lambda x^2}{2a} = 10$	B1
	HL $T = \frac{\lambda x}{a} = 50$	M1
	Leading to $x = 0.4$	M1A1
	$OP = 0.5 + 0.4 = 0.9\,(\text{m})$	A1ft
		(10)
		(10 marks)

Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

Question	Scheme	Marks
4(a)		
	Components parallel to the plane unchanged: $\left(\begin{pmatrix}0\\-7\end{pmatrix}\bullet\frac{1}{5}\begin{pmatrix}-4\\-3\end{pmatrix}=\begin{pmatrix}-a\\1\end{pmatrix}\bullet\frac{1}{5}\begin{pmatrix}-4\\-3\end{pmatrix}\right)$ $\Rightarrow 21=4a-3$	M1
	$a=6$	A1
		(2)
(b)	Component of -7**j** perpendicular to the plane $=\frac{1}{5}\begin{pmatrix}0\\-7\end{pmatrix}\bullet\begin{pmatrix}-3\\4\end{pmatrix}$	M1A1
	Component of $-a\mathbf{i}+\mathbf{j}$ perpendicular to the plane $=\frac{1}{5}\begin{pmatrix}-6\\1\end{pmatrix}\bullet\begin{pmatrix}-3\\4\end{pmatrix}$	M1A1
	Impact law: $e=\dfrac{\frac{1}{5}\times 22}{\frac{1}{5}\times 28}=\dfrac{22}{28}=\dfrac{11}{14}(=0.786)$	DM1 A1
		(6)
		(8 marks)

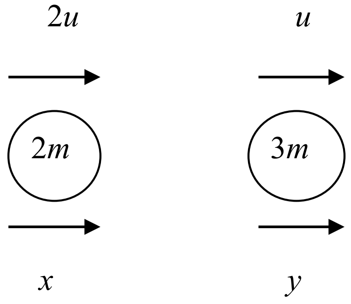
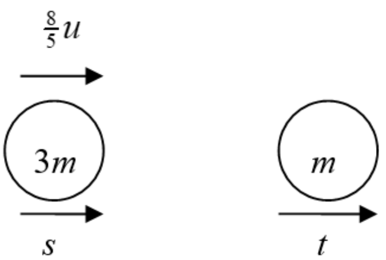
Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme	Marks
5		**B1**
	CLM : $r+2s=3u\cos\theta-2u\cos\theta\left(=u\cos\theta\right)$	**M1A1**
	Impact: $s-r=e\times 4u\cos\theta\left(=\dfrac{u\cos\theta}{2}\right)$	**M1A1**
	$\Rightarrow r=0, s=\dfrac{u\cos\theta}{2}$	**dM1 A1**
	After the collision: $\left(3u\sin\theta\right)^2+r^2=4\left(\left(u\sin\theta\right)^2+s^2\right)$	**M1 A1ft**
	$9u^2\sin^2\theta=4u^2\sin^2\theta+4.\dfrac{u^2}{4}\cos^2\theta$	**A1**
	$\tan^2\theta=\dfrac{1}{5},\quad \theta=24.1(°)$ (0.421 radians)	**dM1 A1**
		(12)
		(12 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 + \frac{3mgx^2}{4a} = mg(a+x)$	M1A2 (1, 0)
	leading to $v^2 = 2g(a+x) - \frac{3gx^2}{2a}$ ∗ cso	A1
		(4)
(b)	Greatest speed is when the acceleration is zero	
	$T = \frac{\lambda x}{a} = \frac{3mgx}{2a} = mg \quad \Rightarrow \quad x = \frac{2a}{3}$	M1A1
	$v^2 = 2g\left(a + \frac{2a}{3}\right) - \frac{3g}{2a} \times \left(\frac{2a}{3}\right)^2 \left(= \frac{8ag}{3}\right)$	M1
	$v = \frac{2}{3}\sqrt{(6ag)}$ accept exact equivalents	A1
		(4)
(c)	$v = 0 \quad \Rightarrow \quad 2g(a+x) - \frac{3gx^2}{2a} = 0$	M1
	$3x^2 - 4ax - 4a^2 = (x-2a)(3x+2a) = 0$	
	$x = 2a$	M1A1
	At D, $m\ddot{x} = mg - \frac{\lambda \times 2a}{a}$ ft their $2a$	M1 A1ft
	$\lvert\ddot{x}\rvert = 2g$	A1
		(6)
		(14 marks)

Mark scheme for Question 7 (Examiner comment) (Return to Question 7)

Question	Scheme	Marks
7(a)		
	LM $4mu + 3mu = 2mx + 3my$	**M1A1**
	NEL $y - x = \frac{1}{2}u$	**B1**
	Solving to $y = \frac{8}{5}u$ * cso	**M1A1**
		(5)
(b)	$x = \frac{11}{10}u$ or equivalent	**B1**
	Energy loss $\frac{1}{2}\times 2m\left((2u)^2 - \left(\frac{11}{10}u\right)^2\right) + \frac{1}{2}\times 3m\left(u^2 - \left(\frac{8}{5}u\right)^2\right)$	**M1 A(2,1,0)**
	$= \frac{9}{20}mu^2$	**A1**
		(5)
(c)		
	LM $\frac{24}{5}mu = 3ms + mt$	**M1A1**
	NEL $t - s = \frac{8}{5}eu$	**B1**
	Solving to $s = \frac{2}{5}u(3-e)$	**M1A1**
	For a further collision $\frac{11}{10}u > \frac{2}{5}u(3-e)$	**M1**
	$e > \frac{1}{4}$ ignore $e \le 1$	**A1**
		(7)
		(17 marks)

Further Mathematics – Further Mechanics 1– Practice Paper 05 –

Examiner report –

Examiner comment for Question 1 **(Mark scheme)** **(Return to Question 1)**

1. (a) This part of the question provided most students with a friendly start to the paper, and many scored full marks. Most students resolved correctly to form an equation of motion involving all three terms, but there were several sign errors. Some students had not noted that the truck was moving down the road, so they had the driving force and the weight acting in opposite directions.

   (b) Many students scored full marks for this part of the question. Here again, the main difficulty was to get the signs correct in the equation of motion. Several students did not score the final mark because the accuracy of their final answer was inappropriate following the use of an approximate value for g - some answers were given as decimals with too many significant figures, and some as exact fractions.

Examiner comment for Question 2 **(Mark scheme)** **(Return to Question 2)**

2. Despite being on a familiar topic, this question was not answered well, possibly because the candidates are used to seeing it presented in vector form. It was common to see candidates trying to use the impulse of 12.5 in an impulse-momentum equation without understanding that it is a vector equation. Many candidates offered the incorrect equation

   $12.5 = 0.25(v - 30)$ leading to $v = 80$.

   The most successful approach was either to adopt vector notation or to deal with the horizontal and vertical components separately. There was some trig confusion in finding the components of the impulse, and there were sign errors in the component parallel to the initial direction. The question does ask for the speed of B after the collision, so answers left as vectors did not earn this mark. Very few candidates adopted the vector triangle approach.

Examiner comment for Question 3 **(Mark scheme)** **(Return to Question 3)**

3. This was probably the best answered question on the paper. Most candidates obtained T (not always shown explicitly) by resolving in two directions. There were a few who drew a triangle of forces. Good candidates were able to “see” the triangle without working and wrote down immediately that $T = 50$. A few unfortunately thought they were dealing with 40g and 30 even though was it clearly stated in the question that the weight was 40 N and not that the mass was 40 kg. The most common mistake was in the arithmetic – $50 = \dfrac{\lambda x}{0.5}$ so $\lambda x = 100$! Very few forgot to complete their solution by adding 0.5 to their extension to find OP.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. Vector methods using scalar products with vectors along or perpendicular to the plane produced elegant and simple solutions to this question but were only attempted by a small minority of candidates.

(a) This application of conservation of momentum parallel to the plane proved elusive for many candidates. A popular equivalent approach was to use the fact that the impulse is perpendicular to the plane. Many tried a trigonometric approach but poor and confusingly labelled diagrams caused problems in identifying the correct angles.

(b) Candidates were more successful in this part; using the given value of a, they were able to find components of velocities perpendicular to the plane and hence find the value of e. Several attempts demonstrated that candidates understood the basic principles, they set up correct equations, but could not solve them.

Examiner comment for Question 5 (Mark scheme) (Return to Question 5)

5. The majority of candidates were confident in applying the impact law and conservation of linear momentum parallel to the line of centres of the spheres. Those who used perpendicular components were generally more successful than those who used a modulus/angle approach. Candidates who introduced two unknown velocities in two unknown directions created additional work for themselves and often had difficulty in finding/eliminating so many unknown values.

A common error was to double the component of the speed parallel to the line of centres, rather than double the overall speed.

Examiner comment for Question 6 (Mark scheme) (Return to Question 6)

6. Most candidates managed to arrive at the required result in part (a), though some unnecessarily split the motion into two parts, considering freefall initially to find the kinetic energy when the string became taut and then proceeding to consider the taut string and others would clearly have failed had not the answer been provided.

Parts (b) and (c) were often difficult to disentangle. Some candidates took an SHM approach from the start of (b); others solved (b) and then resorted to SHM for (c) alone. Either approach was acceptable, but candidates should take note of the instruction on the front of the paper "You should show sufficient working to make your methods clear to the Examiner". The main fault was not when to start considering SHM but not establishing a correct equation to prove that the motion was SHM; no credit is given for making assumptions of this nature. A fully correct solution using SHM was rare, the equations frequently being unsatisfactory due to using x for the distance from the equilibrium point and confusing it with x as defined in the question to be the extension of the string.

For the non-SHM solutions, in part (b) many candidates assumed that the maximum speed occurred when $x = 0$ rather than when $a = 0$. In part (c) most substituted $v = 0$ in the result from part (a). Some did not expect to obtain a quadratic and so stopped working (or ran out of time?). Of those who obtained a solution for their quadratic equation, some would then try incorrectly to use their value for x as the amplitude in SHM instead of using an equation of motion and Hooke's law. Many equations of motion omitted the weight of the particle.

Examiner comment for Question 7 (Mark scheme) (Return to Question 7)

7. This question was generally well understood and answered. Most errors were caused by poor presentation leading to carelessness. Candidates who kept all the velocities in the direction of the original velocities usually fared better than those who reversed one or more velocity. The clearest solutions included clearly annotated diagrams which made the relative directions of motion very clear. In the weaker solutions it was sometimes difficult to work out the candidate's thoughts about what happened in each collision - the question did not give them names for the speeds after the initial collision and this gave rise to problems for some candidates who often gave the same name to more than one variable. Candidates with an incorrect or inconsistent application of Newton's Experimental Law lost a lot of time trying to obtain the given answer for the speed of Q after the first collision. In part (b) although most candidates attempted to form a valid expression for the change in kinetic energy, the m and u^2 were too often discarded along the way.

In part (c), and to a lesser extent in part (a), the tendency to want to solve simultaneous equations by substitution, rather than by elimination, produced untidy and unwieldy expressions which often led to arithmetical errors; a shame when the original equations were correct. Most candidates interpreted the final part correctly, although too many wanted to substitute 11/10u rather than tackle an inequality - it was clear that many candidates were not confident in setting up an inequality. Some problems did occur where students assumed the reversal of the direction of motion of Q following the collision but failed to take account of this in setting up their inequality.