IYGB - SYNOPTIC PAPER H - QUESTION 1

a) GRADIENT BC

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{14-0} = \frac{-2}{14} = -\frac{1}{7}$$

EQUATION OF LINE THROUGH B & C

$$y - y_0 = m(x - x_0)$$

$$y = -\frac{1}{7}x + 3 \qquad \text{(USING B)}$$

$$7y = -x + 21$$

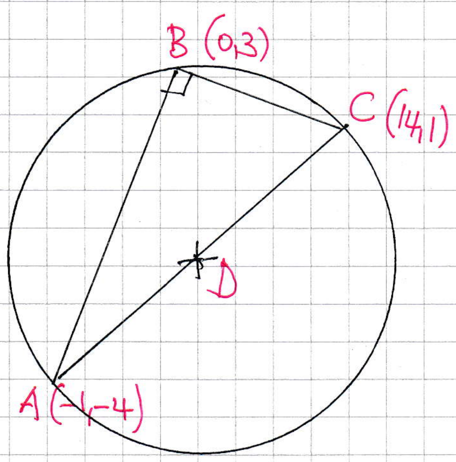
$$x + 7y = 21$$

b) GRADIENT OF AB

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-4)}{0-(-1)} = \frac{7}{1} = 7$$

AS 7 & $-\frac{1}{7}$ ARE NEGATIVE RECIPROCALS $AB \perp BC$

c) USING CIRCLE THEOREMS



CENTRE IS THE MIDPOINT OF AC

CENTRE AT $\left(\frac{14-1}{2}, \frac{1-4}{2}\right)$

I.E. $D\left(\frac{13}{2}, -\frac{3}{2}\right)$

d) FINALLY THE RADIUS - FIND $|AD|$ OR $|CD|$ OR $\frac{1}{2}|AB|$

$$\text{RADIUS} = \frac{1}{2}\sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$$

$$= \frac{1}{2}\sqrt{(-4-1)^2 + (-1-14)^2}$$

$$= \frac{1}{2}\sqrt{250} = \frac{1}{2} \times \sqrt{25}\sqrt{10} = \frac{5}{2}\sqrt{10} \qquad \text{IE } k = \frac{5}{2}$$

IYGB - SYNOPTIC PAPER H - QUESTION 2

LOOKING AT THE DIAGRAM

AREA OF PATH + LAWN $= (x+4)^2$

AREA OF LAWN $= x^2$

AREA OF PATH $= (x+4)^2 - x^2$

FORMING AN INEQUALITY

"AREA OF LAWN" < AREA OF THE PATH

$$x^2 < (x+4)^2 - x^2$$

$$x^2 < \cancel{x^2} + 8x + 16 - \cancel{x^2}$$

$$x^2 - 8x - 16 < 0$$

SOLVING THE INEQUALITY BY COMPLETING THE SQUARE

$$\Rightarrow x^2 - 8x - 16 < 0$$

$$\Rightarrow (x-4)^2 - 16 - 16 < 0$$

$$\Rightarrow (x-4)^2 - 32 < 0$$

$$\Rightarrow (x-4)^2 < 32$$

$$\Rightarrow -\sqrt{32} < x - 4 < \sqrt{32}$$

$$\Rightarrow 4 - \sqrt{32} < x < 4 + \sqrt{32}$$

$$\Rightarrow 4 - 4\sqrt{2} < x < 4 + 4\sqrt{2}$$

BUT x CANNOT BE NEGATIVE

$$\therefore \quad 0 < x < 4 + 4\sqrt{2}$$

IYGB - SYNOPTIC PAPER H - QUESTION 3

a) COMPLETING THE SQUARE

$$f(x) = 2x^2 - 4x + 5$$
$$f(x) = 2\left[x^2 - 2x + \frac{5}{2}\right]$$
$$f(x) = 2\left[(x-1)^2 - 1^2 + \frac{5}{2}\right]$$
$$f(x) = 2\left[(x-1)^2 + \frac{3}{2}\right]$$
$$f(x) = 2(x-1)^2 + 3$$

b) $\left[\frac{6}{f(x)}\right]_{MAX}$ OCCURS WHEN $f(x)$ IS MIN

$$\left[f(x)\right]_{MIN} = 3$$

$\therefore$ MAX VALUE OF $\frac{6}{f(x)} = \frac{6}{3} = 2$

c) USING PART (a)

$$\Rightarrow f(x) = 13$$
$$\Rightarrow 2(x-1)^2 + 3 = 13$$
$$\Rightarrow 2(x-1)^2 = 10$$
$$\Rightarrow (x-1)^2 = 5$$
$$\Rightarrow x - 1 = \begin{cases} \sqrt{5} \\ -\sqrt{5} \end{cases}$$
$$\Rightarrow x = \begin{cases} 1+\sqrt{5} \\ 1-\sqrt{5} \end{cases}$$

IYGB – SYNOPTIC PAPER H – QUESTION 4

DIFFERENTIATING IMPLICITLY W.R.T x

$$\Rightarrow \frac{d}{dx}\left(2\cos 3x \sin y\right) = \frac{d}{dx}(1)$$

$$\Rightarrow -6\sin 3x \sin y + 2\cos 3x \cos y \frac{dy}{dx} = 0$$

AT $\left(\frac{\pi}{12}, \frac{\pi}{4}\right)$ WE OBTAIN

$$\Rightarrow -6\sin\frac{\pi}{4}\sin\frac{\pi}{4} + 2\cos\frac{\pi}{4}\cos\frac{\pi}{4}\left.\frac{dy}{dx}\right|_P = 0$$

$$\Rightarrow -6\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)\left.\frac{dy}{dx}\right|_P = 0$$

$$\Rightarrow -3 + \left.\frac{dy}{dx}\right|_P = 0$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_P = 3$$

FINALLY WE HAVE

$$y - y_0 = m(x - x_0)$$

$$y - \frac{\pi}{4} = 3\left(x - \frac{\pi}{12}\right)$$

$$y - \frac{\pi}{4} = 3x - \frac{\pi}{4}$$

$$y = 3x$$

AS REQUIRED

IYGB – SYNOPTIC PAPER H – QUESTION 5

START BE OBTAINING THE nTH TERM OF THE SEQUENCE (BY INSPECTION)

$$3, 8, 15, 24, 35, 48 \ldots$$
$$(4, 9, 16, 25, 36, 49 \ldots)$$

$$\therefore \quad u_n = (n+1)^2 - 1 = n^2 + 2n$$

NOW FIND THE PRODUCT BETWEEN CONSECUTIVE TERMS

$$u_{n+1} \times u_n = \left[(n+2)^2 - 1\right] \times \left[(n+1)^2 - 1\right]$$
$$= (n^2 + 4n + 4 - 1)(n^2 + 2n + 1 - 1)$$
$$= (n^2 + 4n + 3)(n^2 + 2n)$$
$$= (n+3)(n+1) \times n(n+2)$$
$$= n(n+1)(n+2)(n+3)$$

AS REQUIRED

IYGB - SYNOPTIC PAPER H - QUESTION 6

DIFFERENTIATE USING THE QUOTIENT RULE

$$y = \frac{2}{2-\sin x} \implies \frac{dy}{dx} = \frac{0 \times (2-\sin x) - 2(-\cos x)}{(2-\sin x)^2}$$

$$\implies \frac{dy}{dx} = \frac{2\cos x}{(2-\sin x)^2}$$

$$\implies \frac{dy}{dx} = \frac{\cos x}{2} \times \frac{4}{(2-\sin x)^2}$$

$$\implies \frac{dy}{dx} = \frac{1}{2}\cos x \times \left(\frac{2}{2-\sin x}\right)^2$$

$$\implies \frac{dy}{dx} = \frac{1}{2}\cos x \times y^2$$

$$\implies \frac{dy}{dx} = \frac{1}{2}y^2\cos x$$

IYGB SYNOPTIC PAPER H - QUESTION 7

a) STARTING FROM THE LEFT HAND SIDE

$$\text{LHS} = (\cos x + \sec x)^2 = \cos^2 x + 2\cos x \sec x + \sec^2 x$$
$$= \cos^2 x + 2\cos x\left(\frac{1}{\cos x}\right) + (1 + \tan^2 x)$$
$$= \cos^2 x + 2 + 1 + \tan^2 x$$
$$= \cos^2 x + \tan^2 x + 3$$
$$= \text{R.H.S.}$$

b) USING THE ABOVE RESULT

$$\Rightarrow \cos^2 x + \tan^2 x = \frac{13}{4}$$
$$\Rightarrow \cos^2 x + \tan^2 x + 3 = \frac{13}{4} + 3$$
$$\Rightarrow (\cos x + \sec x)^2 = \frac{25}{4}$$
$$\Rightarrow \cos x + \sec x = \pm \frac{5}{2}$$
$$\Rightarrow \cos x + \frac{1}{\cos x} = \pm \frac{5}{2}$$
$$\Rightarrow \cos^2 x + 1 = \pm \frac{5}{2}\cos x$$
$$\Rightarrow 2\cos^2 x + 1 = \pm 5\cos x$$
$$\Rightarrow 2\cos^2 x \pm 5\cos x + 1 = 0$$

FACTORIZING THE QUADRATIC

$$\Rightarrow (2\cos x + 1)(\cos x + 2) = 0$$

OR

$$(2\cos x - 1)(\cos x - 2) = 0$$

$$\Rightarrow \cos x = \begin{cases} \frac{1}{2} \\ -\frac{1}{2} \end{cases} \quad \text{OR} \quad \cancel{\cos x = \begin{cases} 2 \\ -2 \end{cases}}$$

IYGB - SYNOPTIC PAPER H - QUESTION 7

$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$$\begin{cases} x = \frac{\pi}{3} \pm 2n\pi \\ x = \frac{5\pi}{3} \pm 2n\pi \end{cases}$$

$\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$$\begin{cases} x = \frac{2\pi}{3} \pm 2n\pi \\ x = \frac{4\pi}{3} \pm 2n\pi \end{cases}$$

$n = 0, 1, 2, 3, 4, \ldots$

HENCE IN THE REQUIRED RANGE

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ //

IYGB - SYNOPTIC PAPER H - QUESTION 8

OBTAIN PARTIAL CALCULATION

IF $x = \frac{5}{12}\sqrt{6} \implies \sqrt{x^2 \pm 1} = \sqrt{\left(\frac{5}{12}\sqrt{6}\right)^2 \pm 1}$

$$= \sqrt{\frac{25}{\cancel{144}_{24}} \times \cancel{6} \pm 1}$$

$$= \sqrt{\frac{25}{24} \pm 1}$$

$$= \sqrt{\frac{25 \pm 24}{24}}$$

$$= \begin{cases} \sqrt{\frac{49}{24}} = \frac{7}{\sqrt{24}} = \frac{7}{2\sqrt{6}} \\ \sqrt{\frac{1}{24}} = \frac{1}{\sqrt{24}} = \frac{1}{2\sqrt{6}} \end{cases}$$

$$= \begin{cases} \frac{7\sqrt{6}}{2 \times 6} = \frac{7}{12}\sqrt{6} \\ \frac{1\sqrt{6}}{2 \times 6} = \frac{1}{12}\sqrt{6} \end{cases}$$

THUS WE NOW HAVE

$$f\left(\frac{5}{12}\sqrt{6}\right) = \frac{\frac{5}{12}\sqrt{6} + \frac{7}{12}\sqrt{6}}{\frac{5}{12}\sqrt{6} + \frac{1}{12}\sqrt{6}} = \frac{\sqrt{6}}{\frac{1}{2}\sqrt{6}} = \frac{1}{\frac{1}{2}} = 2$$

IYGB – SYNOPTIC PAPER H – QUESTION 9

a) DESCRIBING THE TRANSFORMATIONS

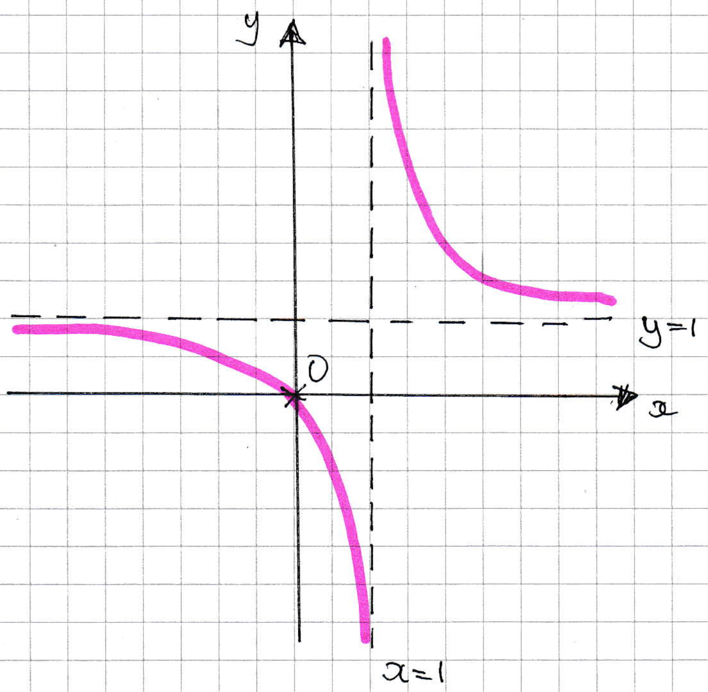
$$\frac{1}{x} \longmapsto \frac{1}{x-1} \longmapsto \frac{1}{x-1}+1$$

TRANSLATION BY ONE UNIT TO THE "RIGHT"

TRANSLATION BY ONE UNIT, "UPWARDS"

i.e TRANSLATION BY THE VECTOR $\begin{pmatrix}1\\1\end{pmatrix}$

b)



WHEN $x=0$

$$y = \frac{1}{0-1}+1 = 0$$

i.e IT PASSES THROUGH THE ORIGIN

c) SOLVING THE REQUIRED EQUATION

$$\Rightarrow \frac{1}{x-1}+1 = x-1$$

$$\Rightarrow \frac{1}{x-1} = x-2$$

$$\Rightarrow 1 = (x-2)(x-1)$$

$$\Rightarrow 1 = x^2-3x+2$$

$$\Rightarrow 0 = x^2-3x+1$$

$$\Rightarrow x = \frac{3 \pm \sqrt{(-3)^2-4\times1\times1}}{2\times1}$$

$$\Rightarrow x = \frac{3\pm\sqrt{5}}{2}$$

i.e. $x = \frac{3}{2} \pm \frac{1}{2}\sqrt{5}$

IYGB - SYNOPTIC PAPER H - QUESTION 10

$$V = 100 + Ae^{-kt}$$

$t=1 \quad V=650$

$$650 = 100 + Ae^{-k}$$
$$550 = Ae^{-k}$$

$t=5 \quad V=350$

$$350 = 100 + Ae^{-5k}$$
$$250 = Ae^{-5k}$$

DIVIDING THE EQUATIONS SIDE BY SIDE

$$\frac{550}{250} = \frac{\cancel{A}e^{-k}}{\cancel{A}e^{-5k}} \Longrightarrow \frac{11}{5} = e^{4k}$$
$$\Longrightarrow 4k = \ln(2.2)$$
$$\Longrightarrow k = \frac{1}{4}\ln(2.2) \approx 0.1971$$

FINDING A AS FOLLOWS

$$e^{4k} = 2.2$$
$$(e^{k})^4 = 2.2$$
$$e^{k} = \sqrt[4]{2.2}$$

$$A = \frac{550}{e^{-k}}$$
$$A = 550e^{k}$$
$$A = 550 \times \sqrt[4]{2.2} \approx 669.84$$

FINALLY WE HAVE

$$V = 100 + 669.84e^{-0.1971t}$$

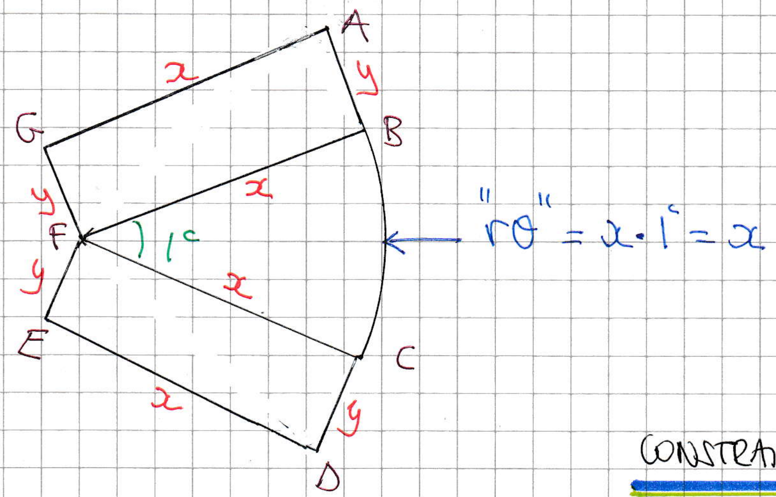
WHEN $t=0$ (NEW)

$$V = 100 + 669.84 \times e^{0}$$
$$V \simeq 770$$

i.e £770

IYGB - SYNOPTIC PAPER H - QUESTION 11

a) LOOKING AT THIS DIAGRAM



CONSTRAINT ON PERIMETER

$\Rightarrow$ PERIMETER $= 40$

$\Rightarrow 2x + 4y + x = 40$

$\Rightarrow 3x + 4y = 40$

$\vdots$

$\Rightarrow 4y = 40 - 3x$

$\Rightarrow \boxed{4xy = 40x - 3x^2}$

AREA OF LOGO

$\Rightarrow A = 2xy + \frac{1}{2}x^2$

$\left(\frac{1}{2}r^2\theta^c\right)$

$\Rightarrow 2A = 4xy + x^2$

$\Rightarrow 2A = (40x - 3x^2) + x^2$

$\Rightarrow 2A = 40x - 2x^2$

$\Rightarrow A = 20x - x^2$

AS REQUIRED

b) DIFFERENTIATE & SOLVE FOR ZERO

$\Rightarrow \frac{dA}{dx} = 20 - 2x$

$\Rightarrow 0 = 20 - 2x$

$\Rightarrow x = 10$

IYGB - SYNOPTIC PAPER H - QUESTION 11

c) USING THE 2^{ND} DERIVATIVE

$$\Rightarrow \frac{dA}{dx} = 20 - 2x$$

$$\Rightarrow \frac{d^2A}{dx^2} = -2$$

$$\Rightarrow \left.\frac{d^2A}{dx^2}\right|_{x=10} = -2 < 0$$

INDEED A MAXIMUM

d) $A = 20x - x^2$

$$\Rightarrow A_{MAX} = 20(10) - 10^2$$

$$\Rightarrow A_{MAX} = 100$$

IYGB - SYNOPTIC PAPER H - QUESTION 12

PROCEED AS FOLLOWS

$$\Rightarrow f(x) = \sqrt{\frac{4-x}{4+x}} = \frac{\sqrt{4-x}}{\sqrt{4+x}} = (4-x)^{\frac{1}{2}}(4+x)^{-\frac{1}{2}}$$

$$= 4^{\frac{1}{2}}\left(1-\tfrac{1}{4}x\right)^{\frac{1}{2}} \times 4^{-\frac{1}{2}}\left(1+\tfrac{1}{4}x\right)^{-\frac{1}{2}}$$

$$= \left(1-\tfrac{1}{4}x\right)^{\frac{1}{2}}\left(1+\tfrac{1}{4}x\right)^{-\frac{1}{2}}$$

EXPAND BINOMIALLY WE OBTAIN

$$\Rightarrow f(x) = \left[1 + \frac{\frac{1}{2}}{1}\left(-\tfrac{1}{4}x\right)^1 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1\times2}\left(-\tfrac{1}{4}x\right)^2 + O(x^3)\right]\left[1 + \frac{-\frac{1}{2}}{1}\left(\tfrac{1}{4}x\right)^1 + \frac{-\frac{1}{2}\left(-\frac{3}{2}\right)}{1\times2}\left(\tfrac{1}{4}x\right)^2 + O(x^3)\right]$$

$$\Rightarrow f(x) = \left[1 - \tfrac{1}{8}x - \tfrac{1}{128}x^2 + O(x^3)\right]\left[1 - \tfrac{1}{8}x + \tfrac{3}{128}x^2 + O(x^3)\right]$$

MULTIPLYING OUT YIELDS

$$\begin{aligned} f(x) = \; & 1 - \tfrac{1}{8}x + \tfrac{3}{128}x^2 + O(x^3) \\ & \quad - \tfrac{1}{8}x + \tfrac{1}{64}x^2 + O(x^3) \\ & \qquad\qquad - \tfrac{1}{128}x^2 + O(x^3) \end{aligned}$$

$$f(x) = 1 - \tfrac{1}{4}x + \tfrac{1}{32}x^2 + O(x^3)$$

ALTERNATIVE TO PART (a)

$$f(x) = \sqrt{\frac{4-x}{4+x}} = \frac{\sqrt{4-x}}{\sqrt{4+x}} = \frac{\sqrt{4-x}\sqrt{4-x}}{\sqrt{4+x}\sqrt{4-x}} = \frac{4-x}{\sqrt{16-x^2}}$$

$$= (4-x)(16-x^2)^{-\frac{1}{2}} = (4-x) \times 16^{-\frac{1}{2}}\left(1-\tfrac{1}{16}x^2\right)^{-\frac{1}{2}}$$

IYGB - SYNOPTIC PAPER H - QUESTION 12

$$= \frac{1}{4}(4-x)\left(1-\frac{1}{16}x^2\right)^{-\frac{1}{2}}$$

EXPANDING UP TO x^2

$$= \frac{1}{4}(4-x)\left[1 + \frac{-\frac{1}{2}}{1}\left(-\frac{1}{16}x^2\right) + O(x^4)\right]$$

$$= \left(1-\frac{1}{4}x\right)\left[1 + \frac{1}{32}x^2 + O(x^4)\right]$$

$$= 1 + \frac{1}{32}x^2 - \frac{1}{4}x + O(x^3)$$

$$= 1 - \frac{1}{4}x + \frac{1}{32}x^2 + O(x^3)$$ AS ABOVE

b) USING THE EXPANSION OF PART (a)

$$\Rightarrow \sqrt{\frac{4-x}{4+x}} \simeq 1 - \frac{1}{4}x + \frac{1}{32}x^2$$

$$\Rightarrow \sqrt{\frac{4-0.5}{4+0.5}} \simeq 1 - \frac{1}{4}\left(\frac{1}{2}\right) + \frac{1}{32}\left(\frac{1}{2}\right)^2$$

$$\Rightarrow \sqrt{\frac{7/2}{9/2}} \simeq 1 - \frac{1}{8} + \frac{1}{128}$$

$$\Rightarrow \sqrt{\frac{7}{9}} \simeq \frac{113}{128}$$

$$\Rightarrow \frac{\sqrt{7}}{3} \simeq \frac{113}{128}$$

$$\Rightarrow \sqrt{7} \simeq \frac{339}{128}$$ AS REQUIRED

IYGB – SYNOPTIC PAPER H – QUESTION 13

a) DIFFERENTIATE x & y WITH RESPECT TO t.

- $\frac{dx}{dt} = -3$
- $\frac{dy}{dt} = \frac{(t+2)\times 1 - (t+6)\times 1}{(t+2)^2} = \frac{t+2-t-6}{(t+2)^2}$

$\frac{dy}{dt} = \frac{-4}{(t+2)^2}$

- $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{-4}{(t+2)^2}}{-3} = \frac{\frac{4}{(t+2)^2}}{\frac{3}{1}} = \frac{4}{3(t+2)^2}$

b) SOLVING SIMULTANEOUSLY THE PARAMETRIC EQUATIONS OF THE CURVE AND THE EQUATION OF L

$$\left.\begin{array}{l} x = 1-3t \\ y = \frac{t+6}{t+2} \\ 4x-3y=1 \end{array}\right\} \Rightarrow 4(1-3t) - 3\left(\frac{t+6}{t+2}\right) = 1$$

$$4 - 12t - \frac{3t+18}{t+2} = 1$$

$$(4-12t)(t+2) - (3t+18) = t+2$$

$$4t + 8 - 12t^2 - 24t - 3t - 18 = t+2$$

$$0 = 12t^2 + 24t + 12$$

$$t^2 - 2t + 1 = 0$$

$$(t+1)^2 = 0$$

REPEATED ROOT ; INDEED L IS A TANGENT

& USING $t=-1$ $\quad x = 4$, $y = 5$

$\therefore$ TANGENCY POINT $(4,5)$

IYGB – SYNOPTIC PAPER H – QUESTION 14

TAKING "TANGENTS" ON BOTH SIDES USING $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\Longrightarrow \arctan\left(\frac{1}{x}\right) + \arctan\left(\frac{1}{x+1}\right) = \frac{\pi}{4}$$

$$\Longrightarrow \tan\left[\arctan\left(\frac{1}{x}\right) + \arctan\left(\frac{1}{x+1}\right)\right] = \tan\frac{\pi}{4}$$

$$\Longrightarrow \frac{\frac{1}{x} + \frac{1}{x+1}}{1 - \frac{1}{x}\left(\frac{1}{x+1}\right)} = 1$$

MULTIPLYING ACROSS

$$\Longrightarrow \frac{1}{x} + \frac{1}{x+1} = 1 - \frac{1}{x(x+1)} \qquad \times\, x(x+1)$$

$$\Longrightarrow (x+1) + x = x(x+1) - 1$$

$$\Longrightarrow x + 1 + x = x^2 + x - 1$$

$$\Longrightarrow 0 = x^2 - x - 2$$

$$\Longrightarrow (x-2)(x+1) = 0$$

$$\Longrightarrow x = \begin{cases} 2 \\ -1 \end{cases}$$

BOTH ARE FINE

- $x = -1$

  $\arctan(-1) + \arctan(\infty) = -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$

- $x = 2$

  $\arctan\frac{1}{2} + \arctan\frac{1}{3} = \frac{\pi}{4}$

IYGB - SYNOPTIC PAPER H - QUESTION 15

$$\boxed{y = x\left(1 - x^{\frac{2}{3}}\right) = x - x^{\frac{5}{3}}}$$

a) OBTAIN THE GRADIENT FUNCTION

$$\frac{dy}{dx} = 1 - \frac{5}{3}x^{\frac{2}{3}}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 1$$

EQUATION OF TANGENT AT $(0,0)$

$$y = x$$

b) OBTAIN THE CO-ORDINATES OF A

$$0 = x\left(1 - x^{\frac{2}{3}}\right)$$

$$1 - x^{\frac{2}{3}} = 0 \qquad (x \neq 0)$$

$$1 = x^{\frac{2}{3}}$$

$$x = 1$$

ie $A(1,0)$

EQUATION OF TANGENT AT $A(1,0)$

$$\left.\frac{dy}{dx}\right|_{x=1} = 1 - \frac{5}{3} \times 1^{\frac{2}{3}} = -\frac{2}{3}$$

$$y - y_0 = m(x - x_0)$$

$$y - 0 = -\frac{2}{3}(x - 1)$$

$$3y = -2x + 2$$

$$3y + 2x = 2$$

AS REQUIRED

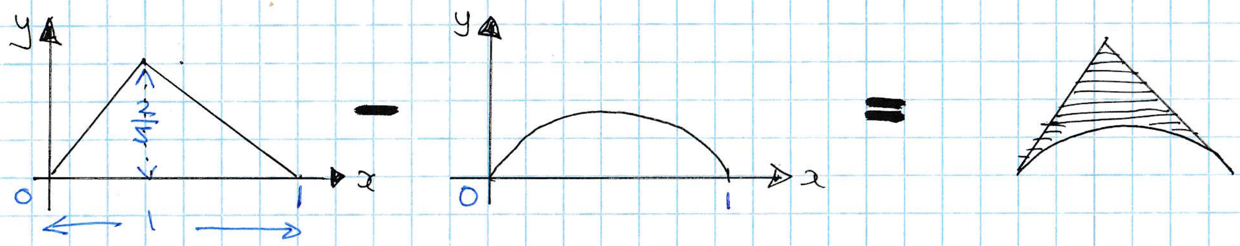
c) START BY FINDING THE CO-ORDS OF B

$$\left.\begin{array}{l} 3y + 2x = 2 \\ y = x \end{array}\right\} \implies 3x + 2x = 2 \implies 5x = 2 \implies x = \frac{2}{5}$$

$$\therefore B\left(\frac{2}{5}, \frac{2}{5}\right)$$

IYGB - SYNOPTIC PAPER H - QUESTION 15

NEXT LOOKING AT THE DIAGRAM BELOW



$\text{AREA} = \frac{1}{2} \times 1 \times \frac{2}{5}$

$\text{AREA} = \frac{1}{5}$

$$\int_0^1 x - x^{\frac{5}{3}}\, dx$$

$$= \left[\frac{1}{2}x^2 - \frac{3}{8}x^{\frac{8}{3}}\right]_0^1$$

$$= \left(\frac{1}{2} - \frac{3}{8}\right) - (0 - 0)$$

$$= \frac{1}{8}$$

REQUIRED AREA $= \frac{1}{5} - \frac{1}{8} = \frac{3}{40}$

IYGB - SYNOPTIC PAPER H - QUESTION 16

METHOD A - USING THE DISCRIMINANT

$$\left.\begin{array}{l} y = 5x + \frac{4}{x} - 3 \\ y = 4x + 1 \end{array}\right\} \Longrightarrow 5x + \frac{4}{x} - 3 = 4x + 1$$

$$\Longrightarrow x + \frac{4}{x} - 4 = 0$$

$$\Longrightarrow x^2 + 4 - 4x = 0$$

$$\Longrightarrow x^2 - 4x + 4 = 0$$

$$\Longrightarrow (x-2)^2 = 0$$

REPEATED ROOT $\therefore$ A TANGENT AT $x=2$

$$\Longrightarrow y = 4 \times 2 + 1$$

$$\therefore (2,9)$$

METHOD B - USING DIFFERENTIATION

- THE LINE HAS GRADIENT 4 $(y = 4x+1)$
- DIFFERENTIATE THE EQUATION OF THE CURVE

$$y = 5x + 4x^{-1} - 3$$

$$\frac{dy}{dx} = 5 - 4x^{-2}$$

- SET THE GRADIENT FUNCTION EQUAL TO 4

$$4 = 5 - 4x^{-2}$$

$$4x^{-2} = 1$$

$$\frac{4}{x^2} = 1$$

$$x^2 = 4$$

IYGB - SYNOPTIC PAPER H - QUESTION 16

$$x = \begin{cases} 2 \\ -2 \end{cases} \qquad y = \begin{cases} 5\times2 + \frac{4}{2} - 3 = 9 \\ 5(-2) + \frac{4}{-2} - 3 = -15 \end{cases}$$

- CHECK EACH POINT

$$y - y_0 = m(x - x_0)$$
$$y - 9 = 4(x - 2)$$
$$y - 9 = 4x - 8$$
$$y = 4x + 1$$

INDEED A TANGENT AT $(2,9)$

$$y - y_0 = m(x - x_0)$$
$$y + 15 = 4(x + 2)$$
$$y + 15 = 4x + 8$$
$$y = 4x - 7$$

A COMPLETELY DIFFERENT TANGENT AT $(-2,-15)$

IYGB - SYNOPTIC PAPER H - QUESTION 17

SEPARATING VARIABLES

$$\Rightarrow \frac{dy}{dx}\cot x = 1-y^2$$

$$\Rightarrow dy\cot x = (1-y^2)\,dx$$

$$\Rightarrow \frac{1}{1-y^2}\,dy = \frac{1}{\cot x}\,dx$$

$$\Rightarrow \int \frac{1}{(1-y)(1+y)}\,dy = \int \tan x\,dx$$

PROCEED WITH PARTIAL FRACTIONS

$$\frac{1}{(1-y)(1+y)} \equiv \frac{A}{1-y} + \frac{B}{1+y}$$

$$1 \equiv A(1+y) + B(1-y)$$

- IF $y=1$: $1 = 2A$, $A = \frac{1}{2}$
- IF $y=-1$: $1 = 2B$, $B = \frac{1}{2}$

RETURNING TO THE O.D.E.

$$\Rightarrow \int \frac{\frac{1}{2}}{1+y} + \frac{\frac{1}{2}}{1-y}\,dy = \int \tan x\,dx$$

$$\Rightarrow \int \frac{1}{1+y} + \frac{1}{1-y}\,dy = \int 2\tan x\,dx$$

$$\int \tan x\,dx = \ln|\sec x| + C$$

INTEGRATING BOTH SIDES SUBJECT TO THE BOUNDARY CONDITION $\left(\frac{\pi}{4}, 0\right)$

$$\Rightarrow \Big[\ln|1+y| - \ln|1-y|\Big]_{y=0}^{y=y} = \Big[2\ln|\sec x|\Big]_{x=\frac{\pi}{4}}^{x=x}$$

IYGB - SYNOPTIC PAPER H - QUESTION 17

$$\Rightarrow \Big[\ln|1+y| - \ln|1-y|\Big] - \cancel{\Big[\ln 1 - \ln 1\Big]} = 2\ln|\sec x| - 2\ln\left|\sec\tfrac{\pi}{4}\right|$$

$$\Rightarrow \ln\left|\frac{1+y}{1-y}\right| = 2\ln|\sec x| - 2\ln(\sqrt{2})$$

$$\Rightarrow \ln\left|\frac{1+y}{1-y}\right| = \ln(\sec^2 x) - \ln 2$$

$$\Rightarrow \ln\left|\frac{1+y}{1-y}\right| = \ln\left(\frac{\sec^2 x}{2}\right)$$

$$\Rightarrow \frac{1+y}{1-y} = \frac{\sec^2 x}{2}$$

$$\Rightarrow 2 + 2y = \sec^2 x - y\sec^2 x$$

$$\Rightarrow 2y + y\sec^2 x = \sec^2 x - 2$$

$$\Rightarrow y(2+\sec^2 x) = \sec^2 x - 2$$

$$\Rightarrow y = \frac{\sec^2 x - 2}{2 + \sec^2 x}$$

$$\Rightarrow y = \frac{\sec^2 x\cos^2 x - 2\cos^2 x}{2\cos^2 x + \sec^2 x\cos^2 x}$$

$$\Rightarrow y = \frac{1 - 2\cos^2 x}{2\cos^2 x + 1}$$

$$\Rightarrow y = \frac{1 - 2\cos^2 x}{1 + 2\cos^2 x}$$

AS REQUIRED

IYGB - SYNOPTIC PAPER H - QUESTION 18

- USING THE FOLLOWING APPROACH

$$S_{n+2} - 2S_{n+1} + S_n$$

$$= \left[S_n + u_{n+1} + u_{n+2}\right] - 2\left[S_n + u_{n+1}\right] + S_n$$

$$= \cancel{S_n} + u_{n+1} + u_{n+2} - \cancel{2S_n} - 2u_{n+1} + \cancel{S_n}$$

$$= u_{n+2} - u_{n+1}$$

$$= \left[u_{n+1} + d\right] - u_{n+1}$$

$$= d$$

AS REQUIRED

- ALTERNATIVE

$$S_{n+2} - S_{n+1} = u_{n+2} = a + (n+1)d = a + nd + d$$

$$S_{n+1} - S_n = u_{n+1} = a + (n)d = a + nd$$

SUBTRACTING THE ABOVE SIDE BY SIDE

$$\Rightarrow S_{n+2} - 2S_{n+1} + S_n = d$$

AS REQUIRED

- LONGER ALTERNATIVE

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}\left[2a + nd - d\right]$$

$$S_{n+1} = \frac{n+1}{2}\left[2a + nd\right] = \left(\frac{n}{2} + \frac{1}{2}\right)(2a + nd) = \frac{n}{2}(2a + nd) + \frac{1}{2}(2a + nd)$$

$$S_{n+2} = \frac{n+2}{2}\left[2a + (n+1)d\right] = \left(\frac{n}{2} + 1\right)(2a + nd + d) = \frac{n}{2}(2a + nd + d) + (2a + nd + d)$$

TIDY EACH EXPRESSION FURTHER

IYGB – SYNOPTIC PAPER H – QUESTION 10

$$S'_n = \frac{n}{2}(2a+nd-d) = \frac{n}{2}(2a+nd) - \frac{1}{2}nd$$

$$S'_{n+1} = \frac{n}{2}(2a+nd) + \frac{1}{2}(2a+nd) = \frac{n}{2}(2a+nd) + a + \frac{1}{2}nd$$

$$S'_{n+2} = \frac{n}{2}(2a+nd+d) + (2a+nd+d) = \frac{n}{2}(2a+nd) + \frac{1}{2}nd + 2a + nd + d$$

$$= \frac{n}{2}(2a+nd) + 2a + \frac{3}{2}nd + d$$

FINALLY TIDYING UP

$$S'_{n+2} - 2S'_{n+1} + S'_n = \begin{array}{l} \cancel{\frac{n}{2}(2a+nd)} + \cancel{2a} + \cancel{\frac{3}{2}nd} + d \\ -\cancel{n(2a+nd)} - \cancel{2a} - \cancel{nd} \\ \cancel{\frac{n}{2}(2a+nd)} \qquad\quad -\cancel{\frac{1}{2}nd} \\ \hline \end{array}$$

$$d$$

AS REQUIRED

IYGB - SYNOPTIC PAPER H - QUESTION 19

LOOKING AT THE RIGHT ANGLED TRIANGLE ABC

$$\frac{|BC|}{|AC|} = \sin\theta$$

$$\frac{1}{|AC|} = \sin\theta$$

$$|AC| = \frac{1}{\sin\theta}$$

LOOKING AT THE RIGHT ANGLED TRIANGLE ACD

$$\frac{|AC|}{|AD|} = \cos\phi$$

$$|AD| = \frac{|AC|}{\cos\phi}$$

$$|AD| = \frac{1}{\sin\theta\cos\phi}$$

FINALLY LOOKING AT THE RIGHT ANGLED TRIANGLE ADE

$$\frac{|AE|}{|AD|} = \cos(\theta+\phi)$$

$$|AE| = |AD|\cos(\theta+\phi)$$

$$|AE| = \frac{\cos(\theta+\phi)}{\sin\theta\cos\phi}$$

$$|AE| = \frac{\cos\theta\cos\phi - \sin\theta\sin\phi}{\sin\theta\cos\phi}$$

$$|AE| = \frac{\cos\theta\cancel{\cos\phi}}{\sin\theta\cancel{\cos\phi}} - \frac{\cancel{\sin\theta}\sin\phi}{\cancel{\sin\theta}\cos\phi}$$

$$|AE| = \cot\theta - \tan\phi$$

AS REQUIRED

IYGB - SYNOPTIC PAPER H - QUESTION 20

a) USING THE SUBSTITUTION GIVEN $u = (1 + x^{n-2})^{\frac{1}{2}}$

$$\Rightarrow u^2 = 1 + x^{n-2} \iff x^{n-2} = u^2 - 1$$

$$\Rightarrow 2u\frac{du}{dx} = (n-2)x^{n-3}$$

$$\Rightarrow 2u\,du = (n-2)x^{n-3}\,dx$$

$$\Rightarrow dx = \frac{2u}{(n-2)x^{n-3}}\,du$$

TRANSFORMING THE INTEGRAL

$$\int \frac{1}{\sqrt{x^2 - x^n}}\,dx = \int \frac{1}{|x|\sqrt{1 - x^{n-2}}}\,dx \qquad (x \geqslant 0)$$

$$= \int \frac{1}{x \times \cancel{u}}\,\frac{2\cancel{u}}{(n-2)x^{n-1}}\,du = \int \frac{2}{(n-2)\,x^{n-2}}\,du$$

$$= \int \frac{2}{(n-2)(u^2-1)}\,du = \frac{1}{n-2}\int \frac{2}{(u-1)(u+1)}\,du$$

// AS REQUIRED

b) PROCEED BY PARTIAL FRACTIONS

$$\frac{2}{(u-1)(u+1)} \equiv \frac{A}{u-1} + \frac{B}{u+1}$$

$$2 \equiv A(u+1) + B(u-1)$$

- IF $u=1$

  $2 = 2A$

  $\underline{A = 2}$

- IF $u=-1$

  $2 = -2B$

  $\underline{B = -1}$

IYGB - SYNOPTIC PAPER H - QUESTION 20

$$\dots = \frac{1}{n-2}\int \frac{1}{u-1} - \frac{1}{u+1}\,du$$

$$= \frac{1}{n-2}\Big[\ln|u-1| - \ln|u+1|\Big] + C$$

$$= \frac{1}{n-2}\ln\left|\frac{u-1}{u+1}\right| + C$$

$$= \frac{1}{n-2}\ln\left|\frac{\sqrt{1+x^{n-2}}-1}{\sqrt{1+x^{n-2}}+1}\right| + C$$

IYGB - SYNOPTIC PAPER H - QUESTION 21

$$f(x) = \begin{cases} -x^2+8x-5 & , x \in \mathbb{R}, x \leq 2 \\ x^2-2x+8 & , x \in \mathbb{R}, x > 2 \end{cases}$$

a) I) AS POLYNOMIALS ARE CONTINUOUS THE ONLY PLACE WHERE DISCONTINUITY MIGHT OCCUR IS AT $x=2$

$f(2) = -2^2+8\times2-5 = -4+16-5 = 7$

$\lim_{x \to 2^-}\left[f(x)\right] = 2^2-2\times2+8 = 4-4+8 = 8$

OR SIMPLY SUBSTITUTE $x=2$ INTO THE "SECOND" SECTION

$\therefore$ NOT CONTINUOUS AS THERE IS A "JUMP" FROM 7 TO 8 AT $x=2$

II) CONSIDERING TWO SEPARATE SECTIONS

- $f_1(x) = -x^2+8x-5, \ x \leq 2$

  $f_1'(x) = -2x+8$

- NOW $x \leq 2$

  $-2x \geq -4$

  $-2x+8 \geq 4$

  $f_1'(x) > 4$

- $f_2(x) = x^2-2x+8, \ x > 2$

  $f_2'(x) = 2x-2$

- NOW $x > 2$

  $2x > 4$

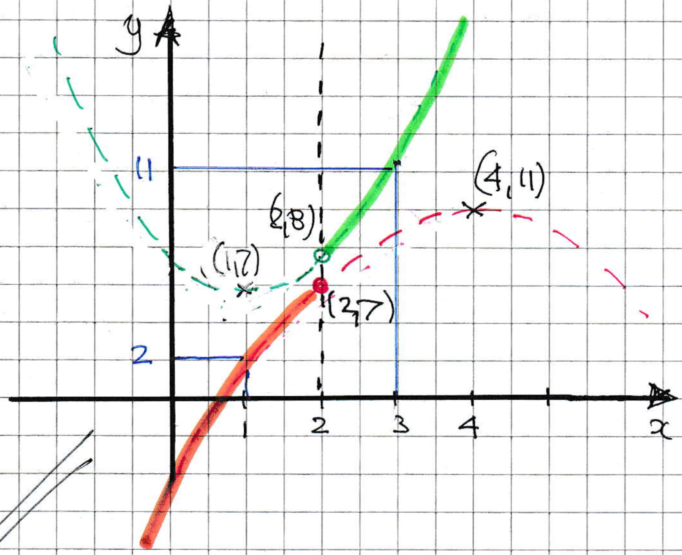
  $2x-2 > 2$

  $f_2'(x) > 2$

$f'(x) > 0$ FOR ALL x, SO f IS INCREASING

IYGB - SYNOPTIC PAPER H - QUESTION 21

b) LOOKING AT THE GRAPH OF f



$f(A)$ CAN TAKE VALUES BETWEEN 2 & 11, EXCLUDING THE GAP

$\therefore\ f(A) \in [2,7] \cup (8,11]$

c) TREATING EACH SECTION SEPARATELY

- $y = -x^2+8x-5,\ x \leq 2$

$\Rightarrow -y = x^2-8x+5$

$\Rightarrow -y = (x-4)^2-11$

$\Rightarrow 11-y = (x-4)^2$

$\Rightarrow x-4 = -\sqrt{11-y}$

$\Rightarrow x = 4-\sqrt{11-y}$

- $y = x^2-2x-5,\ x>2$

$\Rightarrow y=(x-1)^2-6$

$\Rightarrow y+6 = (x-1)^2$

$\Rightarrow x-1 = +\sqrt{y+1}$

$\Rightarrow x = 1+\sqrt{y+1}$

$$\therefore\ f^{-1}(x) = \begin{cases} 4-\sqrt{11-x} & x \leq 7 \\ 1+\sqrt{1+x} & x>8 \end{cases}$$

IYGB - SYNOPTIC PAPER H - QUESTION 22

$\underline{a} = \left(\frac{1}{2}x^2+y^2+3\right)\underline{i} + 4\underline{j}$ $\quad$ $\underline{b} = (x+y)\underline{i} + 2\underline{j}$

AS THE VECTORS ARE PARALLEL

$$\Rightarrow \frac{\frac{1}{2}x^2+y^2+3}{x+y} = \frac{4}{2}$$

$$\Rightarrow x^2+2y^2+6 = 4x+4y$$

$$\Rightarrow x^2-4x+6+2y^2-4y = 0$$

$$\Rightarrow (x-2)^2-4+6+2(y^2-2y) = 0$$

$$\Rightarrow (x-2)^2+2+2\left[(y-1)^2-1\right] = 0$$

$$\Rightarrow (x-2)^2+\cancel{2}+2(y-1)^2-\cancel{2} = 0$$

$$\Rightarrow (x-2)^2+2(y-1)^2 = 0$$

$\therefore$ $x=2$ & $y=1$ //