1. A sample of size 15 was taken from a Normally distributed population with unknown mean and standard deviation 4. If the mean of the sample was 22, test at a 5% significance level the hypothesis that the population mean is 20 **[4 marks]**

2. A Normally distributed population has a variance of 8. A sample of size 25 was taken and had a mean of 38.1. Stating clearly your null and alternative hypotheses, test at the 3% significance level whether the mean is less than 40 **[4]**

3. A sample of size 10 is taken from a Normal population and gives the following values:

   18.2, 19.6, 24.1, 19.3, 21.5, 22.6, 23.3, 20.9, 21.7, 20.3

   Using a significance level of 3%, test whether the population mean is less than 22

   Assume that the standard deviation of the population is 1.8 **[4]**

4. From extensive experience, a manufacturer knows that their halogen light bulbs have a mean lifetime of 1930 hours with a standard deviation of 245 hours. The firm introduces a new type of light bulb. The lifetimes of 20 of the new bulbs are given in the table.

Lifetime (l hours)	Frequency
$1400 \le l < 1800$	3
$1800 \le l < 2000$	5
$2000 \le l < 2100$	6
$2100 \le l < 2300$	4
$2300 \le l < 2700$	2

   a Find the mean lifetime of these bulbs. **[1]**

   b Assuming that the variation in the lifetimes of the bulbs remains unchanged, test at a 5% significance level whether the lifetimes have increased. **[3]**

5. A Normally distributed population has variance 20 and a mean believed to be 12. A sample of size 32 was taken from the population and gave a sample mean of 10.3. By finding the probability of the sample mean taking a value less than 10.3, test the hypothesis that the population mean is 12 against the alternative hypothesis that it is less than 12. You should use a significance level of 5%. **[4]**

6. The average daily maximum temperature in a country can be modelled by a Normal distribution with mean 17.7 °C and variance 11.3 °C². In one particular region, it is thought that the temperature may be higher than the rest of the country. It is assumed that the variance in temperature is the same in the region as in the whole country. A sample of 25 random measurements of the daily maximum temperature is taken for the region and found to have a mean value of 18.6 °C.

   a State the null and alternative hypotheses for this test. **[1]**

   b Calculate the test statistic and the critical value at the 10% significance level. **[2]**

   c State, with a reason, whether the null hypothesis is accepted or rejected and determine the conclusion in context. **[2]**

7. A sample of 8 weather stations is used to see if there is any positive correlation between daily total rainfall and daily maximum relative humidity.

   a State null and alternative hypotheses for this test. **[1]**

   The hypotheses are being considered at the 10% significance level. The PMCC is found to be 0.5101, which has a p-value of 0.0983 for a one-tailed test.

   b State, with a reason, whether H_0 is accepted or rejected and determine the conclusion in context. **[2]**

8. Two scientists have an experiment whose results are modelled by a Normal distribution with variance 17.6 but they think the mean could be different to the intended −13.6. They both

8 Two scientists have an experiment whose results are modelled by a Normal distribution with variance 17.6 but they think the mean could be different to the intended −13.6. They both separately take samples of size 72 to test the hypotheses $H_0 : \mu = -13.6$ and $H_1 : \mu \neq -13.6$ at the 5% level. The critical value is ±1.96

   a One scientist's sample has test statistic −2.04. State, with a reason, whether the null hypothesis is accepted or rejected. [1]

   b The other scientist's sample has test statistic 2.13. State, with a reason, whether the null hypothesis is accepted or rejected. [1]

   c Determine the conclusion the two scientists reach. [2]

9 According to EU legislation, one of the few products that is allowed to be labelled in Imperial measure is milk sold in returnable containers. 30 '1 pint' bottles filled at a dairy farm have the following volumes, given to the nearest ml.

   569 567 568 570 569 568 563 568 572 571
   568 573 570 572 569 571 568 574 567 569
   570 572 564 566 568 572 576 566 571 570

   It is required that the mean of the volume in each bottle must be greater than 1 pint (568 ml). Based on this sample, do you believe that this requirement is being met?
   You should assume that the sample standard deviation gives a good estimate of the population standard deviation and test at a significance level of 5%. [5]

10 The correlation coefficient for two variables, X and Y, is 0.31 based on a sample size of 19. Given that the critical value is ±0.468, test at the 5% significance level whether the population correlation coefficient is zero against the alternative hypothesis that it is not zero. [2]

11 At Heathrow, the daily maximum temperature and the daily total sunshine have a sample correlation coefficient of 0.51. You wish to test the suggestion that the population correlation coefficient between the variables is positive.

   a Write down the null and alternative hypotheses for the test. [1]

   b Given that the critical value is ±0.367, perform the test at a significance level of 2.5% given that the sample size was 12 [3]

12 Given that the sample product moment correlation coefficient between variables X and Y is −0.17 based on a sample of size 46, and the critical value is −0.248, test the hypothesis that the population correlation coefficient, ρ, is less than zero at the 5% significance level. You should state your null and alternative hypotheses. [3]

13 A high-speed fabric weaving machine increases in temperature as it is operated. The number of flaws per square metre is measured at various temperatures and these variables are found to have a correlation coefficient of -0.42 based on a sample of size 30. The manufacturer claims that the number of flaws is independent of the temperature. Given that the critical value is ± 0.367, test at a significance level of 5% the manufacturer's claim. [3]

14 a A Normal random variable X has an unknown mean μ and known standard deviation σ. A sample of size n is taken from the population and gives a sample mean of $\bar{x}$. A test at significance level 2% is to be carried out on whether the population mean has increased from a value μ_0. Find, in terms of μ_0, σ and n, the set of $\bar{x}$ values which would lead to the belief that the mean had increased. [4]

b Bars of steel of diameter 2 cm are known to have a mean breaking point of 80 kN with a standard deviation of 2.1 kN. An increase in the bars' diameter of 0.2 cm is thought to increase the mean breaking point. A sample of 40 bars with the greater diameter have a mean breaking point of 80.9 kN. Test at a significance level of 2% whether the bars with the greater diameter have a greater mean breaking point. State any assumptions used. [3]